“Salience and Consumer Choice”
Appendices for Online Publication

A Extension: Goods with Multiple Quality Attributes

In this Appendix, we extend the model to the case where goods are characterized by multiple attributes. We then study in detail the case where goods differ along two quality attributes.

As in Section 2, a consumer evaluates all $N > 1$ goods in a choice set $\mathbf{C}_{choice} \equiv \{\mathbf{q}_k\}_{k=1,...,N}$. Each good k is a vector $\mathbf{q}_k = (q_{1k}, \ldots, q_{mk}, p_k) \in \mathbb{R}^m$ of $m > 1$ quality attributes, where q_{ik} $(i = 1, \ldots, m)$ measures the utility that attribute i generates for the consumer. The case studied in the main text has $m = 1$.[1]

The consumer has full information about the attributes of each good and, absent salience distortions, evaluates $\mathbf{q}_k$ with a separable utility function:

$$u(\mathbf{q}_k) = \sum_{i=1}^{m} \theta_i q_{ik} - \theta_p p_k, \tag{1}$$

where θ_i is the weight attached to quality attribute i and θ_p is the weight attached to the numeraire in the valuation of the good. We normalize $\theta_1 + ... + \theta_m + \theta_p = 1$. Parameter θ_i captures the importance of attribute i for the overall utility of the good (i.e., the strength/frequency with which a certain attribute is experienced during consumption), and θ_i/θ_j is the rational rate of substitution among attributes j and i.

The choice context is defined similarly to the case where $m = 1$.

Definition 1 *The choice context is the set $\mathbf{C} = \mathbf{C}_{choice} \cup \mathbf{C}_e$, where $\mathbf{C}_{choice}$ is the externally given choice set while $\mathbf{C}_e = \{\mathbf{q}^e_k\}_{k=1,...,N}$ is the set of goods the consumer expects to find in the choice setting. We assume that:*

i) $\mathbf{q}^e_k$ shares the same non-price attributes of choice option $\mathbf{q}_k$, namely $q^e_{ik} = q_{ik}$ for $i = 1, ..., m$. The expected price p^e_k is the rational expectation of p_k, namely $p^e_k \equiv \mathbb{E}[p_k]$.

ii) The choice context is summarized by a reference good $\overline{\mathbf{q}} = \{\overline{q}_1, \ldots, \overline{q}_m, \overline{p}\}$, where the

[1] The extension to a case where a good has multiple price components is straightforward.

reference (or normal) level of attribute i is the average value of that attribute in $\mathbf{C}$*, namely* $\overline{q}_i = \frac{1}{2N}\sum_k (q_{ik} + q^e_{ik})$ *and* $\overline{p} = \frac{1}{2N}\sum_k (p_k + p^e_k)$*. The reference good* $\overline{\mathbf{q}}$ *need not be in* $\mathbf{C}$*.*

Given a salience function $\sigma(\cdot,\cdot)$, the salience distortions of decision weights are again defined similarly to the case where $m = 1$.

Definition 2 *Quality attribute i is more salient than quality attribute j for good* $\mathbf{q}_k$ *if and only if $\sigma(q_{ik}, \overline{q}_i) > \sigma(q_{jk}, \overline{q}_j)$. Quality attribute i is more salient than price for good* $\mathbf{q}_k$ *if and only if $\sigma(q_{ik}, \overline{q}_i) > \sigma(p_k, \overline{p})$. Let r_{ik} be the salience ranking of quality attribute i and r_{pk} the salience ranking of price for good* $\mathbf{q}_k$*, where the most salient attribute has rank 1. Attributes with equal salience receive the same (lowest possible) ranking. The salient thinker evaluates good* $\mathbf{q}_k$ *by transforming the weights θ_i attached to quality attribute $i \in \{1, ..., m\}$ and the weight θ_p attached to the numeraire into:*

$$\widehat{\theta}^k_i = \theta_i \cdot \frac{\delta^{r_{ikt}}}{\sum_j \theta_j \delta^{r_{jk}} + \theta_p \delta^{r_{pk}}} \equiv \theta_i \omega^k_i, \qquad \widehat{\theta}^k_p = \theta_p \cdot \frac{\delta^{r_{pk}}}{\sum_j \theta_j \delta^{r_{jt}} + \theta_p \delta^{r_{pk}}} \equiv \theta_i \omega^t_i \tag{2}$$

where $\delta \in (0, 1]$. The salient thinker's evaluation of good $\mathbf{q}_k$ *is given by:*

$$u^S(\mathbf{q}_t) = \sum_{i=1}^{m} \widehat{\theta}^t_i \cdot q_{it} - \widehat{\theta}_p \cdot p_k. \tag{3}$$

