Appendix: For Online Publication Only

A. Proofs

Proposition 1. The data generating process is $x_t = \rho x_{t-1} + u_t$, where $u_t \sim \mathcal{N}(0, \sigma_u^2)$ i.i.d. over time and $\rho > 0$. Forecaster i observes a noisy signal $s_t^i = x_t + \epsilon_t^i$, where $\epsilon_t^i \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is i.i.d. analyst specific noise. Rational expectations are obtained iteratively:

$$f(x_t|S_t^i) = f(x_t|S_{t-1}^i)\frac{f(s_t^i|x_t)}{f(s_t^i)}$$

The rational estimate thus follows $f(x_t|S_t^i) \sim \mathcal{N}\left(x_{t|t}^i, \frac{\Sigma_{t|t-1}\sigma_\epsilon^2}{\Sigma_{t|t-1}+\sigma_\epsilon^2}\right)$ with

$$x_{t|t}^i = x_{t|t-1}^i + \frac{\Sigma_{t|t-1}}{\Sigma_{t|t-1}+\sigma_\epsilon^2}\left(s_t^i - x_{t|t-1}^i\right),$$

where $\Sigma_{t|t-1}$ is the variance of the prior $f(x_t|S_{t-1}^i)$. The variance of $f(x_{t+1}|S_t^i)$ is:

$$\Sigma_{t+1|t} \equiv var_t(\rho x_t + u_{t+1}) = \rho^2 \frac{\Sigma_{t|t-1}\sigma_\epsilon^2}{\Sigma_{t|t-1}+\sigma_\epsilon^2} + \sigma_u^2,$$

where the steady state variance $\Sigma = \Sigma_{t+1|t} = \Sigma_{t|t-1}$ is equal to:

$$\Sigma = \frac{-(1-\rho^2)\sigma_\epsilon^2 + \sigma_u^2 + \sqrt{[(1-\rho^2)\sigma_\epsilon^2 - \sigma_u^2]^2 + 4\sigma_\epsilon^2\sigma_u^2}}{2}$$

Beliefs about the current state are then described by $f(x_t|S_t^i) \sim \mathcal{N}\left(x_{t|t}^i, \frac{\Sigma\sigma_\epsilon^2}{\Sigma+\sigma_\epsilon^2}\right)$, where:

$$x_{t|t}^i = x_{t|t-1}^i + \frac{\Sigma}{\Sigma+\sigma_\epsilon^2}\left(s_t^i - x_{t|t-1}^i\right)$$

Note that there is a discontinuity at $\rho = 0$ for contemporaneous beliefs in steady state. When shocks have zero persistence, steady state beliefs are constant and described by $f(x_t|S_t^i) \sim \mathcal{N}(0, \Sigma)$ where the steady state variance is $\Sigma = \sigma_u^2$. In particular, the contemporaneous Kalman gain is zero.[1] This also implies that for $\rho = 0$ there are no diagnosticity distortions, because $f(x_t|S_t^i) = f\left(x_t|S_{t-1}^i \cup \{x_{t|t-1}^i\}\right)$, so that $f^\theta(x_t|S_t^i) = f(x_t|S_t^i)$.

Let us now construct diagnostic expectations for $\rho > 0$. For $s_t^i = x_{t|t-1}^i$ we have $x_{t|t}^i = x_{t|t-1}^i = \rho x_{t-1|t-1}^i$, so that $f\left(x_t|S_{t-1}^i \cup \{x_{t|t-1}^i\}\right) \sim \mathcal{N}\left(\rho x_{t-1|t-1}^i, \frac{\Sigma\sigma_\epsilon^2}{\Sigma+\sigma_\epsilon^2}\right)$. In light of the definition of diagnostic expectations in Equation (7), we have that the diagnostic distribution $f^\theta(x_t|S_t^i)$ fulfils:

[1] Expectations for future realizations satisfy $x_{t+h|t}^i = \rho^h x_{t|t-1}^i + \rho^h \frac{\Sigma}{\Sigma+\sigma_\epsilon^2}\left(s_t^i - x_{t|t-1}^i\right)$. The Kalman gain, and therefore the expectations, are now continuous at $\rho = 0$.

$$\ln f^{\theta}(x_t|S_t^i) \propto -\frac{\left(x_t - x_{t|t}^i\right)^2}{2\frac{\Sigma\sigma_\epsilon^2}{\Sigma+\sigma_\epsilon^2}} - \theta\frac{\left(x_t - x_{t|t}^i\right)^2 - \left(x_t - x_{t|t-1}^i\right)^2}{2\frac{\Sigma\sigma_\epsilon^2}{\Sigma+\sigma_\epsilon^2}}$$

$$= -\frac{1}{2\frac{\Sigma\sigma_\epsilon^2}{\Sigma+\sigma_\epsilon^2}}\left[x_t^2 - 2x_t\left(x_{t|t}^i + \theta\left(x_{t|t}^i - x_{t|t-1}^i\right)\right) + \left(x_{t|t}^i\right)^2(1+\theta) - \theta\left(x_{t|t-1}^i\right)^2\right]$$

Given the normalization $\int f^{\theta}(x|S_t^i)dx = 1$, we find $f^{\theta}(x_t|S_t^i) \sim \mathcal{N}\left(x_{t|t}^{i,\theta}, \frac{\Sigma\sigma_\epsilon^2}{\Sigma+\sigma_\epsilon^2}\right)$ with $x_{t|t}^{i,\theta} = x_{t|t}^i + \theta\left(x_{t|t}^i - x_{t|t-1}^i\right)$. Using the definition of the Kalman filter $x_{t|t}^i$ we can write:

$$x_{t|t}^{i,\theta} = x_{t|t-1}^i + (1+\theta)\frac{\Sigma}{\Sigma+\sigma_\epsilon^2}\left(s_t^i - x_{t|t-1}^i\right). \blacksquare$$

Proposition 2. Denote by $K = \Sigma/(\Sigma+\sigma_\epsilon^2)$ the contemporaneous Kalman gain for $\rho > 0$. The rational consensus estimate for the current state is then equal to $\int x_{t|t}^i\, di \equiv x_{t|t} = x_{t|t-1} + K\left(x_t - x_{t|t-1}\right)$. The consensus forecast error under rationality is then equal to $x_t - x_{t|t} = \frac{1-K}{K}\left(x_{t|t} - x_{t|t-1}\right)$. The diagnostic filter for an individual analyst is equal to $x_{t|t}^{i,\theta} = x_{t|t}^i + \theta(x_{t|t}^i - x_{t|t-1}^i)$, which implies a consensus equation $x_{t|t}^{\theta} = x_{t|t} + \theta(x_{t|t} - x_{t|t-1})$. We thus have:

$$x_t - x_{t|t}^{\theta} = \left(\frac{1-K}{K} - \theta\right)\left(x_{t|t} - x_{t|t-1}\right).$$

Note, in addition, that the diagnostic consensus forecast revision is equal to:

$$x_{t|t}^{\theta} - x_{t|t-1}^{\theta} = (1+\theta)\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right).$$

Therefore, the consensus CG coefficient is given by:

$$\beta = \frac{cov\left(x_{t+h} - x_{t+h|t}^{\theta}, x_{t+h|t}^{\theta} - x_{t+h|t-1}^{\theta}\right)}{var\left(x_{t+h|t}^{\theta} - x_{t+h|t-1}^{\theta}\right)}$$

$$= \left(\frac{1-K}{K} - \theta\right) \cdot \frac{cov\left[x_{t|t} - x_{t|t-1}, (1+\theta)\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right)\right]}{var\left[(1+\theta)\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right)\right]}.$$

Where we have that:

$$cov\left[x_{t|t} - x_{t|t-1}, (1+\theta)\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right)\right]$$
$$= (1+\theta)var\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho cov\left(x_{t|t} - x_{t|t-1}, x_{t-1|t-1} - x_{t-1|t-2}\right),$$

and

$$var\left[(1+\theta)\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right)\right]$$
$$= [(1+\theta)^2 + \theta^2\rho^2]var\left(x_{t|t} - x_{t|t-1}\right)$$
$$- 2\theta(1+\theta)\rho cov\left(x_{t|t} - x_{t|t-1}, x_{t-1|t-1} - x_{t-1|t-2}\right).$$

To compute the covariance between adjacent rational revisions, note that $x_{t|t} = x_{t|t-1} + K(x_t - x_{t|t-1})$ and $x_{t|t-1} = x_{t|t-2} + K(\rho x_{t-1} - x_{t|t-2})$ imply that:

$$x_{t|t} - x_{t|t-1} = (1-K)\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right) + Ku_t.$$

As a result,

$$cov\left(x_{t|t} - x_{t|t-1}, x_{t-1|t-1} - x_{t-1|t-2}\right) = (1-K)\rho \cdot var\left(x_{t|t} - x_{t|t-1}\right)$$

Therefore:

$$\beta = \left(\frac{1-K}{K} - \theta\right) \cdot \frac{(1+\theta) - \theta\rho^2(1-K)}{[(1+\theta)^2 + \theta^2\rho^2] - 2\theta(1+\theta)\rho^2(1-K)},$$

which is positive if and only if $1 - K > \theta K$, namely, $\theta < \sigma_\epsilon^2/\Sigma$.

Consider individual level forecasts. The coefficient (at the individual level) of regressing forecast error on forecast revision is equal to:

$$\beta^p = \frac{cov\left(x_{t+h} - x_{t+h|t}^{i,\theta}, x_{t+h|t}^{i,\theta} - x_{t+h|t-1}^{i,\theta}\right)}{var\left(x_{t+h|t}^{i,\theta} - x_{t+h|t-1}^{i,\theta}\right)} =$$

$$\frac{cov\left(x_{t|t} - x_{t|t}^{i,\theta}, x_{t|t}^{i,\theta} - x_{t|t-1}^{i,\theta}\right)}{var\left(x_{t|t}^{i,\theta} - x_{t|t-1}^{i,\theta}\right)}$$

where $x_{t|t}^{i,\theta} - x_{t|t-1}^{i,\theta} = (1+\theta)\left(x_{t|t}^i - x_{t|t-1}^i\right) - \theta\rho\left(x_{t-1|t-1}^i - x_{t-1|t-2}^i\right)$. Because at the individual level $cov\left(x_{t|t}^i - x_{t|t-1}^i, x_{t|t-1}^i - x_{t|t-2}^i\right) = 0$, we immediately have that:

$$\beta^p = -\frac{\theta(1+\theta)}{(1+\theta)^2 + \rho^2\theta^2}.$$

Overreaction is larger (β^p is more negative) for series with lower persistence. Intuitively, when persistence is low, rational beliefs respond less to news (the denominator $var\left(x_{t|t}^{i,\theta} - x_{t|t-1}^{i,\theta}\right)$ is smaller) and there is more scope for overreaction.

For completeness, consider the case of $\rho = 0$. In this case, all forecasters hold the same beliefs, which are independent of their idiosynchratic signals s_t^i. Thus, consensus and individual forecasts are the same, $x_{t|t}^{i,\theta} = x_{t|t}^{\theta}$. Moreover, these forecasts are not revised (as under the rational benchmark) so that the CG coefficients are zero. Thus, because contemporaneous beliefs are discontinuous at $\rho = 0$, so are the CG coefficients.

Finally, we extend the analysis to the case where the degree of diagnosticity varies across forecasters, so that forecaster i's beliefs are given by

$$x_{t|t}^{i,\theta} = x_{t|t}^i + \theta^i\left(x_{t|t}^i - x_{t|t-1}^i\right)$$

Consider first consensus beliefs. We have:

$$\frac{1}{I}\sum\nolimits_i x_{t|t}^{i,\theta} = x_{t|t} + \frac{1}{I}\sum_i \theta^i\left(x_{t|t}^i - x_{t|t-1}^i\right)$$

where I denotes the number of forecasters. Because the revision $x_{t|t}^i - x_{t|t-1}^i$ is uncorrelated with θ^i, then for large I the consensus becomes $x_{t|t} + \theta\left(x_{t|t} - x_{t|t-1}\right)$ as in the case of homogeneous forecasters. As a consequence, Equation (12) goes through.

Consider now the effect of pooling heterogeneous forecasters on the individual level CG coefficient. To do so, write $FR_t^{i,\theta} = x_{t|t}^{i,\theta} - x_{t|t-1}^{i,\theta}$ and $FE_t^{i,\theta} = x_t - x_{t|t}^{i,\theta}$, where $t = 1, \dots T$, as well as $FR^{\theta} = \frac{1}{I}\frac{1}{T}\sum_i \sum_t FR_t^{i,\theta}$ and $FE^{\theta} = \frac{1}{I}\frac{1}{T}\sum_i \sum_t FE_t^{i,\theta}$. In a pooled estimation, we have:

$$\beta_1^p = \frac{\sum_i \sum_t \left(FR_t^{i,\theta} - FR^{\theta}\right)\left(FE_t^{i,\theta} - FE^{\theta}\right)}{\sum_i \sum_t \left(FR_t^{i,\theta} - FR^{\theta}\right)^2}$$

Because the series of shocks is uncorrelated with forecaster heterogeneity, this can be written as:

$$\beta_1^p = \frac{\sum_i \beta_1^i var_t\left(FR_t^{i,\theta}\right)}{var(FR^{i,\theta}) + \sum_i var_t\left(FR_t^{i,\theta}\right)} + \frac{cov\left(FR^{i,\theta}, FE^{i,\theta}\right)}{var(FR^{i,\theta}) + \sum_i var_t\left(FR_t^{i,\theta}\right)}$$

where β_1^i is the coefficient of the CG regression on forecaster i, and $FR^{i,\theta}, FE^{i,\theta}$ are the (time) average forecast error and forecast revision of forecaster i.

Clearly, in the case of homogeneous forecasters, this coefficient is unbiased. However, under heterogeneity two forces bias the coefficient upwards, towards zero, provided forecasters differ in their average forecast revision, $var\left(FR^{i,\theta}\right) > 0$. When this is the case, then the first term, which pools the individual β_1^is, is dampened below a weighted average of the latter. Heterogeneity in the size of forecast revisions directly dampens the role of individual overreaction in the pooled estimate because the pooled variance is now larger than the sum of individual variances. Second, to the extent that they are positively correlated, heterogeneity in forecast revisions and errors also pushes up the pooled coefficient. This is the usual heterogeneity mechanism whereby forecasters who are more optimistic make both more positive mistakes and more positive revisions, leading to a spurious positive correlation between revision and error in the pooled sample.

Thus, in general forecaster heterogeneity biases the pooled estimates against our predictions. Equivalently, to find negative coefficients in a pooled estimate it is necessary that (sufficiently many) forecasters overreact and have negative β_1^i.■

Corollary 1. Denote by p_i the precision of the private signal, by p the precision of the public signal, by p_f the precision of the lagged rational forecast $x_{t|t-1}^i$. The diagnostic filter at time t is:

$$x_{t|t}^{i,\theta} = x_{t|t-1}^i + (1+\theta)\frac{p_i}{p_i + p + p_f}\left(s_t^i - x_{t|t-1}^i\right) + (1+\theta)\frac{p}{p_i + p + p_f}\left(s_t - x_{t|t-1}^i\right).$$

The precision p_f of the forecast depends on the sum of the precisions $(p_i + p)$ and hence stays constant as we vary the relative precision of the public versus private signal.

Denote the Kalman gains as $K_1 = \frac{p_i}{p_i+p+p_f}$ and $K_2 = \frac{p}{p_i+p+p_f}$, and $K = K_1 + K_2$. The consensus Kalman filter can then be written as $x_{t|t} = x_{t|t-1} + K\left(x_t - x_{t|t-1}\right) + K_2 v_t$, while the diagnostic filter can be written as $x_{t|t}^{\theta} = x_{t|t} + \theta\left(x_{t|t} - x_{t|t-1}\right)$. The consensus coefficient is then:

$$\frac{cov\left(x_{t+h} - x_{t+h|t}^{\theta}, x_{t+h|t}^{\theta} - x_{t+h|t-1}^{\theta}\right)}{var\left(x_{t+h|t}^{\theta} - x_{t+h|t-1}^{\theta}\right)} = \frac{\rho^{2h} cov\left(x_t - x_{t|t}^{\theta}, x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right)}{\rho^{2h} var\left(x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right)}.$$

Consider first the numerator. Denote by $FR_t \equiv x_{t|t} - x_{t|t-1}$ the revision of the rational forecast of x_t between t and $t-1$. Then:

$$x_t - x_{t|t}^{\theta} = \left(\frac{1-K}{K} - \theta\right) FR_t - \frac{K_2}{K} v_t,$$

$$x_{t|t}^{\theta} - x_{t|t-1}^{\theta} = (1+\theta) FR_t - \theta\rho FR_{t-1}.$$

The difference between $x_{t|t} = x_{t|t-1} + K\left(x_t - x_{t|t-1}\right) + K_2 v_t$ and $x_{t|t-1} = x_{t|t-2} + K\big(\rho x_{t-1} - x_{t|t-2}\big) + K_2 \rho v_{t-1}$ reads:

$$FR_t = (1-K)\rho FR_{t-1} + K u_t + K_2(v_t - \rho v_{t-1}),$$

which in turn implies:

$$cov(FR_t, FR_{t-1}) = (1-K)\rho \cdot var(FR_t) - \rho K_2^2 \sigma_v^2. \qquad (A.1)$$

It is also immediate to find that:

$$var(FR_t) = \frac{K^2\sigma_u^2 + [(1+\rho^2) - 2\rho^2(1-K)]K_2^2\sigma_v^2}{1 - [(1-K)\rho]^2}.$$

The numerator of the CG coefficient is then equal to:

$$cov\left(x_t - x_{t|t}^{\theta}, x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right) = \left(\frac{1-K}{K} - \theta\right) cov[FR_t, (1+\theta)FR_t - \theta\rho FR_{t-1}] - \frac{K_2}{K}(1+\theta)K_2\sigma_v^2$$

$$= \left(\frac{1-K}{K} - \theta\right)\left[[1+\theta-\theta\rho^2(1-K)]var(FR_t) + \theta\rho^2 K_2^2\sigma_v^2\right] - \frac{(1+\theta)K_2^2\sigma_v^2}{K} \qquad (A.2)$$

The denominator of the CG coefficient equals:

$$var\left(x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right) = var[(1+\theta)FR_t - \theta\rho FR_{t-1}]$$

$$= [(1+\theta)^2 + \theta^2\rho^2]var(FR_t) - 2\theta(1+\theta)\rho cov(FR_t, FR_{t-1})$$

which implies that:

$$\frac{var\left(x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right)}{[(1+\theta)^2 + \theta^2\rho^2]} + \frac{2\theta(1+\theta)\rho}{[(1+\theta)^2 + \theta^2\rho^2]} cov(FR_t, FR_{t-1}) = var(FR_t). \qquad (A.3)$$

Putting $(A.3)$ together with $(A.1)$ one obtains:

$$cov(FR_t, FR_{t-1}) =$$

$$= \frac{(1-K)\rho var\left(x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right)}{\left[1 - \frac{2\theta(1-K)(1+\theta)\rho^2}{[(1+\theta)^2+\theta^2\rho^2]}\right][(1+\theta)^2 + \theta^2\rho^2]} - \frac{\rho K_2^2\sigma_v^2}{\left[1 - \frac{2\theta(1-K)(1+\theta)\rho^2}{[(1+\theta)^2+\theta^2\rho^2]}\right]} \qquad (A.4)$$

Using Equations $(A.2)$ and $(A.4)$ we find:

$$cov\left(x_t - x_{t|t}^{\theta}, x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right)$$

$$= \left(\frac{1-K}{K} - \theta\right)\left[(1+\theta)\frac{var\left(x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right)}{(1+\theta)^2 + \theta^2\rho^2}\right.$$

$$\left. + \theta\rho\left(\frac{2(1+\theta)^2}{(1+\theta)^2 + \theta^2\rho^2} - 1\right) cov(FR_t, FR_{t-1})\right] - \frac{(1+\theta)K_2^2\sigma_v^2}{K} =$$

$$= \beta_\infty var\left(x^\theta_{t|t} - x^\theta_{t|t-1}\right) - K_2^2\sigma_v^2\left[\frac{\theta\rho^2\left(\frac{1-K}{K}-\theta\right)\left(\frac{2(1+\theta)^2}{(1+\theta)^2+\theta^2\rho^2}-1\right)}{\left[1-\frac{2\theta(1-K)(1+\theta)\rho^2}{(1+\theta)^2+\theta^2\rho^2}\right]}+\frac{(1+\theta)}{K}\right],$$

where β_∞ is the consensus coefficient obtained when the public signal is fully uninformative, namely $\sigma_u^2 \to \infty$ and thus $K_2 \to 0$. On the other hand using equation (A.3) this can be rewritten as:

$$var\left(x^\theta_{t|t} - x^\theta_{t|t-1}\right) = \frac{[(1+\theta)^2+\theta^2\rho^2-2\theta(1+\theta)(1-K)\rho^2]K^2\sigma_u^2}{1-[(1-K)\rho]^2} + AK_2^2\sigma_v^2,$$

where A is a suitable positive coefficient. The CG coefficient is then equal to:

$$\frac{cov\left(x_t - x^\theta_{t|t}, x^\theta_{t|t} - x^\theta_{t|t-1}\right)}{var\left(x^\theta_{t|t} - x^\theta_{t|t-1}\right)} = \beta_\infty - \frac{\left[\frac{\theta\rho^2\left(\frac{1-K}{K}-\theta\right)\left(\frac{2(1+\theta)^2}{(1+\theta)^2+\theta^2\rho^2}-1\right)}{1-\frac{2\theta(1-K)(1+\theta)\rho^2}{(1+\theta)^2+\theta^2\rho^2}}+\frac{(1+\theta)}{K}\right]K_2^2\sigma_v^2}{\frac{[(1+\theta)^2+\theta^2\rho^2-2\theta(1+\theta)(1-K)\rho^2]K^2\sigma_u^2}{1-[(1-K)\rho]^2} + AK_2^2\sigma_v^2}.$$

For given total informativeness K, the above expression falls in the precision of the public signal, namely as K_2^2 grows, if and only if:

$$\left[\frac{\theta\rho^2\left(\frac{1-K}{K}-\theta\right)\left(\frac{2(1+\theta)^2}{(1+\theta)^2+\theta^2\rho^2}-1\right)}{1-\frac{2\theta(1-K)(1+\theta)\rho^2}{(1+\theta)^2+\theta^2\rho^2}}+\frac{(1+\theta)}{K}\right] > 0.$$

A sufficient condition for this to hold is that $\left(\frac{1-K}{K}-\theta\right) > 0$, which is equivalent to $\beta_\infty > 0$.

■

Lemma A.1 *Suppose that forecasters observe individual signals as well as the lagged variable. Formally, they receive a signal vector* $\left(s_t^i, y_t^i\right)$ *given by:*

$$\begin{cases} s_t^i = x_t + \epsilon_t^i \\ y_t = x_{t-1} \end{cases}$$

Then the individual CG coefficient is given by Equation (12), and there exists a positive threshold θ^* *such that the consensus CG coefficient is positive for* $\theta \in [0, \theta^*)$ *and negative for* $\theta > \theta^*$.

Proof Consider first updating under rational expectations. After observing $\left(s_{t-1}^i, y_{t-1}^i\right)$ at $t-1$, forecaster i's belief about x_{t-1} is normal with mean

$$x^i_{t-1|t-1} = \rho x_{t-2} + \frac{\sigma_u^2}{\sigma_u^2+\sigma_\epsilon^2}\left(s_{t-1}^i - \rho x_{t-2}\right)$$

and variance σ_ϵ^2 (because uncertainty about x_{t-1} is restricted to uncertainty about u_t which can be written $u_t = s_t^i - \rho x_{t-1} - \epsilon_t^i$). In fact, under rational expectations beliefs are invariant over the timing of the signal, and can be easily derived in the specification where the individual signal about the current state s_{t-1}^i follows the fully revealing signal about the lagged state x_{t-2}.

Consider now diagnostic expectations, in which the believed probability of a current realization x_t is distorted by its representativeness relative to news at t. From Equation (6), we have:

$$R(x_t) = \frac{f\left(x_t | \left(s_t^i, y_t\right)\right)}{f\left(x_t | \left(\rho x_{t-1|t-1}^i, x_{t-1|t-1}^i\right)\right)}$$

Here $\left(\rho x_{t-1|t-1}^i, x_{t-1|t-1}^i\right)$ is the signal at t that is expected at $t-1$, and the expression highlights the fact that, because the lagged state is fully revealed, forecasters optimally ignore any previous signals. $R(x_t)$ compares two normal distributions characterized by the same variance, namely σ_ϵ^2, but different means, namely $x_{t|t}^i$ and $\rho x_{t-1|t-1}^i$. Diagnostic beliefs $f^\theta\left(x_t | \left(s_t^i, y_t^i\right)\right)$, defined by Equation (7), are then normally distributed with variance σ_ϵ^2 and mean:

$$x_{t|t}^{i,\theta} = x_{t|t}^i + \theta\left(x_{t|t}^i - \rho x_{t-1|t-1}^i\right) = \rho x_{t-1} + K\left(s_t^i - \rho x_{t-1}\right) + \theta\left[K\left(s_t^i - \rho x_{t-1}\right) + \rho\left(x_{t-1} - x_{t-1|t-1}^i\right)\right]$$

with $K = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_\epsilon^2}$. Relative to rationality, there are now two distortions: the second and third terms exhibit the diagnostic Kalman filter which captures overreaction to the signal s_{t-1}^i relative to expectations conditional on the true lagged state ρx_{t-1}; the last term captures overreaction to surprise about the lagged state itself. The relative weights of the two distortions are given by the respective impact on the signals on beliefs, K and ρ.

Thus, expectations are too optimistic provided

$$K\left(s_t^i - \rho x_{t-1}\right) + \rho\left(x_{t-1} - x_{t-1|t-1}^i\right) > 0$$

This can be rewritten:

$$K\left(u_t + \epsilon_t^i\right) + \rho\left(u_{t-1} - K\left(u_{t-1} + \epsilon_{t-1}^i\right)\right) > 0$$

Thus, overoptimism at t depends on the sequence of shocks at $t-1$ and t. In particular, because ϵ_t^i is mean zero, this condition is more likely to hold when the process has received two positive fundamental shocks, $u_{t-1}, u_t > 0$. In contrast, if a good shock follows a bad shock, overreaction to the latter is dampened by the realization that the lagged state was not as good as expected. The same intuition holds for consensus forecasts, for which we find:

$$x_{t|t}^\theta = \rho x_{t-1} + K(x_t - \rho x_{t-1}) + \theta[K u_t + (1-K)\rho u_{t-1}]$$

We now derive the Coibion-Gorodnichenko coefficients. Consider first the consensus specification. We have:

$$x_{t|t}^\theta = x_{t|t} + \theta\left(x_{t|t} - x_{t|t-1}\right)$$

where $x_{t|t} = \rho x_{t-1} + K(x_t - \rho x_{t-1})$. Using $x_t = \frac{1}{K}\left(x_{t|t} - \rho x_{t-1}\right) + \rho x_{t-1}$ and

$$x_{t|t}^\theta = \rho x_{t-1} + (1+\theta)K(x_t - \rho x_{t-1}) + \theta\rho(1-K)(x_{t-1} - \rho x_{t-2})$$

the forecast error reads:

$$x_t - x_{t|t}^\theta = \left(1 - K(1+\theta)\right)(x_t - \rho x_{t-1}) - \theta\rho(1-K)(x_{t-1} - \rho x_{t-2})$$

The forecast revision is:

$$x^{\theta}_{t|t} - x^{\theta}_{t|t-1} = \rho[1+\theta(1-K)-(1+\theta)K](x_{t-1}-\rho x_{t-2}) + (1+\theta)K(x_t - \rho x_{t-1}) - \theta\rho^2(1-K)(x_{t-2}-\rho x_{t-3})$$

So the consensus coefficient is:

$$\frac{cov\left(x_t - x^{\theta}_{t|t}, x^{\theta}_{t|t} - x^{\theta}_{t|t-1}\right)}{var\left(x^{\theta}_{t|t} - x^{\theta}_{t|t-1}\right)} = \frac{(1-(1+\theta)K)(1+\theta)K - \theta\rho^2(1-K)[1+\theta(1-K)-(1+\theta)K]}{(\rho[1+\theta(1-K)-(1+\theta)K])^2 + (1+\theta)^2K^2 + \theta^2\rho^4(1-K)^2}$$

This is positive if and only if

$$(1-K)K + \theta[K(1-2K) - \rho^2(1-K)^2] - \theta^2[K^2 + \rho^2(1-K)(1-2K)] > 0$$

This holds for $\theta \in [0, \theta^*)$ (since the quadratic coefficient is positive) where

$$\theta^* = \frac{-[K(1-2K)-\rho^2(1-K)^2] + \sqrt{[K(1-2K)-\rho^2(1-K)^2]^2 - 4(1-K)K[K^2+\rho^2(1-K)(1-2K)]}}{2[K^2+\rho^2(1-K)(1-2K)]}$$

Consider now the individual level forecast. The forecast revision reads:

$$x^{i,\theta}_{t|t} - x^{i,\theta}_{t|t-1} = (1+\theta)\left(x^i_{t|t} - \rho x^i_{t-1|t-1}\right) - \theta\rho\left(x^i_{t-1|t-1} - \rho x^i_{t-2|t-2}\right)$$

The forecast error reads: $x_t - x^{i,\theta}_{t|t} = x_t - x^i_{t|t} - \theta\left(x^i_{t|t} - \rho x^i_{t-1|t-1}\right)$

So:

$$cov\left(x_t - x^{i,\theta}_{t|t}, x^{i,\theta}_{t|t} - x^{i,\theta}_{t|t-1}\right) = -\theta(1+\theta)var\left(x^i_{t|t} - x^i_{t|t-1}\right)$$

since $cov\left(x_t - x^i_{t|t}, x^i_{t|t} - \rho x^i_{t-1|t-1}\right) = 0$ by definition of the Kalman filter and similarly $cov\left(x_t - x^i_{t|t}, x^i_{t-1|t-1} - \rho x^i_{t-2|t-2}\right) = 0$. Moreover:

$$var\left(x^{i,\theta}_{t|t} - x^{i,\theta}_{t|t-1}\right) = [(1+\theta)^2 + (\theta\rho)^2]var\left(x^i_{t|t} - x^i_{t|t-1}\right)$$

So the coefficient is:

$$\frac{cov\left(x^i_{t|t} - x^{i,\theta}_{t|t}, x^{i,\theta}_{t|t} - x^{i,\theta}_{t|t-1}\right)}{var\left(x^{i,\theta}_{t|t} - x^{i,\theta}_{t|t-1}\right)} = -\frac{\theta(1+\theta)}{(1+\theta)^2 + (\theta\rho)^2}$$

As in the baseline case of Proposition 2. ■

B. Variable Definitions

For each variable, we report the data source, the survey time, the question format, and the definitions of forecasts, revisions, and actuals.

1. NGDP_SPF

- Variable: Nominal GDP. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of nominal GDP in the current quarter and the next 4 quarters.
- Forecast: Nominal GDP growth from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and *x* is the level of nominal GDP in a given quarter; x_{t-1} uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data published in quarter *t+4*.

2. RGDP_SPF

- Variable: Real GDP. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real GDP in the current quarter and the next 4 quarters.
- Forecast: Real GDP growth from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and *x* is the level of real GDP in a given quarter; x_{t-1} uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data published in quarter *t*+4.

3. RGDP_BC

- Variable: Real GDP. Source: Blue Chip.
- Time: End of the middle month in the quarter/beginning of the last month in the quarter.
- Question: Real GDP growth (annualized rate) in the current quarter and the next 4 to 5 quarters.
- Forecast: Real GDP growth from end of quarter *t*-1 to end of quarter *t*+3: $F_t[(z_t/4+1) * (z_{t+1}/4+1) * (z_{t+2}/4+1) * (z_{t+3}/4+1) - 1]$, where *t* is the quarter of forecast and z_t is the annualized quarterly GDP growth in quarter *t*. Using simple average $F_t(z_t + z_{t+1} + z_{t+2} + z_{t+3})/4$ produces similar results.
- Revision: $F_t[(z_t/4+1) * (z_{t+1}/4+1) * (z_{t+2}/4+1) * (z_{t+3}/4+1)] - F_{t-1}[(z_t/4+1) * (z_{t+1}/4+1) * (z_{t+2}/4+1) * (z_{t+3}/4+1)]$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data published in quarter *t*+4.

4. PGDP_SPF

- Variable: GDP price deflator. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of GDP price deflator in the current quarter and the next 4 quarters.

- Forecast: GDP price deflator inflation from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and *x* is the level of GDP price deflator in a given quarter; x_{t-1} uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter t.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro published in quarter *t*+4.

5. CPI_SPF

- Variable: Consumer Price Index. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: CPI growth rate in the current quarter and the next 4 quarters.
- Forecast: CPI inflation from end of quarter *t*-1 to end of quarter *t*+3: $F_t[(z_t/4+1)*(z_{t+1}/4+1)*(z_{t+2}/4+1)*(z_{t+3}/4+1)-1]$, where t is the quarter of forecast and z_t is the annualized quarterly CPI inflation in quarter *t*. Using simple average $F_t(z_t+z_{t+1}+z_{t+2}+z_{t+3})/4$ produces similar results.
- Revision: $F_t[(z_t/4+1)*(z_{t+1}/4+1)*(z_{t+2}/4+1)*(z_{t+3}/4+1)] - F_{t-1}[(z_t/4+1)*(z_{t+1}/4+1)*(z_{t+2}/4+1)*(z_{t+3}/4+1)]$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data published in quarter *t*+4. Real time data is not available before 1994Q3. For actual periods prior to this date, we use data published in 1994Q3 to measure the actual outcome.

6. RCONSUM_SPF

- Variable: Real consumption. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real consumption in the current quarter and the next 4 quarters.
- Forecast: Growth of real consumption from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and *x* is the level of real consumption in a given quarter; x_{t-1} uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data published in quarter *t*+4.

7. INDPROD_SPF

- Variable: Industrial production index. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The average level of the industrial production index in the current quarter and the next 4 quarters.
- Forecast: Growth of the industrial production index from quarter *t*-1 to quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and *x* is the level of real consumption in a given quarter; x_{t-1} uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data published in quarter *t*+4.

8. RNRESIN_SPF

- Variable: Real non-residential investment. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real non-residential investment in the current quarter and the next 4 quarters.
- Forecast: Growth of real non-residential investment from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and *x* is the level of real non-residential investment in a given quarter; x_{t-1} uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data published in quarter *t*+4.

9. RRESIN_SPF

- Variable: Real residential investment. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real residential investment in the current quarter and the next 4 quarters.
- Forecast: Growth of real residential investment from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and *x* is the level of real residential investment in a given quarter; x_{t-1} uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data published in quarter *t*+4.

10. RGF_SPF

- Variable: Real federal government consumption. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real federal government consumption in the current quarter and the next 4 quarters.
- Forecast: Growth of real federal government consumption from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and *x* is the level of real federal government consumption in a given quarter; x_{t-1} uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data: initial realease of x_{t+3} published in quarter *t*+4.

11. RGSL_SPF

- Variable: Real state and local government consumption. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real state and local government consumption in the current quarter and the next 4 quarters.
- Forecast: Growth of real state and local government consumption from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and *x* is the level of real state and local

government consumption in a given quarter; x_{t-1} uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.

- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data published in quarter *t*+4.

12. HOUSING_SPF

- Variable: Housing starts. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of housing starts in the current quarter and the next 4 quarters.
- Forecast: Growth of housing starts from quarter *t*-1 to quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and *x* is the level of housing starts in a given quarter; x_{t-1} uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data published in quarter *t*+4.

13. UNEMP_SPF

- Variable: Unemployment rate. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average unemployment rate in the current quarter and the next 4 quarters.
- Forecast: Average quarterly unemployment rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and *x* is the level of unemployment rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: x_{t+3}, using real time macro data published in quarter *t*+4.

14. FF_BC

- Variable: Federal funds rate. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average federal funds rate in the current quarter and the next 4 quarters.
- Forecast: Average quarterly 3-month federal funds rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and *x* is the level of federal funds rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: x_{t+3}.

15. TB3M_SPF

- Variable: 3-month Treasury rate. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average 3-month Treasury rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly 3-month Treasury rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and *x* is the level of 3-month Treasury rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: x_{t+3}.

16. TB3M_BC

- Variable: 3-month Treasury rate. Source: Blue Chip.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average 3-month Treasury rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly 3-month Treasury rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and *x* is the level of 3-month Treasury rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: x_{t+3}.

17. TN5Y_BC

- Variable: 5-year Treasury rate. Source: Blue Chip.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average 5-year Treasury rate in the current quarter and the next 4 quarters.
- Forecast: Average quarterly 5-year Treasury rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and *x* is the level of 5-year Treasury rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: x_{t+3}.

18. TN10Y_SPF

- Variable: 10-year Treasury rate. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average 10-year Treasury rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly 10-year Treasury rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and *x* is the level of 10-year Treasury rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: x_{t+3}.

19. TN10Y_BC

- Variable: 10-year Treasury rate. Source: Blue Chip.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average 10-year Treasury rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly 10-year Treasury rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and *x* is the level of 10-year Treasury rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: x_{t+3}.

20. AAA_SPF

- Variable: AAA corporate bond rate. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average AAA corporate bond rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly AAA corporate bond rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and *x* is the level of AAA corporate bond rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: x_{t+3}.

21. AAA_BC

- Variable: AAA corporate bond rate. Source: Blue Chip.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average AAA corporate bond rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly AAA corporate bond rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and *x* is the level of AAA corporate bond rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: x_{t+3}.

22. BAA_BC

- Variable: BAA corporate bond rate. Source: Blue Chip.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average BAA corporate bond rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly BAA corporate bond rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and *x* is the level of BAA corporate bond rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: x_{t+3}.

C. Robustness Checks

Table C1. Different Forecast Horizons

This table shows the forecast error on forecast revision regressions, using different horizons for forecast errors ($h = 2$: $x_{t+2} - x^i_{t+2|t-1}$) and forecast revisions ($h = 3$: $x^i_{t+3|t} - x^i_{t+3|t-1}$). Columns (1) to (3) report results of the time series regressions using consensus (mean) forecast in each quarter. Columns (4) to (9) report results of individual level panel regressions. Columns (10) to (12) report results of forecaster-by-forecaster regressions (median in the data and the 2.5th and 97.5th percentile of the median based on block bootstrap).