Having examined the tradeoff between quality and price in the main text, we now consider the trade-off between two quality dimensions. We show that diminishing sensitivity naturally creates a taste for goods delivering balanced utilities across different attributes: for unbalanced goods, the salient attributes are their shortcomings rather than their strengths. This mechanism is richer than loss aversion accounts and yields novel predictions.

Consider goods (q_{1k}, q_{2k}, p) that differ in their qualities but not in their prices, so that price is the least salient dimension (we assume price is deterministic, so $p = \overline{p}$). For notational convenience, we omit the price. In this setup, Definition 1 implies that q_{1k} is more salient than q_{2k} for good k if and only if $\sigma(q_{1k}, \overline{q}_1) > \sigma(q_{2k}, \overline{q}_2)$. Once more, the salience ranking of a good in quality-quality space is determined by its location relative to the reference $\overline{\mathbf{q}} = (\overline{q}_1, \overline{q}_2)$. Suppose that $q_{1k} > \overline{q}_1$ and $q_{2k} < \overline{q}_2$. Then, homogeneity of degree zero implies that the upside q_{1k} of good k is salient whenever $\sigma(q_{1k}/\overline{q}_1, 1) > \sigma(1, \overline{q}_2/q_{2k})$, which is equivalent to:

$$q_{1k} \cdot q_{2k} > \overline{q}_1 \cdot \overline{q}_2.$$

The salience ranking is determined by the quality-quality product $q_{1k} \cdot q_{2k}$. In this regard, a version of Proposition 1 carries through: if a good is neither dominated by nor dominates the reference good, its relative advantage is salient if and only if it has a higher quality-quality product than the reference good.

Consider now how salience affects choice along a rational indifference curve. In a quality-quality trade-off, rational indifference curves are downward sloping. Unbalanced goods, which increase the level of one attribute at the cost of weakening the other, have low values of $q_1 \cdot q_2$. Balanced goods, whose strengths and weaknesses are comparable, have high values of $q_1 \cdot q_2$. We then show:

Proposition 1 *Let all goods in the choice context be located on a rational indifference curve, with reference good* $\overline{\mathbf{q}} = (\overline{q}_1, \overline{q}_2)$. *The consumer chooses the good* k *which is furthest from* $\overline{\mathbf{q}}$, *i.e. maximizes* $|q_{1k} - \overline{q}_1|$, *conditional on being more balanced than* $\overline{\mathbf{q}}$, *i.e.* $q_{1k} \cdot q_{2k} > \overline{q}_1 \cdot \overline{q}_2$. *If all goods are less balanced than* $\overline{\mathbf{q}}$, *the salient thinker chooses the most balanced good* k, *namely the good that maximizes* $q_{1k} \cdot q_{2k}$.

The salient thinker picks the good that is most specialized relative to the reference good, provided that good's weakness is not so bad that it is noticed. This choice trades off two forces. On the one hand, keeping the salience ranking fixed, the salient thinker tries to maximize the salient quality along the rational indifference curve. If the good is more balanced than the reference, its salient quality is its advantage relative to the reference. The salient thinker chooses the good which maximizes this advantage, which is measured by the distance $|q_{1k} - \overline{q}_1| = |q_{2k} - \overline{q}_2|$ from the reference. On the other hand, as the good's strength becomes more pronounced at the expense of its weakness, the latter becomes increasingly salient due to diminishing sensitivity.[2] These effects imply that the consumer tends to be

[2]Thus, in quality-quality tradeoffs the salient thinker does not go all the way to the extreme good, as he does in quality-price trade-offs. In fact, along a quality-price indifference curve, an increase in quality is matched by an increase in price, so that diminishing sensitivity causes both attributes to become less salient (Proposition 2). In contrast, along a quality-quality indifference curve one quality increases at the expense

attracted toward goods that are closer to the reference good $\overline{\mathbf{q}}$.