	Consensus			Individual						By Forecaster		
				No fixed effects			With fixed effects			Median	p 2.5	p 97.5
	β_1	s.e.	*p*-val	β_1^p	s.e.	*p*-val	β_1^p	s.e.	*p*-val	β_1^i	β_1^i	β_1^i
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Nominal GDP (SPF)	0.42	0.15	0.01	-0.16	0.06	0.01	-0.17	0.06	0.00	-0.16	-0.28	-0.01
Real GDP (SPF)	0.29	0.17	0.09	-0.14	0.09	0.12	-0.13	0.08	0.12	-0.05	-0.27	0.09
Real GDP (BC)	0.44	0.24	0.06	0.08	0.18	0.67	0.00	0.17	1.00	-0.01	-0.29	0.12
GDP Price Index Inflation (SPF)	0.82	0.14	0.00	0.06	0.09	0.46	-0.01	0.07	0.91	-0.11	-0.32	0.00
CPI (SPF)	0.12	0.17	0.48	-0.18	0.11	0.10	-0.24	0.12	0.05	-0.15	-0.37	-0.07
Real Consumption (SPF)	0.19	0.18	0.30	-0.19	0.08	0.02	-0.22	0.08	0.00	-0.25	-0.40	-0.06
Industrial Production (SPF)	0.58	0.22	0.01	-0.12	0.09	0.17	-0.12	0.08	0.14	-0.17	-0.30	0.03
Real Non-Residential Investment (SPF)	0.80	0.25	0.00	0.05	0.13	0.68	0.03	0.12	0.83	0.05	-0.34	0.17
Real Residential Investment (SPF)	1.02	0.23	0.00	0.00	0.08	0.99	-0.04	0.07	0.59	-0.07	-0.27	0.04
Real Federal Government Consumption (SPF)	-0.10	0.22	0.66	-0.43	0.08	0.00	-0.43	0.08	0.00	-0.38	-0.62	-0.22
Real State & Local Govt Consumption (SPF)	0.48	0.25	0.05	-0.35	0.03	0.00	-0.37	0.03	0.00	-0.32	-0.41	-0.27
Housing Start (SPF)	0.38	0.22	0.09	-0.18	0.07	0.01	-0.19	0.06	0.00	0.16	-0.14	0.32
Unemployment (SPF)	0.60	0.14	0.00	0.24	0.11	0.03	0.21	0.11	0.06	-0.22	-0.32	-0.05
Fed Funds Rate (BC)	0.38	0.16	0.01	0.12	0.07	0.06	0.11	0.07	0.09	0.12	0.03	0.22
3M Treasury Rate (SPF)	0.44	0.18	0.01	0.16	0.07	0.02	0.13	0.07	0.05	0.16	0.03	0.24
3M Treasury Rate (BC)	0.41	0.17	0.02	0.12	0.06	0.05	0.10	0.06	0.09	0.10	0.01	0.19
5Y Treasury Rate (BC)	-0.07	0.18	0.70	-0.14	0.09	0.10	-0.19	0.08	0.02	-0.22	-0.35	-0.09
10Y Treasury Rate (SPF)	-0.07	0.23	0.75	-0.21	0.09	0.02	-0.24	0.09	0.01	-0.27	-0.48	-0.14

10Y Treasury Rate (BC)	-0.13	0.20	0.53	-0.19	0.10	0.06	-0.24	0.10	0.02	-0.27	-0.46	-0.19
AAA Corporate bond Rate (SPF)	0.06	0.19	0.74	-0.19	0.05	0.00	-0.22	0.05	0.00	-0.27	-0.36	-0.16
AAA Corporate Bond Rate (BC)	0.09	0.16	0.58	-0.11	0.05	0.04	-0.14	0.05	0.01	-0.20	-0.33	-0.13
BAA Corporate Bond Rate (BC)	-0.12	0.23	0.60	-0.24	0.07	0.00	-0.27	0.07	0.00	-0.27	-0.36	-0.20

Table C2. Results using Latest Release of Actuals

This table shows the baseline CG (forecast error on forecast revision) regressions, using the latest release of the actual outcome x_{t+3}. Columns (1) to (3) report results of the time series regressions using consensus (mean) forecast in each quarter. Columns (4) to (9) report results of individual level panel regressions. Columns (10) to (12) report results of forecaster-by-forecaster regressions (median in the data and the 2.5th and 97.5th percentile of the median based on block bootstrap).

	Consensus			Individual						By Forecaster		
				No fixed effects			With fixed effects			Median	p 2.5	p 97.5
	β_1	s.e.	*p*-val	β_1^p	s.e.	*p*-val	β_1^p	s.e.	*p*-val	β_1^i	β_1^i	β_1^i
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Nominal GDP (SPF)	0.69	0.22	0.00	-0.17	0.08	0.04	-0.21	0.08	0.01	-0.15	-0.38	0.03
Real GDP (SPF)	0.33	0.24	0.17	-0.21	0.09	0.03	-0.21	0.09	0.02	-0.07	-0.39	0.07
Real GDP (BC)	0.78	0.41	0.06	0.23	0.22	0.28	0.07	0.19	0.72	0.04	-0.41	0.20
GDP Price Index Inflation (SPF)	1.30	0.21	0.00	0.13	0.11	0.21	-0.02	0.08	0.82	-0.15	-0.35	0.02
CPI (SPF)	0.30	0.22	0.16	-0.19	0.12	0.12	-0.26	0.12	0.03	-0.10	-0.39	-0.08
Industrial Production (SPF)	0.61	0.31	0.05	-0.15	0.09	0.08	-0.19	0.08	0.03	-0.25	-0.40	-0.08
Real Consumption (SPF)	0.36	0.31	0.25	-0.16	0.12	0.19	-0.21	0.10	0.04	-0.24	-0.48	-0.01
Real Non-Residential Investment (SPF)	0.81	0.32	0.01	-0.04	0.13	0.76	-0.08	0.12	0.51	-0.11	-0.43	0.11
Real Residential Investment (SPF)	1.41	0.42	0.00	0.06	0.11	0.62	-0.03	0.10	0.80	-0.03	-0.39	0.13
Real Federal Government Consumption (SPF)	-0.85	0.19	0.00	-0.64	0.07	0.00	-0.64	0.06	0.00	-0.58	-0.73	-0.48
Real State & Local Govt Consumption (SPF)	1.25	0.43	0.00	-0.36	0.06	0.00	-0.42	0.05	0.00	-0.34	-0.51	-0.26
Housing Start (SPF)	0.40	0.29	0.18	-0.23	0.09	0.01	-0.26	0.08	0.00	0.21	-0.11	0.38
Unemployment (SPF)	0.81	0.21	0.00	0.33	0.12	0.01	0.28	0.12	0.02	-0.27	-0.43	-0.10
Fed Funds Rate (BC)	0.61	0.23	0.01	0.20	0.09	0.03	0.18	0.09	0.06	0.22	0.08	0.37

3M Treasury Rate (SPF)	0.60	0.25	0.01	0.27	0.10	0.01	0.23	0.10	0.02	0.28	0.07	0.41
3M Treasury Rate (BC)	0.64	0.25	0.01	0.21	0.09	0.02	0.18	0.09	0.04	0.17	0.03	0.32
5Y Treasury Rate (BC)	0.03	0.22	0.88	-0.11	0.10	0.29	-0.18	0.10	0.08	-0.17	-0.31	-0.08
10Y Treasury Rate (SPF)	-0.02	0.27	0.95	-0.19	0.10	0.06	-0.23	0.09	0.01	-0.24	-0.37	-0.18
10Y Treasury Rate (BC)	-0.08	0.24	0.73	-0.18	0.11	0.11	-0.26	0.11	0.02	-0.29	-0.41	-0.17
AAA Corporate bond Rate (SPF)	-0.01	0.23	0.95	-0.22	0.07	0.00	-0.26	0.07	0.00	-0.32	-0.43	-0.19
AAA Corporate Bond Rate (BC)	0.21	0.20	0.29	-0.14	0.06	0.02	-0.18	0.06	0.00	-0.27	-0.42	-0.19
BAA Corporate Bond Rate (BC)	-0.18	0.27	0.50	-0.29	0.09	0.00	-0.33	0.09	0.00	-0.32	-0.46	-0.26

Table C3. Horizon h=0

This table shows the forecast error on forecast revision regressions, using forecast horizon h=0 for both forecast errors and forecast revisions. Columns (1) to (3) report results of the time series regressions using consensus (mean) forecast in each quarter. Columns (4) to (9) report results of individual level panel regressions. Columns (10) to (12) report results of forecaster-by-forecaster regressions (median in the data and the 2.5^{th} and 97.5^{th} percentile of the median based on block bootstrap).

	Consensus			Individual						By Forecaster		
				No fixed effects			With fixed effects			Median	p 2.5	p 97.5
	β_1	s.e.	*p*-val	β_1^p	s.e.	*p*-val	β_1^p	s.e.	*p*-val	β_1^i	β_1^i	β_1^i
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Nominal GDP (SPF)	-0.03	0.09	0.75	-0.36	0.10	0.00	-0.37	0.09	0.00	-0.31	-0.43	-0.16
Real GDP (SPF)	0.02	0.10	0.86	-0.32	0.10	0.00	-0.31	0.10	0.00	-0.25	-0.40	-0.09
Real GDP (BC)	0.19	0.15	0.22	-0.04	0.10	0.72	-0.07	0.10	0.47	-0.08	-0.22	0.01
GDP Price Index Inflation (SPF)	-0.17	0.10	0.09	-0.43	0.07	0.00	-0.46	0.06	0.00	-0.45	-0.60	-0.35
CPI (SPF)	0.52	0.08	0.00	0.04	0.13	0.76	0.03	0.14	0.84	0.03	-0.16	0.12
Real Consumption (SPF)	-0.03	0.15	0.83	-0.38	0.11	0.00	-0.39	0.11	0.00	-0.37	-0.60	-0.16
Industrial Production (SPF)	0.30	0.12	0.01	-0.20	0.10	0.05	-0.20	0.10	0.04	-0.17	-0.34	-0.01
Real Non-Residential Investment (SPF)	0.10	0.18	0.58	-0.34	0.13	0.01	-0.36	0.13	0.00	-0.35	-0.65	-0.09
Real Residential Investment (SPF)	0.70	0.17	0.00	-0.19	0.07	0.00	-0.21	0.06	0.00	-0.11	-0.43	0.03
Real Federal Government Consumption (SPF)	0.03	0.21	0.90	-0.56	0.11	0.00	-0.53	0.11	0.00	-0.54	-0.74	-0.40
Real State & Local Govt Consumption (SPF)	-0.24	0.29	0.39	-0.59	0.04	0.00	-0.61	0.04	0.00	-0.58	-0.71	-0.52

Housing Start (SPF)	0.22	0.11	0.06	-0.28	0.05	0.00	-0.26	0.05	0.00	0.09	-0.03	0.13
Unemployment (SPF)	0.25	0.04	0.00	0.09	0.03	0.00	0.08	0.03	0.01	-0.23	-0.34	-0.15
Fed Funds Rate (BC)	-0.02	0.03	0.55	-0.04	0.04	0.29	-0.04	0.04	0.26	-0.04	-0.21	0.01
3M Treasury Rate (SPF)	0.17	0.02	0.00	0.01	0.03	0.66	0.00	0.03	0.91	0.02	-0.04	0.09
3M Treasury Rate (BC)	0.01	0.02	0.72	-0.03	0.02	0.09	-0.04	0.02	0.06	-0.04	-0.06	-0.01
5Y Treasury Rate (BC)	0.12	0.04	0.00	0.03	0.03	0.36	0.02	0.03	0.55	0.02	-0.03	0.05
10Y Treasury Rate (SPF)	0.15	0.04	0.00	0.03	0.03	0.31	0.01	0.03	0.57	0.03	-0.02	0.05
10Y Treasury Rate (BC)	0.05	0.03	0.10	0.00	0.02	0.81	-0.01	0.02	0.65	0.01	-0.05	0.02
AAA Corporate bond Rate (SPF)	0.08	0.05	0.15	-0.07	0.03	0.01	-0.09	0.02	0.00	-0.08	-0.12	-0.02
AAA Corporate Bond Rate (BC)	-0.10	0.04	0.01	-0.12	0.03	0.00	-0.13	0.03	0.00	-0.10	-0.16	-0.06
BAA Corporate Bond Rate (BC)	0.05	0.03	0.16	-0.04	0.02	0.02	-0.05	0.02	0.01	-0.05	-0.08	-0.03

Table C4. Controlling for Lagged Deviation from Consensus

This table presents results of the individual level forecast error on forecast revision regressions, controlling for each individual's deviation from consensus forecasts in the last quarter $(x_{t+3|t-1}^{i} - x_{t+3|t-1})$. Coefficient on individual level forecast revisions are reported. Columns (1) to (6) show results of individual level panel regressions. Columns (7) to (9) report results of forecaster-by-forecaster regressions (median in the data and the 2.5th and 97.5th percentile of the median based on block bootstrap).

	Individual						By Forecaster		
	No fixed effects			With fixed effects			Med	p 2.5	p97.5
	β_1^p	s.e.	p-val	β_1^p	s.e.	p-val	β_1^i	β_1^i	β_1^i
Variable	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Nominal GDP (SPF)	-0.51	0.09	0.00	-0.54	0.09	0.00	-0.47	-0.31	0.06
Real GDP (SPF)	-0.42	0.11	0.00	-0.39	0.11	0.00	-0.25	-0.60	-0.12
Real GDP (BC)	-0.07	0.20	0.73	-0.18	0.18	0.32	-0.22	-0.62	-0.05
GDP Price Index Inflation (SPF)	-0.06	0.18	0.76	-0.21	0.14	0.14	-0.47	-0.68	-0.27
CPI (SPF)	-0.43	0.16	0.01	-0.45	0.17	0.01	-0.48	-0.65	-0.32
Real Consumption (SPF)	-0.53	0.12	0.00	-0.57	0.11	0.00	-0.48	-0.89	-0.38
Industrial Production (SPF)	-0.49	0.11	0.00	-0.48	0.11	0.00	-0.54	-0.67	-0.25
Real Non-Residential Investment (SPF)	-0.15	0.18	0.40	-0.17	0.16	0.30	-0.14	-0.62	0.12
Real Residential Investment (SPF)	-0.26	0.13	0.04	-0.35	0.11	0.00	-0.37	-0.73	-0.22
Real Federal Government Consumption (SPF)	-0.88	0.07	0.00	-0.88	0.07	0.00	-0.87	-1.04	-0.70
Real State & Local Govt Consumption (SPF)	-0.81	0.04	0.00	-0.83	0.04	0.00	-0.79	-0.92	-0.72
Housing Start (SPF)	-0.56	0.12	0.00	-0.57	0.11	0.00	0.03	-0.35	0.22
Unemployment (SPF)	0.18	0.15	0.25	0.16	0.16	0.31	-0.52	-0.70	-0.33
Fed Funds Rate (BC)	0.00	0.12	1.00	-0.01	0.12	0.94	0.09	-0.11	0.26
3M Treasury Rate (SPF)	0.07	0.13	0.61	0.05	0.13	0.71	0.14	-0.15	0.32
3M Treasury Rate (BC)	0.00	0.11	0.97	-0.01	0.11	0.92	0.03	-0.12	0.20
5Y Treasury Rate (BC)	-0.35	0.13	0.01	-0.38	0.12	0.00	-0.35	-0.52	-0.23
10Y Treasury Rate (SPF)	-0.48	0.12	0.00	-0.49	0.11	0.00	-0.49	-0.63	-0.34
10Y Treasury Rate (BC)	-0.41	0.14	0.00	-0.46	0.13	0.00	-0.45	-0.59	-0.32
AAA Corporate Bond Rate (SPF)	-0.59	0.09	0.00	-0.61	0.09	0.00	-0.67	-0.78	-0.43
AAA Corporate Bond Rate (BC)	-0.48	0.08	0.00	-0.49	0.08	0.00	-0.49	-0.69	-0.42
BAA Corporate Bond Rate (BC)	-0.19	0.28	0.49	-0.63	0.11	0.00	-0.59	0.11	0.00

D. Non-Normal Shocks and Particle Filtering

In the main text, we assume that both the innovations of the latent process, u_t, and the measurement error for each expert, ϵ_t, follow normal distributions. In this case, as all the posterior distributions are normal, the Kalman filter provides the closed form expression for the posterior densities. However, many processes for macro and financial variables may have heavy tails and more closely follow, for example, a t-distribution. In this appendix, we relax the normality assumption and verify the model predictions with fundamental shocks following fat tailed t-distributions.

In the non-normal case, while the point estimates of the Kalman filter still minimize mean-squared error (MSE), the mean and covariance estimates of the filter are no longer sufficient to determine the posterior distribution. Given that our formulation of diagnostic expectations involves a reweighting of the likelihood function, we require more than the posterior mean and variance to properly compute the diagnostic expectation distribution. Accordingly, we apply particle filtering to analyze expectations under non-normal shocks.

In Section D.1 we describe the sample-importance resample (SIR) algorithm underlying particle filtering. In Section D.2 we adapt it to the case of diagnostic expectations, and in Section D.3 we run it on on simulated data and find that the qualitative results of the model go through. Diagnostic expectations overreact to information, and if anything CG coefficients are more negative than under the normal case (both for individual and consensus). The estimation results using particle filtering are presented in Appendix F.

D.1 Particle Filtering: Motivation and Set-Up

We start with the processes in Equations (3) and (4):

$$s_t^i = x_t + \epsilon_t^i, \quad x_t = \rho x_{t-1} + u_t$$

where x_t is the fundamental process and s_t^i is forecaster i's noisy signal. In Section 3, the shocks to these processes are assumed to be normal. In the following, we analyze the case where the shock to the fundamental process u_t follows a t-distribution.

Since the t-distribution is no longer conjugate to normal noise, one can no longer get closed form solutions. Instead, we draw from the posterior distribution in a Monte Carlo approach using the particle filter, a popular algorithm for simulating Bayesian inference on Hidden Markov Models (Doucet, de Freitas, and Gordon, 2001; Doucet and Johansen 2011). We first briefly describe this approach, then formulate the application to diagnostic expectations, and finally show simulation results for the CG forecast error on forecast revision regressions.

Particle filtering builds on the idea of importance sampling. Specifically, suppose we wish to estimate the expectation of $f(x)$, where x is distributed according to p; we are not able to sample from p, or in general unable to compute its precise density, but can compute p up to a proportionality constant: $p(x) = \frac{1}{Z}\tilde{p}(x)$, where $Z = \int \tilde{p}(x)\, dx$ is the (unknown) normalizing constant. If we can sample from an arbitrary density q, we can use the following importance sampling mechanism to indirectly sample from p: for each sample from q, x_n, compute the importance weight $w_n = \frac{\tilde{p}(x_n)}{q(x_n)}$ and resample from $x_n$according to probability proportional to the weights. It is easy to see that the average of the weights estimates the proportionality factor Z : $\frac{1}{N}\sum_{n=1}^{N} w(x_n) \to \int \frac{\tilde{p}(x)}{q(x)} \cdot q(x)dx = \int \tilde{p}(x)\, dx = Z$. Consequently, one can easily derive that the resampled $x_n$converge in distribution to p: given any measurable function ϕ, the expectation of $\phi(x)$for the resampled x converges to $E_P\phi$: $\int \sum_{i=1}^{N} \phi(x_i) \frac{w(x_i)}{N}\frac{q(x_{1:N})}{Z}\, dx_{1:N} = \frac{1}{Z}\frac{1}{N}\sum_{i=1}^{N} \int \phi(x_i) \frac{\tilde{p}(x_i)}{q(x_i)} q(x_i)q(x_{-i})\, dx_{1:N} = \frac{1}{N}\sum_{i=1}^{N} E_p[\phi(x)] = E_p\phi$

The algorithm above, called the sample-importance resample (SIR) algorithm, can be summarized in the following steps:

1. Sample N particles from q, denoted as $x_{1:N}$
2. For each x_i, compute$w_i = \frac{\tilde{p}(x_i)}{q(x_i)}$.
3. Resample according to probability $\propto w_i$

For the hidden Markov Process model, the above idea generalizes to give us a quick algorithm to sample from the filtering density $p(x_n|s_{1:n})$. Like the Kalman filter, the idea is to proceed inductively, using the following forward equation:

$$p(x_n|s_{1:n}) = \frac{g(s_n|x_n)\, p(x_n|s_{1:n-1})}{p(s_n|\, s_{1:n-1})} = \frac{\int g(s_n|x_n)\, f(x_n|x_{n-1})\, p(x_{n-1}|s_{1:n-1}) ds_{1:n-1} dx_{n-1}}{p(s_n|s_{1:n-1})}$$

By induction, suppose that we have samples from the previous filtered distribution $p(x_{n-1}|s_{1:n-1})$. Now, given a (conditional) proposal $q(x_n|x_{n-1}, s_{1:n})$for each sample, the recursive equality above suggests the resampling weights: $w(x_n \mid x_{n-1}) = \frac{g(s_n|x_n) f(x_n|x_{n-1})}{q(x_n \mid x_{n-1}, s_{1:n})}$. For the base case, where we have only seen the data point s_1, our filtered density $p(x_1|s_1)$is the standard Bayesian posterior, which can be sampled via importance sampling.

The particle filter algorithm refers to this extension of the SIR algorithm to the sequential setting. The procedure is as follows:

1. At time n = 1, generate Ni.i.d. samples from a default proposal q.
2. Compute for each sample the weights $w(x_i) = \frac{\mu(x_i)\, g(s_1 \mid x_i)}{q(x_i)}$
3. Resample according to the weights, and store the sample.
4. For $n \geq 2$: for each x_{n-1}^i in the sample, propose x_n^i according to $q(x_n|x_{n-1} = {x^i}_{n-1}, s_{1:n})$
5. Compute for each x_n^i the weights $w({x_n}^i) = \frac{g(s_n|{x_n}^i)\, f({x_n}^i|{x_{n-1}}^i)}{q(x_n|x_{n-1} = {x^i}_{n-1}, s_{1:n})}$
6. Resample according to the weights, save as x_n^i.

Finally, we need to specify the proposal density $q(x_n|x_{n-1} = {x^i}_{n-1}, s_{1:n})$. It is well-known that the optimal proposal density should be the conditional distribution $p(x_n|x_{n-1} = {x^i}_{n-1}, s_n)$. If the latent Markov process is a simple AR(1) process with normal innovation, one can analytically derive the optimal proposal density $p(x_n|x_{n-1} = {x^i}_{n-1}, s_n)$.

$$x_n|x_{n-1}, s_n \sim N(\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_u^2} \rho\, x_{n-1} + \frac{\sigma_u^2}{\sigma_\epsilon^2 + \sigma_u^2}\, s_n, \frac{\sigma_\epsilon^2 \sigma_u^2}{\sigma_\epsilon^2 + \sigma_u^2}) = N(\bar{\mu}, \bar{\Sigma})$$

While this result is only precise for normal processes, we shall still use $\bar{\mu}, \bar{\Sigma}$as location and scale parameters for our proposal, which is now t-distributed. If the original innovations have $d$degrees of freedom, our proposal will have $\frac{d+2}{2}$degrees of freedom, which have much thicker tails.

D.2 Application to Diagnostic Expectations

To analyze the case of diagnostic expectations, we only need to re-adjust the resampling weights by a simple likelihood ratio, as given by the following proposition:

Proposition D1 *Let $s^*(s_{1:n-1})$ be the predictive expectation of s_n given $s_{1:n-1}$. The representativeness $R(x_n|s_{1:n}) = \frac{p(x_n|\, s_{1:n})}{p(x_n|\, s_{1:n-1}, s^*)}$ can be simplified to the likelihood ratio $\frac{g(s_n|x_n)}{g(s^*|x_n)}$, up to a proportionality constant independent of x_n.*

Proof. By Bayes' rule: $R(x_n|s_{1:n}) = \frac{p(x_n|\, s_{1:n})}{p(x_n|\, s_{1:n-1}, s^*)} = \frac{p(s_n|s_{1:n-1}, x_n)\cdot p(x_n|\, s_{1:n-1})}{p(s_n\,|\, s_{1:n-1})} \cdot (\frac{p(s^*\,|\, s_{1:n-1})\cdot p(x_n|\, s_{1:n-1})}{p(s^*|\, s_{1:n-1})})^{-1}$.

Due to the Markov property, $p(s_n|s_{1:n-1}, x_n) = g(s_n|x_n)$ and $p(s_n = s^*\,|s_{1:n-1}, x_n) = g(s^*|x_n)$. Plugging this in, we obtain:

$$R(x_n\,|s_{1:n}) = \frac{g(s_n\,|\,x_n)\cdot p(x_n|s_{1:n-1})}{p(s_n|s_{1:n-1})}\cdot(\frac{g(s^*|x_n)\cdot p(x_n\,|\,s_{1:n-1})}{p(s^*|s_{1:n-1})})^{-1} = \frac{g(s_n|x_n)}{g(s^*\,|x_n)}\cdot\frac{p(s^*|s_{1:n-1})}{p(s_n|s_{1:n-1})}$$

The latter term $\frac{p(s^*|s_{1:n-1})}{p(s_n|s_{1:n-1})}$ is constant with respect to x_n, as desired.

As we have assumed that g is a normal density, the likelihood ratio simplifies to:

$$R(x_n|s_{1:n}) \propto exp\left(-\frac{(x_n - s_n)^2}{2\sigma_\epsilon{}^2} + \frac{(x_n - s^*)^2}{2\sigma_\epsilon{}^2}\right) = exp\left(\frac{(s_n - s^*)x_n}{\sigma_\epsilon{}^2}\right)$$

Hence, if the observed signal s_n is greater than s^*(a positive news), the forecaster puts an exponentially heavier weight on larger values of x_n, and for negative news, he overweights smaller values of x_n, which is in line with over-reaction to most recent news.

With the particle filter, we get the exponential reweighting by multiplying the original weights $w(x_n{}^i) = \frac{g(s_n|x_n{}^i)\, f(x_n{}^i|x_{n-1}{}^i)}{q(x_n|x_{n-1}=x^i{}_{n-1}, s_{1:n})}$ with the extra exponential factor $exp(\frac{(s_n - s^*)x_n}{\sigma_\epsilon{}^2})$. As with the basic particle filter algorithm discussed above, we need to specify our proposal density q to target regions of high density. We would like to target $\tilde{q} \propto exp(\frac{(s_n - s^*)x_n}{\sigma_\epsilon{}^2})p(x_n|x_{n-1,}s_n)$, which we estimate by first assuming the normal model. Given that $x_n|x_{n-1}, s_n \sim N(\bar{\mu}, \bar{\Sigma})$ in the normal model, the diagnostic expectation is given by a shift of the posterior density by $\frac{\theta\cdot\bar{\Sigma}\cdot(s_n - s^*)}{\sigma_\epsilon{}^2}$. Thus we set the location and scale parameter of our proposals as $\mu_{diag} = \bar{\mu} + \frac{\theta\cdot\bar{\Sigma}(s_n - s^*)}{\sigma_\epsilon{}^2}$, $\Sigma_{diag} = \bar{\Sigma}$, where $\bar{\mu}, \bar{\Sigma}$ are the location and scale parameters for our original proposal. As before, we have $df_q = \frac{df+2}{2}$ to ensure that our proposal has heavier tails than the target distribution. To summarize, the algorithm is as follows:

1. From the original particle filter, estimate $s^* = \rho\mu_{n-1}$, with μ_{n-1} our predictive mean $E[x_{n-1}|\, s_{1:n-1}]$, estimated by the mean of our particles $x^i{}_{n-1}$.
2. Propose according to a t-distribution with location parameter $\mu_{diag} = \bar{\mu} + \frac{\theta \cdot \bar{\Sigma}(s_n - s^*)}{\sigma_\epsilon{}^2}$, $\Sigma_{diag} = \bar{\Sigma}, \quad df_q = \frac{df+2}{2}$.
3. For each proposal, resample with weights$w_{diag}(x_n|x_{n-1,}s_n) =$ $\frac{g(s_n|x_n{}^i)\, f(x_n{}^i|x_{n-1}{}^i)}{q(x_n|x_{n-1}=x^i{}_{n-1,}s_{1:n})} exp(\frac{(s_n - s^*)x_n}{\sigma_\epsilon{}^2})$

D.3 Simulations

In the simulations below, we set $\rho = 0.9, \sigma_u = 0.2, \sigma_\epsilon = 0.2$, and $0 \le \theta \le 1.5$. We find that the basic qualitative characteristics of diagnostic expectations are robust to fat tails. As Figure D1 shows, the diagnostic expectation over-reacts to news, relative to the rational benchmark.

We then check the results of the CG forecast error on forecast revision regressions. Figure D2 shows the distribution of bootstrapped regression coefficients. Panel A first checks the case with normal shocks, the particle filter simulation agrees with the predicted coefficients $-\frac{\theta(1+\theta)}{(1+\theta)^2 + \theta^2\rho^2}$ using the Kalman filter. Panel B then shows the case where the shocks are heavy-tailed. We see that the coefficients for the heavy-tailed shocks are more negative compared to the predicted values for the normal case. Specifically, as the rational posterior exhibits heavier tail, the exponential reweighting of the diagnostic expectation results in greater mass located on the extreme values of the exponential weight, and hence greater shift in the diagnostic expectation. This effect is only present for diagnostic expectations — for rational expectations i.e. $\theta = 0$, we do not observe a divergence between normal and fat tailed distributions.

Figure D1. Response to News under Rational and Diagnostic Expectations

This plot shows the belief distribution in response to news, with fat tailed fundamental shocks and particle filtering. The black line plots the distribution with no news. The dashed red line plots the distribution in response to news with rational expectations. The dotted blue line plots the distribution in response to news with diagnostic expectations.

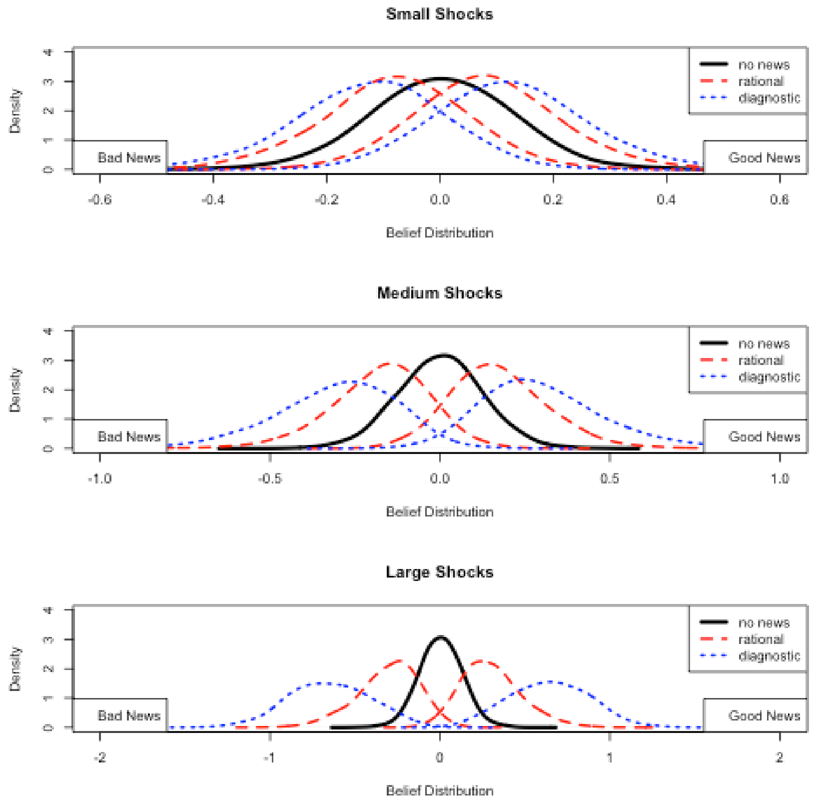
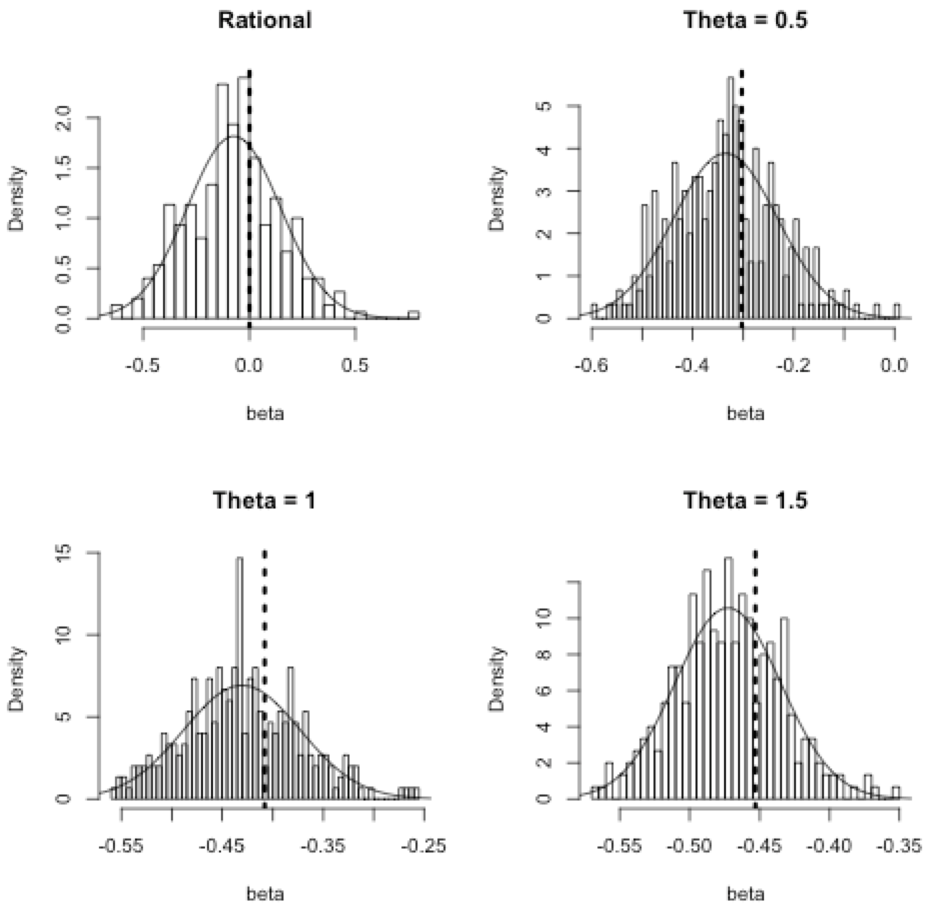


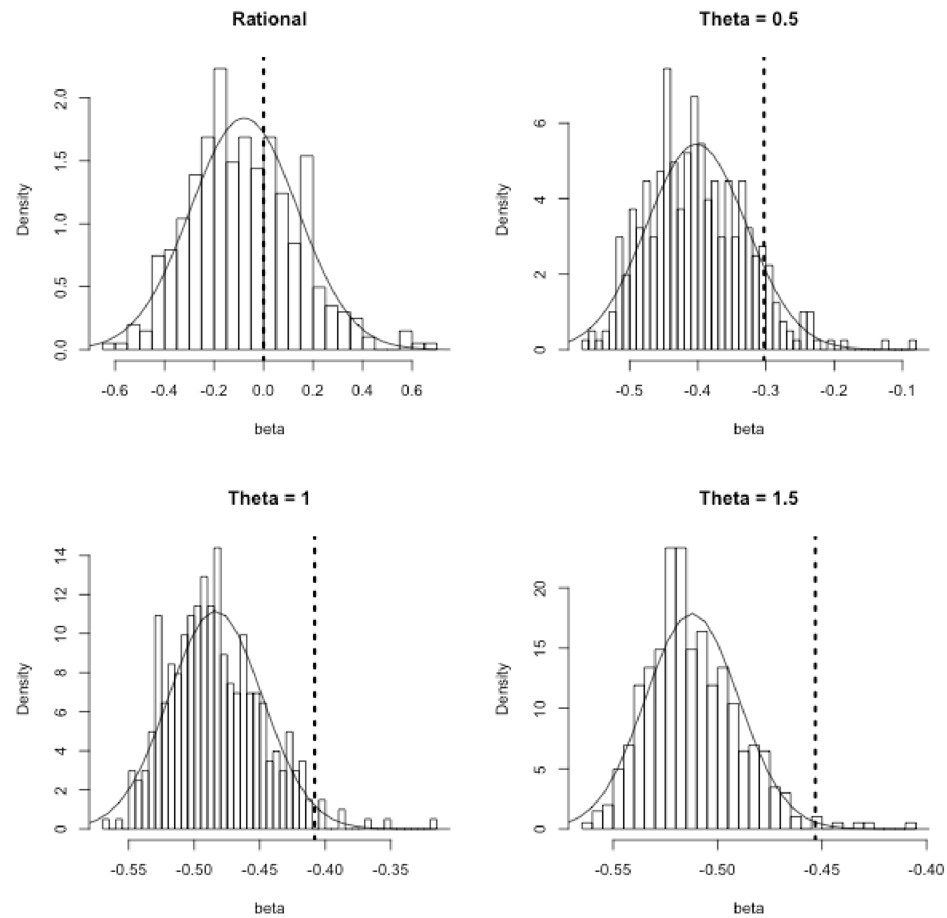
Figure D2. Individual CG Coefficients with Normal and Fat Tailed Shocks

This plot shows the distribution of coefficients from individual level (pooled panel) CG regressions. Panel A analyzes the case for normal shocks and Panel B analyzes the case for fat tailed shocks, both using the particle filter. Each simulation has 80 time periods and each plot shows the coefficients from 300 simulations. The dashed vertical line indicates $-\frac{\theta(1+\theta)}{(1+\theta)^2+\theta^2\rho^2}$, which is the coefficient predicted by normal shocks and Kalman filtering.