This effect is again different from loss aversion (Tversky and Simonson 1993, Bodner and Prelec 1994) in that consumers do not mechanically prefer middle-of-the-road options. They instead prefer goods that are somewhat specialized in favor of their salient upsides. Unlike in Koszegi and Szeidl (2013)'s "bias towards concentration", specialization here cannot be excessive, because a severe lack of quality in any dimension is highly salient. An uncommonly spacious back seat may enhance consumers' valuation of a car, but not if this comes at the cost of an extremely small trunk. Producers often specialize a little, rarely a lot.

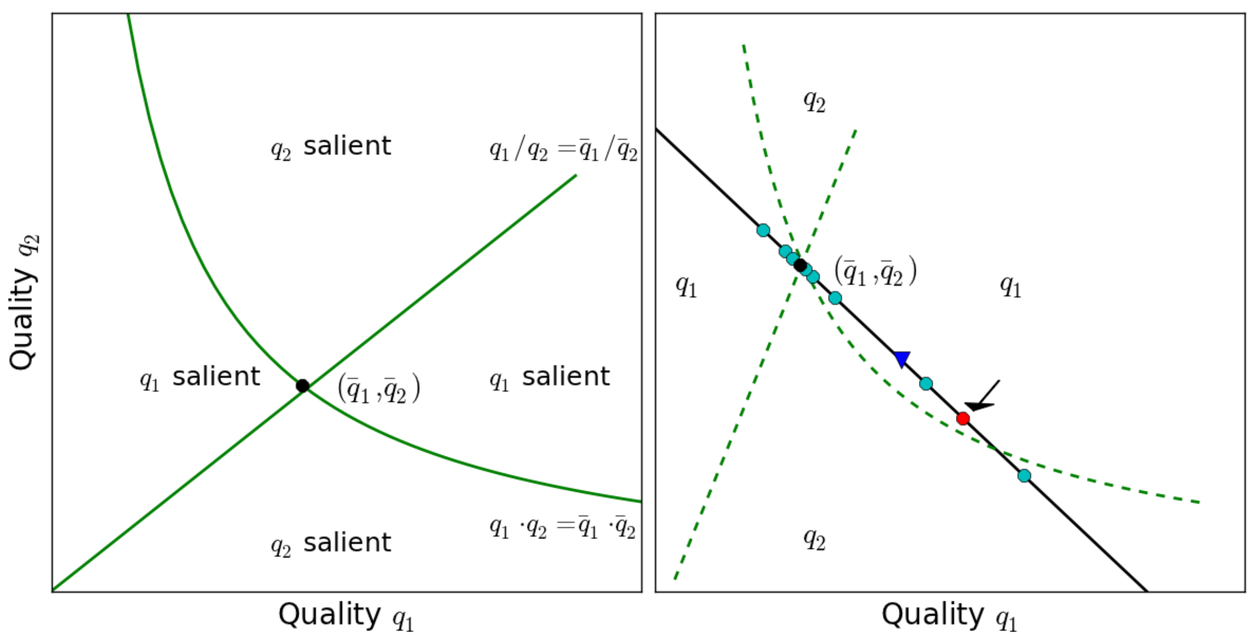

Figure 1: Salience ranking of goods with two quality attributes.