Panel A. Normal Shocks



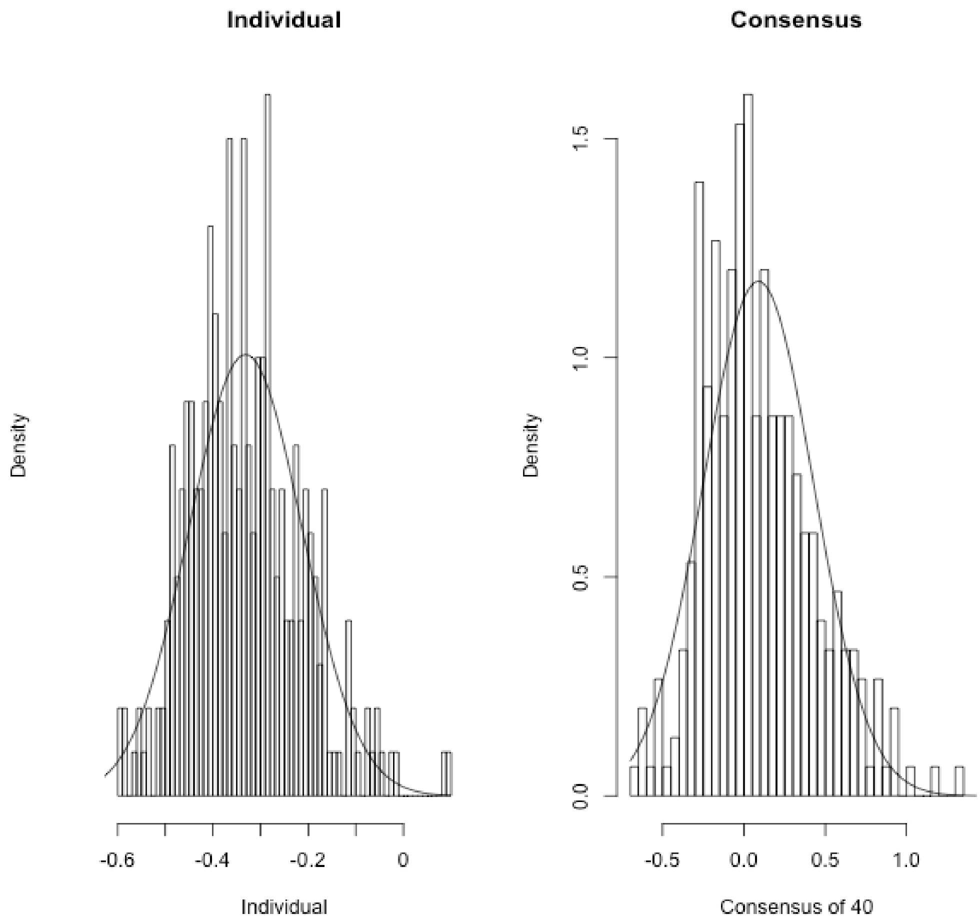
Panel B. Fat Tailed Shocks, df = 2.5



Finally, Figure D3 replicates the results for the contrast between regressions using individual and consensus forecasts. The general qualitative result is that there is much less over-reaction in consensus forecasts. On average, we get slight under-reaction in consensus forecasts. Under-reaction occurs when the noise ${\sigma_\epsilon}^2$is sufficiently high and individual over-reaction parameter θ is sufficiently low. Figure D3 plots the case where $\sigma_\epsilon = 1, \theta = 0.1$, which shows robustly positive consensus regression coefficients for 40 forecasters and 80 time periods.

Figure D3. Individual vs. Consensus Diagnostic Expectations

This plot shows the distribution of coefficients from individual level (pooled panel) and consensus CG regressions, using fat tailed shocks and particle filtering. The left panel shows the coefficients from pooled individual level regressions, and the right panel shows the coefficients from consensus regressions. Each draw has 40 forecasters and 80 time periods; there are 300 draws.



E. Kernel of Truth Assessment

We develop two tests of the kernel of truth property of Diagnostic Expectations. In Section E.1, we run a cross sectional test based on the persistence of the different series, which allows us to compare Diagnostic Expectations with Adaptive Expectations. In Section E.2 we assess whether, for series that feature hump-shaped dynamics, beliefs over-react both to short-term news and to longer-term reversals.

E.1 Kernel of Truth in the Cross Section: Persistence Tests

Under Noisy Rational and Diagnostic Expectations, forecast revision at t satisfies:

$$x^i_{t+h|t} - x^i_{t+h|t-1} = \rho\left(x^i_{t+h-1|t} - x^i_{t+h-1|t-1}\right).$$

The revision h periods ahead reflects the forecast revision about the same variable $h-1$ periods ahead, adjusted by the persistence ρ of the series. The idea is simple: when forecasts are forward looking, more persistent series should witness more correlated revisions across different forecast horizons.

Under Adaptive Expectations, in contrast, updating is mechanical and should not depend on the true persistence of the forecasted process. Formally, in this case:

$$x^i_{t+h|t} - x^i_{t+h|t-1} = \mu\left(x^i_{t+h-1|t} - x^i_{t+h-1|t-1}\right),$$

where μ is a positive constant independent of ρ. [2]

To assess this prediction, we fit an AR(1) for the actuals of each series and estimate ρ. The actuals have the same format as the forecast variables, and we use the exact time period for which the forecasts are available. The estimates of ρ are presented in Figure E1. We run the following individual level regression using forecast revisions for different horizons:

$$x^i_{t+3|t} - x^i_{t+3|t-1} = \gamma^p_o + \gamma^p_1\left(x^i_{t+2|t} - x^i_{t+2|t-1}\right) + \epsilon^i_{t+3},$$

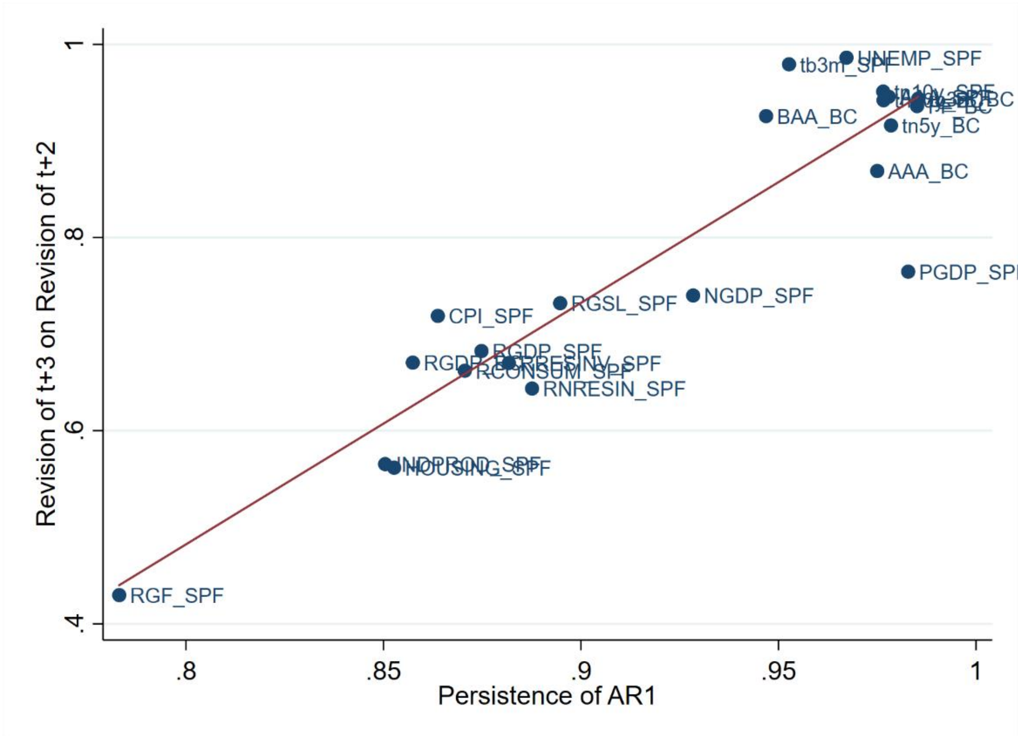
and repeat the same specification at the consensus level.

[2] This formula is based on the Error-Learning model, a generalization of adaptive expectations for longer horizons (Pesaran and Weale 2006). This model postulates $x^i_{t+s|t} - x^i_{t+s|t-1} = \mu_s\left(x_t - x^i_{t|t-1}\right)$, so that $\mu = \mu_h/\mu_{h-1}$.

Finally, we study the relationship between the slope coefficient γ_1^p and the persistence ρ of each series. The results are reported in Figure E1, which show that the more persistent series display more correlated forecast revisions. While we only have 22 series (18 different variables), the correlation is statistically different from zero with a p-value less than 0.001. [3] In line with forward-looking models, forecasters see more persistent impact of news for more persistent series. The positive relationship between the slope coefficient γ_1^p and the persistence ρ of each series depends only on the first autocorrelation lag, and so holds also for series with richer dynamics than AR(1). This evidence is inconsistent with adaptive expectations, in which updating does not depend on persistence, in which case the line in Figure E1 should be flat.

Figure E1. Properties of Forecast Revisions and Actuals

The y-axis is the coefficient γ_1^p from regression $x_{t+3|t}^i - x_{t+3|t-1}^i = \gamma_o^p + \gamma_1^p(x_{t+2|t}^i - x_{t+2|t-1}^i) + \epsilon_{t+3}^i$. The x-axis is the persistence measured from an AR(1) regression of the actuals corresponding to the forecasts. For each variable, the AR(1) regression uses the same time period as when the forecast data is available.



[3] The results in Figure E1 are similar if we exclude the Blue Chip series that are also available in SPF (e.g. real GDP, 3-month Treasuries, 10-year Treasuries, AAA corporate bond rate).

E.2. Kernel of Truth in the Time Series

We now allow the forecasted series to be described by an AR(2) process. As shown by Fuster, Laibson and Mendel (2010), several macroeconomic variables follow hump-shaped dynamics with short-term momentum and longer-term reversals. Considering this possibility is relevant for two reasons. First, under the kernel of truth, forecasters should exaggerate true features of the data generating process, including the presence of long-term reversals. Taking longer term dynamics into account may thus lead to a clearer picture of overreaction relative to Table 3. Second, this analysis also allows us to compare Diagnostic Expectations to the model of Natural Expectations proposed by Fuster, Laibson and Mendel (2010), in which agents forecast an AR(2) process "as if" it was AR(1) in changes, and in particular neglect longer lags.

E.2.1 Diagnostic Expectations with AR(2) Processes

Suppose that forecasters seek to forecast an AR(2) process:

$$x_{t+3} = \rho_2 x_{t+2} + \rho_1 x_{t+1} + u_{t+3}. \qquad (E1)$$

If $\rho_2 > 0$ and $\rho_1 < 0$, the variable displays short-term momentum and long-term reversal. Each forecaster now observes two signals, one about the current state $s^i_{t,t} = x_t + \epsilon^i_t$ and another about the past state $s^i_{t-1,t} = x_{t-1} + v^i_t$. The presence of two signals implies that the current forecast revisions for x_{t+1} and x_{t+2} are not perfectly collinear, which is necessary for out test.

We now show that diagnostic forecasts about $t+1$ and $t+2$ overweigh each signal, so that forecast revisions are excessive. First note that the diagnostic forecast about $t+3$ can be written $x^{i,\theta}_{t+3|t} = x^i_{t+3|t} + \theta FR^i_{t+3|t}$, where $FR^i_{t+3|t} = \left(x^i_{t+3|t} - x^i_{t+3|t-1}\right)$. Similar to rational expectations, the diagnostic forecast of x_{t+3} is then a linear combination of the forecasts of x_{t+2} and x_{t+1} with weights given by the autoregressive parameters ρ_1 and ρ_2:

$$x^{i,\theta}_{t+3|t} = \rho_2 x^{i,\theta}_{t+2|t} + \rho_1 x^{i,\theta}_{t+1|t}.$$

This formula suggests a way to test for over-reaction, generalizing Equation (2) to AR(2). To do so, simply predict forecast errors in the long term using forecast revisions about shorter term:

$$x_{t+3} - x^i_{t+3|t} = \delta^p_0 + \delta^p_2 FR^i_{t,t+2} + \delta^p_1 FR^i_{t,t+1} + \epsilon_{t,t+h}, \qquad (E2)$$

where $FR^i_{t,t+1}$ and $FR^i_{t,t+2}$ stand for the surveyed forecast revisions at for $t+1$ and $t+2$, respectively. Under diagnostic expectations, estimates of (12) satisfy the following property.

Proposition 3. *Under the Diagnostic Kalman filter, the estimated coefficients $\hat{\delta}_1^p$ and $\hat{\delta}_2^p$ in Equation $(E2)$ are proportional to the negative of the AR(2) coefficients:*

$$\hat{\delta}_1^p \propto -\rho_1\theta, \qquad (E3)$$

$$\hat{\delta}_2^p \propto -\rho_2\theta. \qquad (E4)$$

Proof. The diagnostic forecast revision $FR^{i,\theta}_{t+3|t} = \left(x^{i,\theta}_{t+3|t} - x^{i,\theta}_{t+3|t-1}\right)$ is equal to:

$$FR^{i,\theta}_{t+3|t} = (1+\theta)FR^i_{t+3|t} - \theta FR^i_{t+3|t-1}.$$

The diagnostic forecast error $FE^{i,\theta}_{t+3|t} \equiv x_{t+3} - x^{i,\theta}_{t+3|t}$ is equal to:

$$FE^{i,\theta}_{t+3|t} = u_{t+3} - \theta FR^i_{t+3|t},$$

where u_{t+3} is white noise. We then have:

$$cov\left(FE^{i,\theta}_{t+3|t}, FR^{i,\theta}_{t+3|t}\right) = -\theta cov\left(FR^i_{t+3|t}, (1+\theta)FR^i_{t+3|t} - \theta FR^i_{t+3|t-1}\right)$$

$$= -\theta(1+\theta)var\left(FR^i_{t+3|t}\right)$$

$$var\left(FR^{i,\theta}_{t+3|t}\right) = (1+\theta)^2 var\left(FR^i_{t+3|t}\right) + \theta^2 var\left(FR^i_{t+3|t-1}\right).$$

As a result, the relationship between forecast error and forecast revision is equal to:

$$FE^{i,\theta}_{t+3|t} = -\frac{\theta(1+\theta)}{(1+\theta)^2 + \theta^2 \frac{var\left(FR^i_{t+3|t-1}\right)}{var\left(FR^i_{t+3|t}\right)}} FR^{i,\theta}_{t+3|t} + v_{t+3}$$

By plugging Equation (13) in the text, we obtain:

$$FE^i_{t+3|t} = -\frac{\rho_2\theta(1+\theta)}{(1+\theta)^2 + \theta^2 \frac{var\left(FR^i_{t+3|t-1}\right)}{var\left(FR^i_{t+3|t}\right)}} FR^i_{t+2|t} - \frac{\rho_1\theta(1+\theta)}{(1+\theta)^2 + \theta^2 \frac{var\left(FR^i_{t+3|t-1}\right)}{var\left(FR^i_{t+3|t}\right)}} FR^i_{t+1|t}$$
$$+ v_{t+3},$$

If $FR^i_{t+2|t}$ and $FR^i_{t+1|t}$ are not collinear, the above equation can be estimated and it satisfies the prediction of Proposition 3. To conclude the proof, we therefore need to prove non-collinearity. Recall that the state follows AR(2) dynamics:

$$x_{t+1} = ax_t + bx_{t-1} + u_{t+1},$$

At time t, the agent observes two signals, one about the current state, $s^i_t = x_t + \epsilon^i_t$, and one about the past state $z^i_t = s^i_{t-1,t} = x_{t-1} + v^i_t$. Signals ϵ^i_t and v^i_t are normal with precision ϵ and v. At time t, the agent forms estimates about x_t and x_{t-1}. He then combines them to forecast about x_{t+k}, $k \geq 1$.

To ease notation we drop superscripts i from the noise and the signals and subscript t from the signals. Conditional on the signals, the density of the current state $f(x_t, x_{t-1}|s_t, z_t)$ satisfies:

$$-\ln f \propto \epsilon(1-\varphi^2)(s_t - x_t)^2 + v(1-\varphi^2)(z_t - x_{t-1})^2 + \left(x_t - x_{t|t-1}\right)^2 p + \left(x_{t-1} - x_{t-1|t-1}\right)^2 q - 2\varphi\sqrt{pq}\left(x_t - x_{t|t-1}\right)\left(x_{t-1} - x_{t-1|t-1}\right)$$

where p is the precision of x_t, q is the precision of x_{t-1}, and φ is their correlation.

Maximizing the likelihood f with respect to x_t and x_{t-1} yields the first order conditions:

$$-2\epsilon(1-\varphi^2)\left(s_t - x_{t|t}\right) + 2p\left(x_{t|t} - x_{t|t-1}\right) - 2\varphi\sqrt{pq}\left(x_{t-1|t} - x_{t-1|t-1}\right) = 0$$

$$-2v(1-\varphi^2)\left(z_t - x_{t-1|t}\right) + 2q\left(x_{t-1|t} - x_{t-1|t-1}\right) - 2\varphi\sqrt{pq}\left(x_{t|t} - x_{t|t-1}\right) = 0$$

which identify the conditional estimates (the Kalman filter):

$$x_{t|t} = \frac{(1-\varphi^2)\frac{\epsilon}{p}s_t + x_{t|t-1} + \varphi\sqrt{\frac{q}{p}}FR_{t-1|t}}{(1-\varphi^2)\frac{\epsilon}{p} + 1},$$

$$x_{t-1|t} = \frac{(1-\varphi^2)\frac{v}{q}z_t + x_{t-1|t-1} + \varphi\sqrt{\frac{p}{q}}FR_{t|t}}{(1-\varphi^2)\frac{v}{q} + 1},$$

Where $FR_{s|t}$ is the forecast revision at t for x_s. This further implies that:

$$FR_{t|t} = \frac{(1-\varphi^2)\frac{\epsilon}{p}\left(s_t - x_{t|t-1}\right) + \varphi\sqrt{\frac{q}{p}}FR_{t-1|t}}{(1-\varphi^2)\frac{\epsilon}{p} + 1},$$

$$FR_{t-1|t} = \frac{(1-\varphi^2)\frac{v}{q}\left(z_t - x_{t-1|t-1}\right) + \varphi\sqrt{\frac{p}{q}}FR_{t|t}}{(1-\varphi^2)\frac{v}{q} + 1}.$$

These equations imply that, provided $\varphi < 1$, the forecast revisions $FR_{t|t}$ and $FR_{t-1|t}$ are linearly independent combinations of the news $s_t - x_{t|t-1}$ and $z_t - x_{t-1|t-1}$:

$$FR_{t|t} = \frac{\left[(1-\varphi^2)\frac{v}{q} + 1\right]\frac{\epsilon}{p}\left(s_t - x_{t|t-1}\right) + \varphi\sqrt{\frac{1}{qp}}v\left(z_t - x_{t-1|t-1}\right)}{\left[(1-\varphi^2)\frac{v}{q} + 1\right]\frac{\epsilon}{p} + \frac{v}{q} + 1},$$

$$FR_{t-1|t} = \frac{\left[(1-\varphi^2)\frac{\epsilon}{p} + 1\right]\frac{v}{q}\left(z_t - x_{t-1|t-1}\right) + \varphi\sqrt{\frac{1}{qp}}\epsilon\left(s_t - x_{t|t-1}\right)}{\left[(1-\varphi^2)\frac{\epsilon}{p} + 1\right]\frac{v}{q} + \frac{\epsilon}{p} + 1}.$$

Therefore, $FR^i_{t|t}$ and $FR^i_{t-1|t}$ are not collinear. Since $FR^i_{t+1|t} = aFR^i_{t|t} + bFR^i_{t-1|t}$ and $FR^i_{t+2|t} = (a^2+b)FR^i_{t|t} + abFR^i_{t-1|t}$, we conclude that $FR^i_{t+2|t}$ and $FR^i_{t+1|t}$ are not collinear. ■

Once again, under rational expectations ($\theta = 0$) individual forecast errors cannot be predicted from any forecast revisions. Due to the kernel of truth property, diagnostic expectations instead imply that forecast errors are predictable, and in particular negatively predictable from revisions (that is, relative to the data generating process). Over-reaction to short term news, $\rho_2 > 0$, implies that upward forecast revisions about x_{t+2} lead to exaggerated optimism about x_{t+3} and thus negative forecast errors. This yields $\hat{\delta}_2^p < 0$. On the other hand, over-reaction to long-term reversal, $\rho_1 < 0$, implies that upward forecast revisions about x_{t+1} lead to exaggerated pessimism about x_{t+3} and thus positive forecast errors, yielding $\hat{\delta}_1^p > 0$.[4]

Before moving to the data, we link this discussion to the model of Natural Expectations, which was proposed to account for expectational errors in AR(2) settings. In this model, forecasters fit to the AR(2) data a process that is AR(1) in changes. Formally, they use the rule $(x_{t+1} - x_t) = \varphi(x_t - x_{t-1}) + v_{t+1}$ with fitted coefficient $\varphi = (\rho_2 - \rho_1 - 1)/2$. For a stationary AR(2) process (i.e. if $\rho_2 - \rho_1 < 1$, $\rho_1 + \rho_2 < 1$ and $|\rho_1| < 1$) this implies that forecasters exaggerate short term momentum and dampen long term reversals. This model entails an exaggeration of the short run persistence of the series and, similarly to Diagnostic Expectations, yields negative predictability of forecast errors at this horizon. On the other hand, Natural Expectations also dampen long-term reversals, contrary to our prediction of over-reaction to long-term reversals ($\hat{\delta}_1^p > 0$). Thus, the two models predict overlapping but distinguishable patterns of predictable forecast errors (not however that Natural Expectations cannot be directly estimated using Equation (12) because it implies that the two forecast revisions are perfectly collinear.)