Proposition 1 is illustrated in the right panel of Figure 1. The solid downward sloping curve represents a rational indifference curve with utility level u, along which the choice set is distributed. The average good has a relatively low value $\bar{q}_1$ and a high value $\bar{q}_2$. The convex (dotted) curve represents the (choice-set dependent) iso-salience curve, namely the set of attribute combinations (q_1, q_2) for which $q_1 \cdot q_2 = \bar{q}_1 \cdot \bar{q}_2$. Crucially, the product $q_1 \cdot q_2$ is

of the other. Due to diminishing sensitivity, the reduction in one quality dimension exerts a stronger effect on salience than the increase in the other quality dimension.

concave along the rational indifference curve, with a maximum at the middle good $\left(\frac{u}{2\theta_1}, \frac{u}{2\theta_2}\right)$ (denoted by a triangle). Thus the convex curve separates three regions of the rational indifference curve: a region of low $q_{1,k}$ where $q_1 \cdot q_2 < \bar{q}_1 \cdot \bar{q}_2$ (goods in this region have their weakest quality q_1 salient), a region of low q_2 where $q_1 \cdot q_2 < \bar{q}_1 \cdot \bar{q}_2$ (goods in this region have their weakest quality q_2 salient), and the intermediate range of goods which lie above the iso-salience curve. These goods have a high (balanced) quality product $q_{1,k} \cdot q_{2,k} > \bar{q}_1 \cdot \bar{q}_2$, as well as a higher level of q_1 than the average good. As a consequence, these goods have their q_1 quality salient. It is easy to see that such goods are overvalued relative to all other goods in the choice set. The most overvalued good is the one with the highest q_1 value (arrow) which is chosen.

To jointly characterize the salience ranking of all goods in a general choice set $\mathbf{C}$ we simply need to compute the reference attribute levels, and then place the goods in a diagram such as that of Figure 1 above. The Figure's left panel clearly shows that a good's quality q_{ik} is salient in regions where it is far from the reference quality level $\overline{q}_i$, with $i = 1, 2$, thus allowing us to develop visual intuitions for the role of salience in explaining choices.[3]

Proof of Proposition 1. Consider an indifference curve characterized by $u(q_1, q_2) = q_1 + q_2 = u$, where for simplicity we set $\theta_1 = \theta_2$. The average good $(\overline{q}_1, \overline{q}_2)$ also lies on the indifference curve, and good k's advantage relative to the reference good is salient whenever $q_{1k} \cdot q_{2k} > \overline{q}_1 \cdot \overline{q}_2$. The central point of the indifference curve $(u/2, u/2)$, which maximizes the product of qualities, satisfies $q_{1k} \cdot q_{2k} \leq \frac{u}{2} \cdot \frac{u}{2}$ for all k.

Let $\mathbf{C}_{bal}$ be the set of goods satisfying $q_{1k} \cdot q_{2k} \geq \overline{q}_1 \cdot \overline{q}_2$, where $\overline{\mathbf{q}}$ is the reference good in the choice set. Goods in $\mathbf{C}_{bal}$ have their advantages relative to $\overline{\mathbf{q}}$ salient. Importantly, since all such goods lie closer to the central point of the indifference curve than $\overline{\mathbf{q}}$, they have the same advantage relative to the reference. By diminishing sensitivity, this coincides with $\overline{\mathbf{q}}$'s weak attribute, namely the quality dimension in which $\overline{\mathbf{q}}$ delivers lower utility. Goods in $\mathbf{C}_{bal}$

[3] To identify the upward sloping curve, note that when $\mathbf{q}_k$ dominates the reference (i.e. $q_{1k} > \overline{q}_1$ and $q_{2k} > \overline{q}_2$), then q_{1k} is salient if and only if $\sigma\left(q_{1k}/\overline{q}_1, 1\right) > \sigma(q_{2k}/\overline{q}_2, 1)$, namely if and only if $q_{1k}/q_{2k} > \overline{q}_1/\overline{q}_2$. Instead, when $\mathbf{q}_k$ is dominated by the reference, its quality q_{1k} is salient if and only if $q_{1k}/q_{2k} < \overline{q}_1/\overline{q}_2$.

The figure thus also illustrates the quality price trade off considered in the main text. If we re-interpret the dimension q_2 as a price dimension, it follows that – in the regions where there is a trade-off between $\mathbf{q}_k$ and the reference good $\overline{\mathbf{q}}$, namely $q_{1k} < \overline{q}_1$, $p_k < \overline{p}$ or $q_{1k} > \overline{q}_1$, $p_k > \overline{p}$ – good $\mathbf{q}_k$'s advantage relative to the reference $\overline{\mathbf{q}}$ if salient if and only if $\mathbf{q}_k$'s quality-price ratio is higher than that of the reference.

maybe undervalued (if their weakness coincides with that of the reference) or overvalued. However, since they lie close to the central point, they are less affected by salience than the good lying outside $\mathbf{C}_{bal}$.

Consider now those goods that are less balanced than $\overline{\mathbf{q}}$, namely which lie outside $\mathbf{C}_{bal}$. Diminishing sensitivity implies that these good's disadvantages relative to $\overline{\mathbf{q}}$ are salient. Since any such good lies farther from the central point than $\overline{\mathbf{q}}$, its disadvantage relative to the reference coincides with its weak dimension. As a result all such goods are undervalued. Note that, within this set of goods, the more balanced goods closer to the centre of the indifference curve (namely, with higher $q_{1k} \cdot q_{2k}$) are preferred to the more extreme goods, because their salient disadvantages are less extreme.

To conclude, if $\mathbf{C}_{bal}$ is non-empty, the consumer chooses the good in $\mathbf{C}_{bal}$ which has the highest value along the reference's weak dimension. If $\mathbf{C}_{bal}$ is empty, then the chooses the good which maximizes $q_{1k} \cdot q_{2k}$. ■

B Continuous Salience Distortions

The dependence of valuation distortions on the salience ranking of different attributes (Definition 2) implies that the salient thinker's valuation can jump discontinuously at attribute values where salience ranking changes. Here we provide a continuous formulation where this behavior does not occur. Continuous salience distortions also allows to rule out non-monotonicity in valuation, which may sometimes arise in the salience ranking specification (which may even lead, in finely tuned examples, to a dominated good being preferred over a dominating good).

Take a choice context $\mathbf{C}$ characterized by a given reference good $(\overline{q}, \overline{p})$. We define the salient thinker's valuation of an individual good (q, p) to be:

$$u(q,p) = q \cdot w(q, \overline{q}) - p \cdot w(p, \overline{p}), \tag{4}$$

where w is a continuous weighting function encoding the properties of salience. We later offer a specification that makes this link transparent. Note that this formulation imposes two restrictions: i) salience weights are determined independently for different attributes, and ii) salient weights have the same functional form for all attributes.

The weighting function satisfies the properties of ordering, symmetry and homogeneity of degree zero. Formally, let $k > 0$ be the level of a good's attribute (either quality or price) and let $\overline{k}$ be the reference level of that attribute in a given choice context. Then:

$$\partial_k w(k, \overline{k})\big|_{k \geq \overline{k}} > 0 > \partial_k w(k, \overline{k})\big|_{k < \overline{k}}, \tag{5}$$

$$w(k, \overline{k}) = w(\overline{k}, k), \tag{6}$$

$$w(k, \overline{k}) = w(\alpha k, \alpha \overline{k}), \text{ for any } \alpha > 0 \tag{7}$$

That is, the weight attached to any attribute (quality or price) increases as the value of that attribute becomes more distant from its reference value. The property of reflection follows from the specification that w takes (positive) prices as arguments. As we saw in the text, ordering and homogeneity of degree zero together imply diminishing sensitivity of the weighting function (for positive quality and price levels). For convenience, we also assume that w is bounded.

Due to the assumed continuity of w, valuation in Equation (4) is continuous at any (q, p). For differentiable w, monotonicity in quality and price read as:

$$\partial_q u(q, p) = w(q, \overline{q}) + q \cdot \partial_q w(q, \overline{q}) \geq 0, \tag{8}$$

$$\partial_p u(q, p) = -w(p, \overline{p}) - p \cdot \partial_p w(p, \overline{p}) \leq 0, \tag{9}$$

We proceed in three steps: first, we derive the conditions under which – keeping the reference good fixed – valuation is monotonic. In other words, a cheaper good is perceived to have a price advantage over a more expensive good. Second, we examine when *valuation* exhibits diminishing sensitivity, namely when the price advantage of the cheaper good becomes less pronounced as prices increase (as in the store vs. restaurant example). Finally, we turn to violations of IIA, and show the workings of the decoy effect and of willingness to pay when salience weighting is continuous.