In the remainder of the section, we test the predictions of Proposition 3.

E.2.2 AR(1) vs AR(2) Dynamics

As a first step, we assess which of our 18 variables is more accurately described by AR(2) rather than AR(1). We do not aim to find the unconstrained optimal ARMA(k, q) specification, which is well

[4] Proposition 3 also implies that the tests of Section 3 may be biased toward finding under-reaction when the AR(2) process has $\rho_2 > 0$ and $\rho_1 < 0$. Positive news at t may then trigger an upward revision of the forecasts for both x_{t+1} and x_{t+2}. The former creates excess pessimism, the latter excess optimism. If the first effect is strong, the test of Section 3 may detect excess pessimism after good news, giving a false impression of under-reaction.

known to be difficult. We only wish to capture the simplest longer lags and see whether expectations react to them as predicted by the model. We fit a quarterly AR(2) process for our 22 series. Figure E2 below plots the estimates for ρ_1 and ρ_2.[5] As before, the actuals have the same format as the forecast variables, and for each series the regression covers the time period when the forecast data are available.

The signs of coefficients point to a positive momentum at short horizons ($\rho_2 > 0$) for all series, and to long-run reversals ($\rho_1 < 0$) for most series, the remaining ones having ρ_1 approximately zero.[6] To assess which dynamics better describe the series, we compare the AR(2) estimates to the AR(1) estimates from Section 5.1. Table E1 shows the Bayesian Information Criterion (BIC) score associated with each fit. For the majority of series, AR(2) is favored over AR(1). The tests favor AR(1) dynamics only for real consumption (SPF), Government consumption (state and federal) and the BAA bond rate (BC), while for the 10-year Treasury rate series the tests are inconclusive.[7] In sum, hump shaped dynamics are a key feature of several series.

Figure E2. AR(2) Coefficients of Actuals

For each variable, the AR(2) regression uses the same time period as when the forecast data is available. The blue circles show the first lag and the red diamonds show the second lag. Standard errors are Newey-West, and the vertical bars show the 95% confidence intervals.

[5] Just like for the case of AR(1), for growth variables we run quarterly AR(2) regressions of growth from $t-1$ to $t+3$. For variables in levels, we run quarterly regressions in levels. We run separate regressions for the variables that occur both in SPF and BC, because they cover slightly different time periods.

[6] We check whether multicollinearity may affect our results in this Section, given that forecasts revisions at different horizons are often highly correlated. The standard issue with multicollinearity is the coefficients are imprecisely estimated, which we do not find to be the case. We also perform simulations to verify that the correlation among the right hand side variables by itself does not mechanically lead to the patterns we observe.

[7] The Akaike Information Criterion (AIC) yields similar results, except that it positively identifies the TN10Y (SPF) series as AR(2). To interpret the IC scores, recall that lower scores represent a better fit. The likelihood ratio $\frac{\Pr(AR2)}{\Pr(AR1)}$ is estimated as $exp\left[-\frac{BIC_{AR2}-BIC_{AR1}}{2}\right]$, so that $\Delta BIC_{2-1} = -2$ means the AR(2) model is 2.7 times more likely than the AR(1) model.

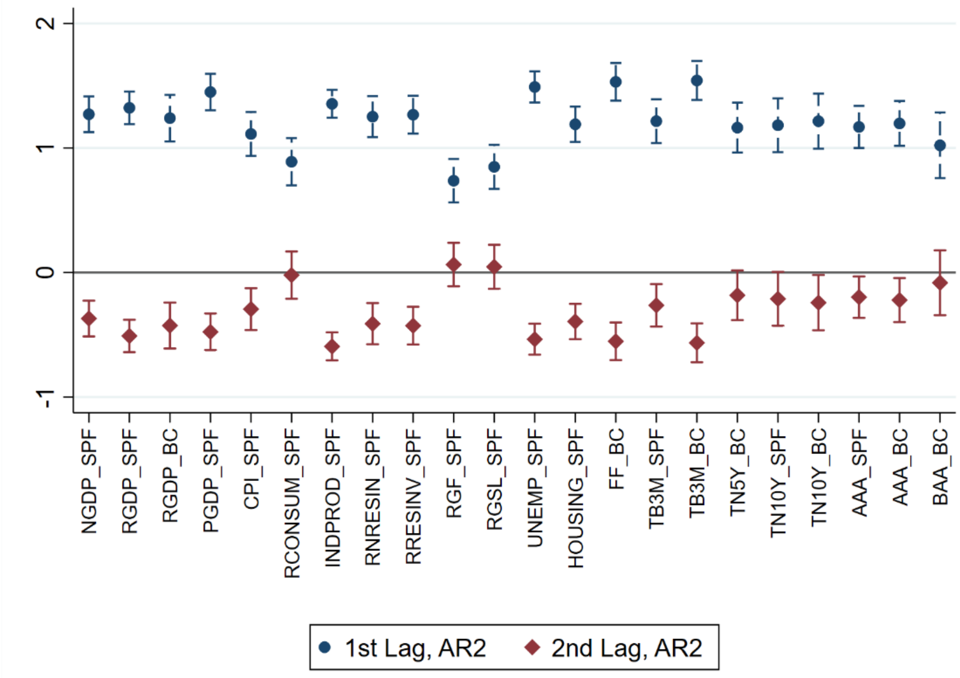


Table E1. BIC of AR(1) and AR(2) Regressions of Actuals

This table shows the BIC statistic corresponding to the AR(1) and AR(2) regressions of the actuals. The final column shows the specification that has a lower BIC (preferred).

Variable	BIC_{AR1}	BIC_{AR2}	$\Delta BIC_{2\text{-}1}$	Model
Nominal GDP (SPF)	-1189.74	-1205.30	-15.56	AR(2)
Real GDP (SPF)	-1176.39	-1222.01	-45.61	AR(2)
Real GDP (BC)	-671.10	-679.81	-8.70	AR(2)
GDP Price Index Inflation (SPF)	-1492.75	-1527.69	-34.95	AR(2)
CPI (SPF)	-1008.02	-1017.30	-9.28	AR(2)
Real Consumption (SPF)	-987.61	-974.69	12.92	AR(1)
Industrial Production (SPF)	-863.32	-935.37	-72.05	AR(2)
Real Non-Residential Investment (SPF)	-547.67	-563.73	-16.07	AR(2)
Real Residential Investment (SPF)	-404.80	-432.10	-27.30	AR(2)
Real Federal Government Consumption (SPF)	-602.66	-594.83	7.83	AR(1)
Real State&Local Govt Consumption (SPF)	-961.46	-951.77	9.69	AR(1)
Housing Start (SPF)	170.22	109.46	-60.76	AR(2)
Unemployment (SPF)	-277.73	-302.34	-24.61	AR(2)
Fed Funds Rate (BC)	195.12	150.76	-44.37	AR(2)
3M Treasury Rate (SPF)	247.57	238.90	-8.67	AR(2)
3M Treasury Rate (BC)	163.88	118.39	-45.49	AR(2)
5Y Treasury Rate (BC)	140.40	139.19	-1.21	AR(2)
10Y Treasury Rate (SPF)	91.29	91.48	0.19	AR(1)
10Y Treasury Rate (BC)	87.99	86.06	-1.93	AR(2)
AAA Corporate Bond Rate (SPF)	132.77	121.32	-11.44	AR(2)
AAA Corporate Bond Rate (BC)	87.49	86.03	-1.46	AR(2)
BAA Corporate Bond Rate (BC)	63.17	66.59	3.42	AR(1)

E.2.3 Empirical Tests of Over-Reaction with AR(2) dynamics

We next restrict the analysis to the series for which AR(2) is favored, and test the prediction of Proposition E1 by estimating Equation (E2). Since our AR(2) series exhibit $\rho_2 > 0$ and $\rho_1 < 0$, under diagnostic expectations the estimated coefficient on medium term forecast revision should be negative, $\hat{\delta}_2^p < 0$, while the estimated coefficient on short term forecast revision should be positive, $\hat{\delta}_1^p > 0$.

Figure E3 shows, for each relevant series, the forecast error regression coefficients $\hat{\delta}_2^p$ and $\hat{\delta}_1^p$ obtained from estimating Equation (E2) with pooled individual data. Table E2 reports these coefficients, together with their corresponding standard errors and *p*-values. In line with the predictions of the model, the signs of the coefficients indicate that the short-term revision positively predicts forecast errors ($\hat{\delta}_1^p > 0$ for all 15 series, 10 of which are statistically significant at the 5% level) while the medium-term revision negatively predicts them ($\hat{\delta}_2^p < 0$ for 12 out of 15 series, 8 of which are statistically significant at the 5% level). To further assess these results, we perform a test of joint significance for $\hat{\delta}_2^p < 0, \hat{\delta}_1^p > 0$. We resample the data using block bootstrap and calculate the fraction of times when $\hat{\delta}_2^p < 0, \hat{\delta}_1^p > 0$ holds, as shown in the last column of Table E2. The probability is greater than 95% for 8 out of the 15 series.

Figure E3. Coefficients in CG Regression AR(2) Version

This plot shows the coefficients δ_2^p (blue circles) and δ_1^p(red diamonds) from the regression in Equation (E2). Standard errors are clustered by both forecaster and time, and the vertical bars shown the 95% confidence intervals.

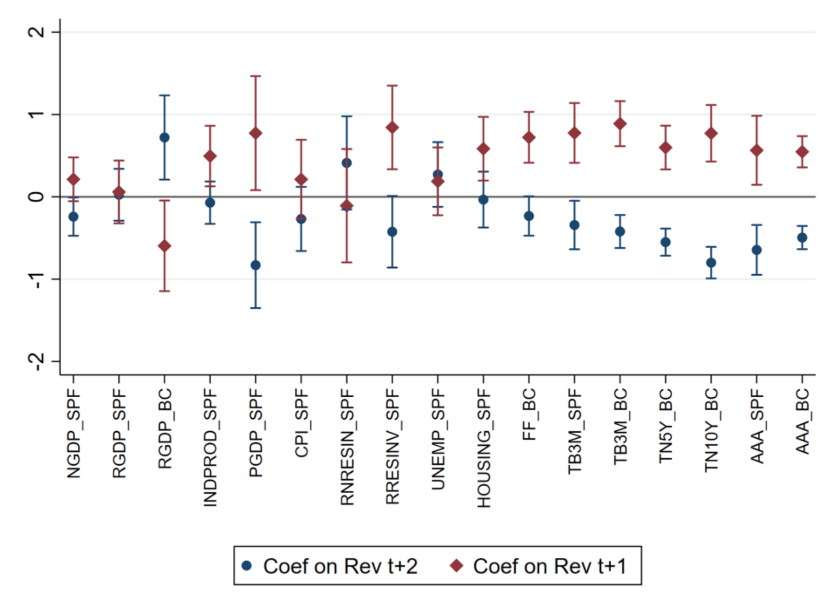


Table E2. Coefficients in CG Regression AR(2) Version

Coefficients δ_2^p and δ_1^p from the regression in Equation (E2), together with the corresponding standard errors and *p*-values. The final column resamples the data using bootstrap (bootstrapping forecasters with replacement) and shows the probability of $\delta_2^p < 0$ and $\delta_1^p > 0$.

Variable	δ_2^p	s.e.	*p*-val	δ_1^p	s.e.	*p*-val	Prob $\delta_2^p < 0$ & $\delta_1^p > 0$
Nominal GDP (SPF)	-0.24	0.12	0.04	0.21	0.14	0.12	0.97
Real GDP (SPF)	0.03	0.16	0.88	0.06	0.19	0.76	0.42
Real GDP (BC)	0.72	0.26	0.01	-0.60	0.28	0.03	0.00
GDP Price Index Inflation (SPF)	-0.07	0.13	0.59	0.50	0.19	0.01	0.74
CPI (SPF)	-0.83	0.27	0.00	0.77	0.35	0.03	1.00
Industrial Production (SPF)	-0.27	0.20	0.18	0.21	0.25	0.39	0.94
Real Non-Residential Investment (SPF)	0.41	0.29	0.15	-0.11	0.35	0.76	0.01
Real Residential Investment (SPF)	-0.42	0.22	0.06	0.84	0.26	0.00	1.00
Housing Start (SPF)	0.27	0.20	0.18	0.19	0.21	0.37	0.03
Unemployment (SPF)	-0.03	0.17	0.85	0.58	0.20	0.00	0.56
Fed Funds Rate (BC)	-0.23	0.12	0.06	0.72	0.16	0.00	0.98
3M Treasury Rate (SPF)	-0.34	0.15	0.02	0.78	0.19	0.00	0.97
3M Treasury Rate (BC)	-0.42	0.10	0.00	0.89	0.14	0.00	1.00
5Y Treasury Rate (BC)	-0.55	0.08	0.00	0.60	0.14	0.00	1.00
10Y Treasury Rate (BC)	-0.80	0.10	0.00	0.77	0.18	0.00	1.00
AAA Corporate Bond Rate (SPF)	-0.64	0.15	0.00	0.56	0.21	0.01	1.00
AAA Corporate Bond Rate (BC)	-0.49	0.07	0.00	0.55	0.10	0.00	1.00

These results are consistent with kernel of truth but are harder to reconcile with Natural Expectations, where forecasters neglect longer lags. Overall, then, the AR(2) analysis confirms and perhaps strengthens the evidence for over-reaction in the data. Four of the seven series (PGDP_SPF, RRESINV_SPF, TN5Y_BC and TN10Y_BC) for which individual level forecast errors seemed unpredictable (Table 3), and thus consistent with Noisy Rational Expectations, show evidence of over-reaction in the AR(2) setting. In addition, the four series that seemed to display under-reaction at the individual level, unemployment, the Fed Funds rate and the 3-months T Bill rate (SPF and BC), now display over-reaction to long-term reversals ($\hat{\delta}_1^p > 0$), and in all cases except unemployment also display significant overreaction in short term forecasts. In all these cases, it is possible that over-reaction to long term reversals moved the individual level coefficient in Table 4 close to zero or above, giving the false impression of rationality or under-reaction. Only for the variable RGDP_SPF, which displayed significant over-reaction under the AR(1) specification loses its significance at conventional level in the AR(2) case.

F. Model Estimation: Supporting Information

In this Section, we provide supporting information for the estimation exercises. After discussing in turn estimation methods 1, 2, and 3 described in Section 5, we perform a sensitivity analysis of the robustness of our results to different estimation methods and assumptions. We conclude by calibrating a model of overconfidence using the approach of method 1, which does a poorer job at matching the data than does diagnostic expectations.

We begin by documenting, in Table F1, the properties of actuals when estimated as AR(1) or as AR(2) processes.

Table F1. Estimates of AR(1) and AR(2) Parameters for Fundamentals

This table shows the autocorrelation and standard deviation parameters of the fundamental processes, for both AR(1) and AR(2) specifications. The parameters are estimated for the same time period when the corresponding forecasts are available.

	AR(1)		AR(2)		
	ρ	σ_u	ρ_1	ρ_2	σ_u
Nominal GDP (SPF)	0.93	1.06	1.27	-0.37	0.99
Real GDP (SPF)	0.87	1.10	1.32	-0.51	0.95
Real GDP (BC)	0.86	0.75	1.24	-0.43	0.68
GDP Price Index Inflation (SPF)	0.98	0.48	1.45	-0.48	0.43
CPI (SPF)	0.86	0.65	1.11	-0.29	0.61
Real Consumption (SPF)	0.87	0.70	0.89	-0.02	0.71
Industrial Production (SPF)	0.85	2.49	1.35	-0.59	2.01
Real Non-Residential Investment (SPF)	0.89	3.35	1.25	-0.41	3.06
Real Residential Investment (SPF)	0.88	5.56	1.27	-0.43	4.90
Real Federal Government Consumption (SPF)	0.78	2.76	0.74	0.06	2.74
Real State&Local Govt Consumption (SPF)	0.89	0.77	0.85	0.05	0.77
Housing Start (SPF)	0.97	0.37	1.49	-0.54	0.31
Unemployment (SPF)	0.85	11.33	1.19	-0.39	10.43
Fed Funds Rate (BC)	0.99	0.49	1.53	-0.55	0.41
3M Treasury Rate (SPF)	0.95	0.56	1.22	-0.26	0.54
3M Treasury Rate (BC)	0.99	0.44	1.54	-0.56	0.37
5Y Treasury Rate (BC)	0.98	0.43	1.16	-0.18	0.42
10Y Treasury Rate (SPF)	0.98	0.37	1.18	-0.21	0.36
10Y Treasury Rate (BC)	0.98	0.37	1.21	-0.24	0.36
AAA Corporate Bond Rate (SPF)	0.98	0.37	1.17	-0.20	0.35
AAA Corporate Bond Rate (BC)	0.97	0.33	1.20	-0.22	0.32
BAA Corporate Bond Rate (BC)	0.95	0.37	1.02	-0.08	0.37

F.1 Method 1

In Method 1 (Section 5.1), we match parameters $(\theta, \sigma_\varepsilon/\sigma_u)$ by fitting, for each series, the variance of analysts' forecast errors and forecast revisions. Within this method, we consider three specifications: i) the baseline AR(1) specification, described in the text; ii) a mixed specification, where series are described by the best fitting AR(1) or AR(2) process, following the classification in Table E1, and iii) an AR(1) specification which allows for fundamental shocks being drawn from a distribution with fat tails, based on the particle filter method described in Appendix D.

AR(1) specification. Kalman inference for AR(1) processes is described in the text, see Equations (8,9). The estimation results were presented in Table 4 and the model performance, in terms of matching the (non-targeted) individual and consensus CG coefficients were shown in Figure 1. Table F2 below documents the match to the target moments.

Table F2. Variance of Forecast Errors and Forecast Revisions: Data and Model (Method 1 AR(1) Specfication)

This table shows forecast error variance, σ^2_{FE}, and forecast revision variance σ^2_{FR} in the data and in the estimated model (Method 1 AR1 version), as well as the absolute log difference between them.