Using homogeneity of degree zero, write $w(q, \overline{q}) = f(q/\overline{q})$. Then the ordering property simply states that $f(x)$ gets larger as x gets further from 1, namely $f'(x) > 0$ for $x > 1$ and

$f'(x) < 0$ for $x < 1$. Moreover, symmetry implies that $f(x) = f(1/x)$ (in particular, $f(x)$ need not be differentiable at $x = 1$).

We now re-write the monotonicity conditions in terms of f and show under what conditions they are satisfied. Consider monotonicity in price. Then (9) becomes $f(p/\overline{p}) + p \cdot \partial_p f\,(p/\overline{p}) > 0$. Note that for $p > \overline{p}$, this condition is guaranteed by the ordering property, namely the second term is positive. As a consequence, monotonicity need only be checked for attribute values below the reference levels for which the second term is negative. Suppose $p < \overline{p}$ and p increases, while $\overline{p}$ stays fixed. Then we get

$$f\,[\overline{p}/p] > \frac{\overline{p}}{p} \cdot f'\,[\overline{p}/p] \tag{10}$$

Since p and $\overline{p}$ are arbitrary, the function $f(x)$ must be concave for $x > 1$.

As an example of a salience weighting function, consider

$$w(p, \overline{p}) = \frac{[1 + \sigma(p, \overline{p})]^{1-\delta}}{2}$$

where $\sigma(\cdot,\cdot)$ is a salience function that satisfies the properties of ordering, symmetry and homogeneity of degree zero (and diminishing sensitivity) which it receives from the weighting function w. Using $f(x) = [1 + \sigma(x, 1)]^{1-\delta}/2$, we can rewrite the monotonicity condition (10) in terms of σ as $x \cdot \partial_x \sigma(x, 1) < \frac{1+\sigma(x,1)}{1-\delta}$ for $x > 1$. Our standard salience function (4) satisfies this condition.

Consider now other properties of the model, starting from the store vs. restaurant comparison. Consider a pairwise choice between goods (q_h, p_h) and (q_l, p_l) where $p_h > p_l$ and where we denote $\overline{p} = (p_h + p_l)/2$. Then a uniform increase Δp in the level of prices induces the consumer to substitute toward the more expensive good provided the difference

$$(p_h + \Delta p) \cdot f\left((p_h + \Delta p)/(\overline{p} + \Delta p)\right) - (p_l + \Delta p) \cdot f\left((\overline{p} + \Delta p)/(p_l + \Delta p)\right)$$

decreases in Δp. Write $R_{\Delta p} = \frac{p_h + \Delta p}{p_l + \Delta p}$, with $\Delta p > 0$. Also, denote $r_{h,\Delta p} = \frac{2R_{\Delta p}}{1+R_{\Delta p}}$ and $r_{l,\Delta p} = \frac{1+R_{\Delta p}}{2}$ the arguments of the salience function for the expensive and cheap good, respectively. Note that $r_{h,\Delta p} < r_{l,\Delta p}$. The above expression can then be rewritten as:

$$(p_h + \Delta p) \cdot f\left(r_{h,\Delta p}\right) - (p_l + \Delta p) \cdot f\left(r_{l,\Delta p}\right), \tag{11}$$

which should decrease with Δp. Differentiating with respect to Δp, we find

$$\begin{aligned} & f\left(r_{h,\Delta p}\right) - f\left(r_{l,\Delta p}\right) + \\ & + \partial_{\Delta p} R_{\Delta p} \cdot \frac{p_l + \Delta p}{1 + R_{\Delta p}} \left[f'\left(r_{h,\Delta p}\right) \cdot r_{h,\Delta p} - f'\left(r_{l,\Delta p}\right) \cdot r_{l,\Delta p} \right] \end{aligned} \tag{12}$$

To analyze this expression, recall that $r_{h,\Delta p} < r_{l,\Delta p}$. The first line is negative by monotonicity of f. This is the direct effect of diminishing sensitivity of the salience function, which ensures that the salience of price is lower for the expensive good than for the cheap good. The second line captures instead that differential effect of price level in the price salience of the two goods. Because f is concave, this effect is larger (more negative) for the more expensive good, $f'\left(r_{h,\Delta p}\right) > f'\left(r_{l,\Delta p}\right)$. In particular, a sufficient condition for (12) to be negative is that

$$\partial_x \left[f'(x) \cdot x \right] \leq 0$$

This holds as long as f grows at most as fast as the logarithmic function. In particular, it holds for our example $f = \left[1 + \sigma(x,1)\right]^{1-\delta}/2$.