	Forecast Error Variance σ^2_{FE}			Forecast Revision Variance σ^2_{FR}		
	Data	Model	Log Dif	Data	Model	Log Dif
Nominal GDP (SPF)	4.33	4.44	0.026	1.60	1.63	0.023
Real GDP (SPF)	3.92	5.07	0.258	1.16	1.32	0.130
Real GDP (BC)	1.79	1.78	0.006	0.37	0.37	0.010
GDP Price Index Inflation (SPF)	2.09	2.01	0.036	0.62	0.60	0.040
CPI (SPF)	1.57	1.61	0.022	0.45	0.45	0.003
Real Consumption (SPF)	1.63	1.67	0.022	0.50	0.50	0.001
Industrial Production (SPF)	18.65	24.28	0.264	3.91	4.78	0.201
Real Non-Residential Investment (SPF)	39.22	39.25	0.001	8.05	8.14	0.012
Real Residential Investment (SPF)	92.69	90.02	0.029	22.04	21.59	0.021
Real Federal Government Consumption (SPF)	13.93	13.77	0.011	4.60	4.59	0.001
Real State&Local Govt Consumption (SPF)	2.43	2.49	0.024	1.03	1.04	0.002
Housing Start (SPF)	459.38	456.10	0.007	127.22	125.24	0.016
Unemployment (SPF)	0.73	0.81	0.097	0.20	0.19	0.045
Fed Funds Rate (BC)	1.28	1.43	0.109	0.55	0.39	0.339
3M Treasury Rate (SPF)	1.31	1.27	0.034	0.44	0.43	0.032
3M Treasury Rate (BC)	1.29	1.33	0.025	0.50	0.34	0.379
5Y Treasury Rate (BC)	0.93	0.89	0.045	0.37	0.37	0.013

10Y Treasury Rate (SPF)	0.65	0.62	0.049	0.26	0.25	0.007
10Y Treasury Rate (BC)	0.68	0.68	0.014	0.27	0.27	0.011
AAA Corporate Bond Rate (SPF)	0.84	0.86	0.033	0.35	0.36	0.018
AAA Corporate Bond Rate (BC)	0.77	0.79	0.023	0.36	0.36	0.009
BAA Corporate Bond Rate (BC)	0.61	0.62	0.007	0.26	0.26	0.002

Mixed specification (AR(1) and AR(2)). We first describe Kalman inference for an AR(2) process. The state variable is a vector $\vec{x}_t = (x_t, x_{t-1})$ which evolves according to $\vec{x}_t = A\vec{x}_{t-1} + W_t$, with transition matrix $A = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}$ and disturbance $W_t = \begin{bmatrix} u_t & 0 \\ 0 & 0 \end{bmatrix}$ with $u_t \sim \mathcal{N}(0, \sigma_u^2)$ i.i.d. across time. The observation equation is $s_t = C\vec{x}_t + \epsilon_t$ with $C = [1 \quad 0]$ and $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ i.i.d. across time. The Kalman filter can then be written:

$$x_{t|t}^{i,\theta} = x_{t|t-1}^i + (1+\theta)\frac{\Sigma_{11}}{\Sigma_{11} + \sigma_\epsilon^2}\left(s_t^i - \rho_1 x_{t-1|t-1}^i - \rho_2 x_{t-2|t-1}^i\right), \qquad \text{(F1)}$$

where Σ_{11} is the first entry of the steady state variance matrix of beliefs at $t-1$ about x_t, which is given by the following condition:

$$\Sigma = \mathrm{A}\Sigma\mathrm{A}^T + W - A\Sigma\mathrm{C}(\mathrm{C}^T\Sigma\mathrm{C} + \sigma_\epsilon^2)^{-1}\mathrm{C}^T\Sigma\mathrm{A}^T$$

where $W = \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & 0 \end{bmatrix}$. The above expression does not have a closed form solution. One can solve for Σ by numerically solving for the unique root of a polynomial, or iterating the above equation until the value stabilizes. In practice, we solve for the root and confirm that the above condition is satisfied. Once we have the value of Σ, one can iterate equation (F1) to generate forecasts for our SMM estimation procedure.

Table F3 presents the estimation results.

Table F3. SMM Estimates of θ and σ_ϵ (Method 1, Mixed AR(2) and AR(1) Specification)

This table shows the estimates of θ and σ_ϵ based on Method 1 with mixed AR(2) and AR(1) processes, as well as the 95% confidence interval using bootstrap (bootstrapping forecasters with replacement). The standard deviation of the noise σ_ϵ is normalized by the standard deviation of innovations in the actual process σ_u. Results for each series are estimated using the AR(1) or AR(2) version of the diagnostic expectations model based on the properties of the actuals according to Table F1.

	θ	95% CI	σ_ϵ/σ_u	95% CI	Consensus CG	Individual CG
Nominal GDP (SPF)	0.14	(0.04, 0.25)	0.36	(0.04, 0.73)	0.08	-0.04
Real GDP (SPF)	0.16	(0.04, 0.37)	0.28	(0.02, 0.73)	0.09	0.01

Real GDP (BC)	0.31	(0.14, 0.56)	1.30	(0.73, 2.20)	0.58	-0.27
GDP Price Index Inflation (SPF)	0.14	(0.05, 0.27)	2.52	(2.20, 3.18)	1.54	0.02
CPI (SPF)	0.80	(0.44, 1.24)	1.57	(0.73, 2.72)	0.39	-0.37
Real Consumption (SPF)	0.99	(0.80, 1.36)	3.32	(2.20, 4.60)	1.36	-0.36
Industrial Production (SPF)	0.31	(0.09, 0.58)	0.69	(0.10, 1.30)	0.30	-0.15
Real Non-Residential Investment (SPF)	0.22	(0.14, 0.34)	1.27	(0.73, 1.52)	1.18	-0.03
Real Residential Investment (SPF)	0.28	(0.11, 0.46)	1.54	(0.73, 2.20)	1.10	-0.15
Real Federal Government Consumption (SPF)	0.89	(0.66, 1.19)	1.39	(0.73, 2.20)	0.32	-0.32
Real State & Local Govt Consumption (SPF)	1.43	(0.99, 2.31)	4.36	(3.18, 6.65)	0.73	-0.46
Housing Start (SPF)	0.91	(0.53, 1.32)	1.64	(0.73, 2.20)	0.56	-0.34
Unemployment (SPF)	0.00	(0.00, 0.02)	1.59	(1.52, 2.20)	1.25	0.12
Fed Funds Rate (BC)	0.00	(0.00, 0.02)	1.26	(1.05, 1.52)	0.77	-0.01
3M Treasury Rate (SPF)	0.16	(0.11, 0.21)	1.11	(1.05, 1.52)	0.96	0.06
3M Treasury Rate (BC)	0.01	(0.00, 0.05)	1.94	(1.52, 2.20)	1.25	0.04
5Y Treasury Rate (BC)	0.26	(0.21, 0.32)	2.20	(2.20, 2.20)	1.02	-0.18
10Y Treasury Rate (SPF)	0.50	(0.41, 0.59)	3.13	(2.20, 3.18)	0.73	-0.37
10Y Treasury Rate (BC)	0.25	(0.20, 0.29)	2.26	(2.20, 3.18)	0.71	-0.30
AAA Corporate Bond Rate (SPF)	0.55	(0.47, 0.65)	4.64	(4.60, 4.60)	1.27	-0.35
AAA Corporate Bond Rate (BC)	0.60	(0.53, 0.71)	4.60	(4.60, 4.60)	1.44	-0.33
BAA Corporate Bond Rate (BC)	0.68	(0.53, 0.79)	2.38	(1.85, 3.18)	0.55	-0.36

The results are similar to those obtained under the AR(1) specification (see Table 4 in the text).

Table F4. Variance of Forecast Errors and Forecast Revisions: Data and Model

(Method 1, Mixed AR(2) and AR(1) Specification)

This table shows forecast error variance, σ_{FE}^2, and forecast revision variance σ_{FR}^2 in the data and in the estimated model (Method 1, mixed AR(2) and AR(1) version), as well as the absolute log difference between them.

	Forecast Error Variance σ_{FE}^2			Forecast Revision Variance σ_{FR}^2		
	Data	Model	Log Dif	Data	Model	Log Dif
Nominal GDP (SPF)	4.33	4.34	0.003	1.60	1.60	0.004
Real GDP (SPF)	3.92	3.91	0.002	1.16	1.17	0.008
Real GDP (BC)	1.79	1.79	0.000	0.37	0.37	0.000
GDP Price Index Inflation (SPF)	2.09	2.07	0.006	0.62	0.63	0.024
CPI (SPF)	1.57	1.56	0.006	0.45	0.44	0.018
Real Consumption (SPF)	1.63	1.66	0.020	0.50	0.50	0.002
Industrial Production (SPF)	18.65	18.67	0.001	3.91	3.95	0.010
Real Non-Residential Investment (SPF)	39.22	39.22	0.000	8.05	8.11	0.008
Real Residential Investment (SPF)	92.69	91.73	0.010	22.04	21.95	0.004
Real Federal Government Consumption (SPF)	13.93	13.94	0.001	4.60	4.61	0.004
Real State&Local Govt Consumption (SPF)	2.43	2.47	0.016	1.03	1.04	0.006
Housing Start (SPF)	459.38	460.06	0.001	127.22	127.20	0.000
Unemployment (SPF)	0.73	0.89	0.201	0.20	0.26	0.275

Fed Funds Rate (BC)	1.28	1.33	0.038	0.55	0.58	0.064
3M Treasury Rate (SPF)	1.31	1.32	0.007	0.44	0.45	0.005
3M Treasury Rate (BC)	1.29	1.33	0.028	0.50	0.51	0.022
5Y Treasury Rate (BC)	0.93	0.94	0.009	0.37	0.37	0.003
10Y Treasury Rate (SPF)	0.65	0.65	0.014	0.26	0.25	0.003
10Y Treasury Rate (BC)	0.68	0.67	0.014	0.27	0.27	0.002
AAA Corporate Bond Rate (SPF)	0.84	0.83	0.009	0.35	0.35	0.003
AAA Corporate Bond Rate (BC)	0.77	0.76	0.011	0.36	0.36	0.012
BAA Corporate Bond Rate (BC)	0.61	0.61	0.003	0.26	0.26	0.005

Overall, these estimation results are similar to those in Table 4 (where all series are estimated based on the AR(1) version of the model. For series that are selected as AR(2) in Table E1, θ estimated using AR(1) and AR(2) versions of Method 1 are 0.63 correlated (*p*-value 0.01), and σ_ϵ are 0.94 correlated (*p*-value 0.00). The levels of these key parameters also generally match. The individual CG coefficients are 0.82 correlated (*p*-value 0.00) and the consensus CG coefficients are 0.85 correlated (*p*-value 0.00).

AR(1) Specification with Particle Filtering. Finally, we present the results of the specification where series are assumed to follow an AR(1) allowing for non-normal shocks. The particle filter procedure used for the estimation is explained in detail in Appendix D.

Table F5. SMM Estimates of θ and σ_ϵ (Method 1, Particle Filtering)

This table shows the estimates of θ and σ_ϵ based on Method 1 with particle filtering, as well as the 95% confidence interval using bootstrap (bootstrapping forecasters with replacement). The standard deviation of the noise σ_ϵ is normalized by the standard deviation of innovations in the actual process σ_u.

	θ	95% CI	σ_ϵ/σ_u	95% CI	Consensus CG	Individual CG
Nominal GDP (SPF)	0.56	(0.48, 0.60)	0.12	(0.02, 0.35)	0.09	-0.01
Real GDP (SPF)	0.53	(0.41, 0.60)	0.26	(0.02, 0.55)	0.40	0.25
Real GDP (BC)	0.38	(0.25, 0.58)	0.41	(0.02, 1.33)	0.13	-0.03
GDP Price Index Inflation (SPF)	0.25	(0.19, 0.32)	3.72	(2.07, 5.63)	1.58	0.09
CPI (SPF)	0.64	(0.35, 1.24)	0.62	(0.03, 1.53)	0.02	-0.15
Real Consumption (SPF)	0.95	(0.80, 1.36)	4.77	(3.86, 6.37)	1.31	-0.35
Industrial Production (SPF)	0.85	(0.35, 1.41)	0.04	(0.01, 0.09)	0.34	0.28
Real Non-Residential Investment (SPF)	0.38	(0.28, 0.56)	0.09	(0.01, 0.18)	0.62	0.38
Real Residential Investment (SPF)	0.31	(0.19, 0.53)	0.13	(0.02, 0.30)	0.72	0.03
Real Federal Government Consumption (SPF)	0.71	(0.53, 0.97)	0.46	(0.17, 0.60)	0.24	-0.30

Real State & Local Govt Consumption (SPF)	1.60	(0.99, 2.49)	5.76	(3.52, 9.57)	0.73	-0.45
Housing Start (SPF)	0.80	(0.44, 1.50)	0.05	(0.00, 0.15)	0.18	-0.05
Unemployment (SPF)	0.29	(0.27, 0.30)	1.31	(1.00, 1.65)	0.90	0.49
Fed Funds Rate (BC)	0.30	(0.30, 0.30)	1.55	(1.25, 2.06)	1.11	0.11
3M Treasury Rate (SPF)	0.27	(0.21, 0.32)	1.76	(1.41, 1.78)	0.56	0.13
3M Treasury Rate (BC)	0.30	(0.27, 0.30)	2.22	(1.37, 2.26)	1.20	0.10
5Y Treasury Rate (BC)	0.49	(0.42, 0.56)	3.88	(3.84, 3.84)	1.16	-0.27
10Y Treasury Rate (SPF)	0.47	(0.41, 0.53)	7.50	(7.42, 7.42)	0.58	-0.38
10Y Treasury Rate (BC)	0.49	(0.41, 0.53)	6.95	(4.48, 7.38)	0.78	-0.35
AAA Corporate Bond Rate (SPF)	0.64	(0.53, 0.85)	10.49	(7.27, 11.98)	0.46	-0.43
AAA Corporate Bond Rate (BC)	0.98	(0.79, 1.24)	12.83	(8.33, 13.73)	1.12	-0.40
BAA Corporate Bond Rate (BC)	0.52	(0.39, 0.62)	6.93	(4.50, 7.42)	0.43	-0.39

Table F6. Variance of Forecast Errors and Forecast Revisions: Data and Model (Method 1, Particle Filtering)

This table shows forecast error variance, σ_{FE}^2, and forecast revision variance σ_{FR}^2 in the data and in the estimated model (Method 1, particle filtering version), as well as the absolute log difference between them.

	Forecast Error Variance σ_{FE}^2			Forecast Revision Variance σ_{FR}^2		
	Data	Model	Log Dif	Data	Model	Log Dif
Nominal GDP (SPF)	4.33	4.49	0.037	1.60	1.63	0.023
Real GDP (SPF)	3.92	5.08	0.260	1.16	1.19	0.031
Real GDP (BC)	1.79	1.78	0.009	0.37	0.37	0.012
GDP Price Index Inflation (SPF)	2.09	2.10	0.005	0.62	0.61	0.020
CPI (SPF)	1.57	1.59	0.011	0.45	0.45	0.008
Real Consumption (SPF)	1.63	1.65	0.009	0.50	0.50	0.002
Industrial Production (SPF)	18.65	24.25	0.263	3.91	4.07	0.042
Real Non-Residential Investment (SPF)	39.22	39.42	0.005	8.05	8.15	0.013
Real Residential Investment (SPF)	92.69	91.18	0.016	22.04	21.64	0.019
Real Federal Government Consumption (SPF)	13.93	13.69	0.017	4.60	4.57	0.006
Real State&Local Govt Consumption (SPF)	2.43	2.46	0.013	1.03	1.03	0.006
Housing Start (SPF)	459.38	452.10	0.016	127.22	125.09	0.017
Unemployment (SPF)	0.73	0.81	0.101	0.20	0.21	0.043
Fed Funds Rate (BC)	1.28	1.60	0.221	0.55	0.54	0.015
3M Treasury Rate (SPF)	1.31	1.24	0.056	0.44	0.42	0.065
3M Treasury Rate (BC)	1.29	1.36	0.049	0.50	0.49	0.021
5Y Treasury Rate (BC)	0.93	0.95	0.025	0.37	0.38	0.007
10Y Treasury Rate (SPF)	0.65	0.63	0.038	0.26	0.25	0.009
10Y Treasury Rate (BC)	0.68	0.66	0.015	0.27	0.26	0.002
AAA Corporate Bond Rate (SPF)	0.84	0.83	0.009	0.35	0.36	0.011
AAA Corporate Bond Rate (BC)	0.77	0.75	0.033	0.36	0.36	0.013
BAA Corporate Bond Rate (BC)	0.61	0.61	0.001	0.26	0.26	0.004

These estimation results using the particle filtering are very similar to the baseline results in Table 4. The θ in these two cases are 0.92 correlated (*p*-value 0.01), and σ_ϵ are 0.90 correlated (*p*-value 0.00). The levels of these key parameters also generally match. The individual CG coefficients are 0.96 correlated (*p*-value 0.00) and the consensus CG coefficients are 0.92 correlated (*p*-value 0.00).

F.2 Method 2

In Method 2 we assume the series are AR(1) and estimate θ by directly by fitting individual level coefficients to the corresponding model prediction (Equation 12). We then estimate σ_ϵ/σ_u by fitting the variance of forecast revisions. Within this method, we consider a pooled specification and forecaster by forecaster specification. The results under the pooled specification are shown in Table 7 in the main text.

Table F7. Variance of Forecast Revisions: Data and Model (Method 2)

This table shows forecast revision variance σ_{FR}^2 in the data and in the estimated model (Method 2), as well as the absolute log difference between them.

	Forecast Revision Variance σ_{FR}^2		
	Data	Model	Log Dif
Nominal GDP (SPF)	1.60	1.22	0.268
Real GDP (SPF)	1.16	0.81	0.353
Real GDP (BC)	0.37	0.19	0.642
GDP Price Index Inflation (SPF)	0.62	0.22	1.043
CPI (SPF)	0.45	0.32	0.337
Real Consumption (SPF)	0.50	0.37	0.288
Industrial Production (SPF)	3.91	3.32	0.161
Real Non-Residential Investment (SPF)	8.05	4.74	0.529
Real Residential Investment (SPF)	22.04	12.71	0.550
Real Federal Government Consumption (SPF)	4.60	21.71	1.552
Real State&Local Govt Consumption (SPF)	1.03	1.06	0.025
Housing Start (SPF)	127.22	82.01	0.439
Unemployment (SPF)	0.20	0.10	0.637
Fed Funds Rate (BC)	0.55	0.23	0.865
3M Treasury Rate (SPF)	0.44	0.17	0.952
3M Treasury Rate (BC)	0.50	0.20	0.932
5Y Treasury Rate (BC)	0.37	0.22	0.531
10Y Treasury Rate (SPF)	0.26	0.18	0.343
10Y Treasury Rate (BC)	0.27	0.18	0.367

AAA Corporate Bond Rate (SPF)	0.35	0.22	0.492
AAA Corporate Bond Rate (BC)	0.36	0.15	0.899
BAA Corporate Bond Rate (BC)	0.26	0.22	0.188

Table F8. Forecaster-by-Forecaster Estimates (Medians)

This table presents the median estimate based on forecaster-level estimation using Method 2. The first two columns show the median forecaster-level estimate of θ and σ_ϵ. The third column shows the median individual level CG coefficient implied by the model. The final three columns show the median forecast revision variance in the data, in the model, and the median absolute difference between the data and the model.

	θ	σ_ϵ/σ_u	Individual CG	Data	Model	Log Dif
Nominal GDP (SPF)	0.08	1.15	-0.16	0.96	0.71	0.106
Real GDP (SPF)	0.09	0.66	0.02	0.77	0.59	0.188
Real GDP (BC)	-0.01	0.50	-0.09	0.34	0.20	0.294
GDP Price Index Inflation (SPF)	0.13	1.51	-0.24	0.36	0.32	0.177
CPI (SPF)	0.27	0.87	-0.28	0.32	0.23	0.138
Real Consumption (SPF)	0.19	0.66	0.06	0.34	0.27	0.229
Industrial Production (SPF)	0.10	0.66	0.16	3.48	2.77	0.132
Real Non-Residential Investment (SPF)	-0.08	0.87	0.42	6.97	4.68	0.341
Real Residential Investment (SPF)	0.10	1.15	-0.08	17.83	13.45	0.112
Real Federal Government Consumption (SPF)	0.80	2.07	-0.36	2.56	2.33	0.092
Real State & Local Govt Consumption (SPF)	0.53	1.51	-0.39	0.55	0.48	0.090
Housing Start (SPF)	0.28	0.87	-0.16	91.03	60.36	0.228
Unemployment (SPF)	-0.02	1.15	0.67	0.18	0.12	0.192
Fed Funds Rate (BC)	-0.14	1.15	0.98	0.43	0.22	0.654
3M Treasury Rate (SPF)	-0.14	1.15	0.38	0.39	0.16	0.606
3M Treasury Rate (BC)	-0.14	1.15	1.09	0.43	0.21	0.727
5Y Treasury Rate (BC)	0.19	1.15	-0.07	0.35	0.26	0.243
10Y Treasury Rate (SPF)	0.29	1.15	-0.29	0.24	0.18	0.151
10Y Treasury Rate (BC)	0.40	1.15	-0.35	0.25	0.21	0.096
AAA Corporate Bond Rate (SPF)	0.38	1.75	-0.36	0.29	0.21	0.145
AAA Corporate Bond Rate (BC)	0.34	1.15	-0.24	0.29	0.18	0.479
BAA Corporate Bond Rate (BC)	0.46	1.51	-0.34	0.25	0.19	0.138

Table F9. Rank Correlations for Forecaster-Level Diagnosticity θ^i across Variables

This table shows the rank correlation for forecaster-level estimates of θ^i across different series, and p-value in parenthesis. Panel A shows results for series and forecasters in SPF. Panel B shows results for series and forecasters in Blue Chip. θ^i for each series is estimated using the AR(1) or AR(2) version of the diagnostic expectations model based on the properties of the actuals according to Table 6.

Panel A. SPF Series

	NGDP	RGDP	PGDP	CPI	RCONSUM	INDPROD	RNRESINV	RRESINV	RGF	RGSL	HOUSING	UNEMP	tb3m	tn10y
RGDP	0.54													
	(0.000)													
PGDP	0.14	0.18												
	(0.270)	(0.201)												
CPI	0.03	-0.21	0.31											
	(0.840)	(0.139)	(0.023)											
RCONSUM	0.43	0.44	0.18	-0.22										
	(0.001)	(0.001)	(0.175)	(0.107)										
INDPROD	0.02	-0.08	0.23	0.10	0.23									
	(0.856)	(0.604)	(0.095)	(0.477)	(0.107)									
RNRESINV	0.45	0.05	0.20	0.04	0.17	0.09								
	(0.001)	(0.708)	(0.170)	(0.783)	(0.236)	(0.586)								
RRESINV	0.26	0.07	0.33	0.30	0.13	0.22	0.03							
	(0.073)	(0.654)	(0.027)	(0.049)	(0.364)	(0.164)	(0.868)							
RGF	-0.12	0.05	0.08	0.17	-0.10	0.07	-0.26	0.11						
	(0.444)	(0.754)	(0.627)	(0.276)	(0.535)	(0.706)	(0.097)	(0.544)						
RGSL	0.09	0.04	0.18	0.05	0.14	0.11	0.16	0.31	0.07					
	(0.547)	(0.779)	(0.246)	(0.719)	(0.344)	(0.477)	(0.321)	(0.065)	(0.672)					
HOUSING	0.32	0.30	0.02	0.14	0.35	-0.14	0.10	0.30	-0.10	0.15				
	(0.014)	(0.028)	(0.873)	(0.308)	(0.011)	(0.331)	(0.527)	(0.046)	(0.579)	(0.325)				
UNEMP	0.13	0.03	0.03	0.07	0.02	0.36	0.60	0.00	-0.21	0.25	0.13			
	(0.353)	(0.832)	(0.845)	(0.657)	(0.890)	(0.030)	(0.000)	(0.998)	(0.284)	(0.142)	(0.413)			
tb3m	0.15	-0.20	0.15	0.03	0.28	0.20	0.26	0.31	-0.01	0.32	0.24	0.38		
	(0.238)	(0.149)	(0.241)	(0.827)	(0.042)	(0.182)	(0.084)	(0.048)	(0.975)	(0.047)	(0.085)	(0.006)		
tn10y	0.06	0.14	0.10	0.28	-0.03	-0.09	0.04	0.03	-0.07	-0.25	0.21	-0.12	0.12	
	(0.652)	(0.326)	(0.496)	(0.055)	(0.813)	(0.547)	(0.782)	(0.854)	(0.675)	(0.122)	(0.158)	(0.486)	(0.426)	
AAA	0.11	0.16	0.00	0.39	0.05	-0.08	0.01	0.05	-0.02	-0.17	0.16	0.00	-0.16	0.56
	(0.421)	(0.300)	(0.986)	(0.004)	(0.713)	(0.601)	(0.967)	(0.752)	(0.906)	(0.282)	(0.282)	(0.977)	(0.254)	(0.000)

Panel B: Blue Chip Series

	RGDPBC	FFBC	tb3mBC	tn5yBC	tn10yBC	AAABC
FFBC	0.49					
	(0.000)					
tb3mBC	0.46	0.72				
	(0.000)	(0.000)				
tb5yBC	0.06	0.09	0.20			
	(0.582)	(0.375)	(0.044)			
tn10yBC	0.08	-0.10	-0.07	0.46		
	(0.454)	(0.357)	(0.495)	(0.000)		
AAABC	0.25	0.07	0.21	0.39	0.51	
	(0.026)	(0.502)	(0.045)	(0.000)	(0.000)	
BAABC	0.22	0.07	0.05	0.14	0.23	0.41
	(0.123)	(0.672)	(0.725)	(0.353)	(0.129)	(0.004)

F.3 Method 3

Method 3 is similar to Method 2 except that θ is restricted to be equal for all series. We consider both an AR(1) specification and a mixed (AR(1) and AR(2)) specification, following the classification in Table E1.

AR(1) specification. The estimation results are shown in Table F10.

Table F10. SMM Estimates of θ and σ_ϵ (Method 3, AR(1))

This table shows the estimates based on Method 3 AR(1) version when $\theta = 0.5$. The standard deviation of the noise σ_ϵ is normalized by the standard deviation of innovations in the actual process σ_u.

	θ	σ_ϵ/σ_u	95% CI	Consensus CG	Individual CG	Forecast Revision Var		
						Data	Model	Log Dif
Nominal GDP (SPF)	0.5	0.75	(0.02, 2.20)	0.32	-0.06	1.60	1.59	0.006
Real GDP (SPF)	0.5	0.64	(0.02, 2.20)	0.63	0.20	1.16	1.14	0.010
Real GDP (BC)	0.5	1.94	(1.52, 2.20)	0.93	-0.29	0.37	0.37	0.002
GDP Price Index Inflation (SPF)	0.5	1.13	(0.37, 1.52)	1.09	0.17	0.62	0.56	0.101
CPI (SPF)	0.5	0.56	(0.02, 1.05)	0.13	-0.14	0.45	0.45	0.009
Real Consumption (SPF)	0.5	0.45	(0.04, 1.52)	0.16	-0.08	0.50	0.45	0.093
Industrial Production (SPF)	0.5	1.36	(0.02, 2.20)	0.85	0.00	3.91	3.92	0.005
Real Non-Residential Investment (SPF)	0.5	2.14	(0.02, 3.18)	1.60	0.02	8.05	7.95	0.012
Real Residential Investment (SPF)	0.5	0.64	(0.02, 3.18)	0.61	0.07	22.04	21.93	0.005
Real Federal Government Consumption (SPF)	0.5	0.08	(0.02, 0.43)	-0.13	-0.04	4.60	3.67	0.226
Real State & Local Govt Consumption (SPF)	0.5	0.65	(0.50, 0.73)	-0.06	-0.28	1.03	0.64	0.482
Housing Start (SPF)	0.5	0.65	(0.50, 0.73)	0.35	-0.03	127.22	105.54	0.187
Unemployment (SPF)	0.5	2.26	(0.02, 6.65)	1.88	0.37	0.20	0.20	0.016
Fed Funds Rate (BC)	0.5	2.32	(0.73, 5.68)	2.16	0.12	0.55	0.54	0.019
3M Treasury Rate (SPF)	0.5	3.03	(0.06, 4.60)	2.07	0.00	0.44	0.45	0.005
3M Treasury Rate (BC)	0.5	1.90	(1.28, 2.20)	1.99	0.16	0.50	0.47	0.054
5Y Treasury Rate (BC)	0.5	1.22	(0.24, 3.18)	0.45	-0.21	0.37	0.37	0.002
10Y Treasury Rate (SPF)	0.5	2.03	(0.03, 4.60)	0.43	-0.31	0.26	0.26	0.001
10Y Treasury Rate (BC)	0.5	1.36	(0.03, 4.60)	0.39	-0.25	0.27	0.26	0.006
AAA Corporate Bond Rate (SPF)	0.5	1.61	(0.02, 2.20)	0.32	-0.31	0.35	0.29	0.205
AAA Corporate Bond Rate (BC)	0.5	1.43	(1.05, 1.52)	0.48	-0.21	0.36	0.23	0.443
BAA Corporate Bond Rate (BC)	0.5	0.51	(0.02, 1.05)	-0.20	-0.29	0.26	0.23	0.127

Mixed (AR(1) and AR(2)) specification. The estimation results are shown in Table F11.

Table F11. SMM Estimates of θ and σ_ϵ (Method 3, Mixed AR(2) and AR(1))

This table shows the estimates based on Method 3 mixed AR(2) and AR(1) version when $\theta = 0.3$. The standard deviation of the noise σ_ϵ is normalized by the standard deviation of innovations in the actual process σ_u.

	θ	σ_ϵ/σ_u	95% CI	Consensus CG	Individual CG	Forecast Revision Var		
						Data	Model	Log Dif
Nominal GDP (SPF)	0.3	1.74	(1.05, 2.20)	0.82	-0.19	1.60	1.59	0.001
Real GDP (SPF)	0.3	0.79	(0.50, 1.05)	0.40	-0.10	1.16	1.15	0.003
Real GDP (BC)	0.3	1.26	(1.05, 1.52)	0.56	-0.27	0.37	0.37	0.010
GDP Price Index Inflation (SPF)	0.3	9.26	(4.60, 13.90)	2.61	-0.20	0.62	0.64	0.036
CPI (SPF)	0.3	0.18	(0.02, 0.50)	-0.02	-0.09	0.45	0.44	0.021
Real Consumption (SPF)	0.3	0.31	(0.03, 0.73)	0.22	0.07	0.50	0.33	0.405
Industrial Production (SPF)	0.3	0.69	(0.50, 0.90)	0.32	-0.15	3.91	3.91	0.000
Real Non-Residential Investment (SPF)	0.3	1.87	(1.52, 2.20)	1.52	-0.13	8.05	7.85	0.024
Real Residential Investment (SPF)	0.3	1.74	(1.05, 2.20)	1.23	-0.18	22.04	21.59	0.021
Real Federal Government Consumption (SPF)	0.3	0.07	(0.02, 0.35)	-0.02	0.10	4.60	2.63	0.559
Real State & Local Govt Consumption (SPF)	0.3	0.63	(0.50, 0.73)	0.06	-0.20	1.03	0.46	0.809
Housing Start (SPF)	0.3	0.02	(0.02, 0.04)	-0.03	0.01	127.22	103.12	0.210
Unemployment (SPF)	0.3	13.25	(9.61, 13.90)	4.92	-0.12	0.20	0.18	0.067
Fed Funds Rate (BC)	0.3	20.09	(20.09, 20.09)	5.83	-0.24	0.55	0.65	0.168
3M Treasury Rate (SPF)	0.3	5.70	(4.60, 6.65)	3.15	-0.06	0.44	0.44	0.006
3M Treasury Rate (BC)	0.3	13.90	(13.90, 13.90)	5.05	-0.18	0.50	0.59	0.172
5Y Treasury Rate (BC)	0.3	4.24	(0.07, 6.65)	2.03	-0.20	0.37	0.38	0.010
10Y Treasury Rate (SPF)	0.3	0.83	(0.50, 1.05)	0.10	-0.24	0.26	0.20	0.234
10Y Treasury Rate (BC)	0.3	5.85	(2.20, 9.61)	2.21	-0.33	0.27	0.26	0.006
AAA Corporate Bond Rate (SPF)	0.3	0.04	(0.02, 0.17)	-0.20	-0.21	0.35	0.33	0.066
AAA Corporate Bond Rate (BC)	0.3	0.05	(0.02, 0.04)	-0.13	-0.15	0.36	0.26	0.328
BAA Corporate Bond Rate (BC)	0.3	0.57	(0.12, 1.05)	-0.10	-0.22	0.26	0.17	0.450

F.4 Sensitivity Analysis

We next assess the robustness of our results to alternative estimation methods.

Table F.12 below shows the correlation between the estimated diagnostic parameters θ in Methods 1 (using AR(1), mixed, and particle versions) and Method 2. In Panel A, we find a very high correlation between the distortions θ_k estimated under the different specifications in Methods 1, above 85%. The correlation with Method 2 is lower due to an outlier variable (RGF in SPF); without it the correlations are all above 0.85. In Panel B, we also find high rank correlations, ranging from 74% to 83%.

The average estimates for θ in the alternative specifications are also very similar (0.59 for Method 1 AR(1), 0.44 for Method 1 mixed, 0.58 for Method 1 AR(1) particle, 0.42 for Method 2).

Table F12. Correlation of θ_k across Different Estimation Methods

This table shows the correlation of θ_k among different estimation methods. Panel A shows raw correlations and Panel B shows rank correlations. For Method 3, θ_k is restricted to be the same across all variables, so it is not included here.

Panel A. Raw Correlations of θ_k

	Method 1 (AR1)	Method 1 (Mix)	Method 1 (Particle)
Method 1 (Mix)	0.86		
Method 1 (Particle)	0.92	0.85	
Method 2	0.42	0.46	0.30

Panel B. Rank Correlations of θ_k

	Method 1 (AR1)	Method 1 (Mix)	Method 1 (Particle)
Method 1 (Mix)	0.74		
Method 1 (Particle)	0.83	0.78	
Method 2	0.82	0.80	0.81

Table F13 below shows the correlation between the empirical CG coefficients and the predicted CG coefficients in all methods considered.

Table F13. CG Coefficients: Data vs Model

This table shows regressions of CG coefficients in the data (LHS) on CG coefficients in the estimated model (RHS) across different series. Panel A uses individual CG coefficient from forecaster-level panel regressions. Panel B uses consensus CG coefficient from time series regressions of consensus forecasts.

Panel A. Individual CG

	Data CG (Individual)		
	(1)	(2)	(3)
Model CG (Method 1)	0.586***		
	(0.087)		
Model CG (Method 2)		0.374***	
		(0.033)	
Model CG (Method 3)			0.695***
			(0.184)
Constant	-0.065*	-0.194***	-0.046
	(0.033)	(0.024)	(0.044)
Observations	22	22	22
R-squared	0.575	0.848	0.331

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Panel B. Consensus CG

	Data CG (Consensus)		
	(1)	(2)	(3)
Model CG (Method 1)	0.171		
	(0.204)		
Model CG (Method 2)		0.462***	
		(0.113)	
Model CG (Method 3)			0.351***
			(0.099)
Constant	0.316*	0.0783	0.179
	(0.164)	(0.104)	(0.116)
Observations	22	22	22
R-squared	0.031	0.418	0.318

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Overall, our structural estimation exercise yields three results. First, diagnostic distortions in professional forecasters' expectations are sizable and in the ballpark of previous estimates obtained in different contexts. Representativeness is thus a promising candidate for a robust psychological distortion in expectation formation. Second, the estimated distortions are quite robust to alternative assumptions. Third, the diagnostic expectation model does a good job at capturing variation in the data.

F.5 Overconfidence

We now discuss a version of the model in Section 4 with overconfidence instead of diagnostic expectations. Here the agent underestimates the standard deviation of the noise in his signal by a factor of α, where $\alpha < 1$. He then substitutes the deflated standard deviation of the noise into the Kalman filter update equation. Formally, setting $\widehat{\sigma^2_{\epsilon,\alpha}} = \alpha^2 \sigma^2_{\epsilon}, \alpha < 1$, the overconfidence Kalman update is given by the following two equations:

$$\widehat{\Sigma_\alpha} = \frac{-(1-\rho^2)\,\widehat{\sigma^2_{\epsilon,\alpha}} + \sigma_u^2 + \sqrt{\left[(1-\rho^2)\,\widehat{\sigma^2_{\epsilon,\alpha}} - \sigma_u^2\right]^2 + 4\,\widehat{\sigma^2_{\epsilon,\alpha}}\sigma_u^2}}{2}$$

$$x_{i,t|t} = x_{i,t|t-1} + \frac{\widehat{\Sigma_\alpha}}{\widehat{\Sigma_\alpha} + \widehat{\sigma^2_{\epsilon,\alpha}}}(s_t^i - x_{i,t|t-1})$$

One can easily derive that the Kalman gain is a decreasing function of α, which needs to be bounded above by one. Intuitively, no matter how overconfident the agent is, he can only give at most full weight to the most recent observation.

On the other hand, for our model with diagnostic expectations, the Kalman gain can be greater than one. Extrapolating beyond the noisy signal is only possible for diagnostic agents. Table F14 below shows Kalman gains calculated from our three estimation methods. In a number of cases, the estimated Kalman gains are greater than one.

Table F14. Diagnostic Kalman Gains

This table shows the implied Kalman gains in the baseline estimation of our model. We report results for all three estimation methods in Section 5 of the paper.

	Method 1	Method 2	Method 3
Nominal GDP (SPF)	1.51	0.92	1.06
Real GDP (SPF)	1.48	0.92	1.13
Real GDP (BC)	1.23	0.82	0.51
GDP Price Index Inflation (SPF)	0.65	0.60	0.86
CPI (SPF)	1.30	1.23	1.19
Real Consumption (SPF)	0.38	1.26	1.28
Industrial Production (SPF)	1.58	0.92	0.70
Real Non-Residential Investment (SPF)	1.25	0.68	0.48
Real Residential Investment (SPF)	0.97	0.68	1.13
Real Federal Government Consumption (SPF)	0.89	0.15	1.49
Real State&Local Govt Consumption (SPF)	0.34	0.52	1.12
Housing Start (SPF)	1.35	1.00	1.11
Unemployment (SPF)	1.09	0.51	0.51
Fed Funds Rate (BC)	0.92	0.48	0.51
3M Treasury Rate (SPF)	0.86	0.40	0.38
3M Treasury Rate (BC)	0.81	0.48	0.60
5Y Treasury Rate (BC)	0.64	0.62	0.82
10Y Treasury Rate (SPF)	0.43	0.82	0.56
10Y Treasury Rate (BC)	0.47	0.80	0.76
AAA Corporate Bond Rate (SPF)	0.36	0.56	0.67
AAA Corporate Bond Rate (BC)	0.38	0.62	0.73
BAA Corporate Bond Rate (BC)	0.50	1.12	1.23