We now turn to the analysis of violations of IIA. We begin with the decoy effect. The workings of the decoy effect follow in a straightforward manner from the ordering property. To see that, consider again the context of a pairwise choice. As before, the price advantage of the cheaper good is given by

$$p_h \cdot f\left(p_h/\overline{p}\right) - p_l \cdot f\left(\overline{p}/p_l\right) \tag{13}$$

Suppose a decoy option (q_d, p_d) is introduced in the choice set, such that $q_d \geq q_h$ and $p_h > p_h$.

The resulting reference price is equal to $\overline{p}' = (p_h + p_l + p_d)/3$ which locates closer to p_h relative to $\overline{p}$. Then the price advantage of the cheaper good strictly decreases because of ordering, since the price p_h becomes less salient (the ratio $p_h/\overline{p}$ goes down) while the price

p_l becomes more salient (the ratio $\overline{p}/p_l$ goes up). Similarly, the quality advantage of the higher quality good decreases: since the reference quality moves closer to q_h, the quality salience of the high quality good decreases relative to that of the low quality good. The net effect on the relative valuation of the goods depends on which effect dominates: if the price advantage decreases more than the quality advantage, then the decoy benefits the high quality good. Intuitively, this holds when the reference price becomes close to p_h, while q_h is still significantly higher than $\overline{q}$.

To proceed, suppose for simplicity that $q_h = p_h$ and $q_l = p_l$. In particular, the consumer is indifferent between the goods in a pairwise choice. Suppose further that the effect of the decoy good is to change the reference good as $\overline{q} \to \lambda_q \overline{q}$ and $\overline{p} \to \lambda_p \overline{p}$, where λ_q, λ_p are small. Then one can show that the price advantage of good l decreases by more than the quality advantage of good h if and only if $\lambda_p > \lambda_q$, in particular if and only if the decoy leads to a drop in the quality price ratio. Note that this setting describes two possible types of decoy: a decoy for the high quality good, where e.g. $q_d \geq q_h$ and $p_d > p_h$, but also a decoy for the low quality good, where e.g. $q_d \leq q_l$ and $p_d > p_l$.

Finally, we turn to the determination of willingness to pay for quality. Let the choice context be $\mathbf{C} = \{(0,0), (q,p), (q,p_\gamma)\}$, where p_γ is the expected price of quality q in context γ (e.g. the store or the resort). Since the reference quality is $\overline{q} = 2q/3$, the salience weight of quality is $f(3/2)$. The salience weight for price is, in turn, $f(p/\overline{p})$, where $\overline{p} = (p + p_\gamma)/3$. According to the definition in the text, the willingness to pay $\text{WTP}(q)$ for quality q in the choice context $\mathbf{C}$ is the maximum price p such that $q \cdot f(3/2) - p \cdot f(p/\overline{p}) \geq 0$. In other words, $\text{WTP}(q)$ satisfies

$$\text{WTP}(q) \cdot f\left(1, \frac{p_\gamma/\text{WTP}(q) + 1}{3}\right) = q \cdot f(3/2) \tag{14}$$

To gain insight into this expression, note that the salience weighting on the LHS reaches its minimum $f(1)$ when $\text{WTP}(q) = p_\gamma/2$. Suppose p_γ is such that $p_\gamma/2 \cdot f(1) = q \cdot f(3/2)$. In this case, the willingness to pay is exactly $p_\gamma/2$, as can be seen by direct substitution into (14). Moreover, if $p_\gamma\ 2 \cdot f(1) < q \cdot f(3/2)$, it must be that $\text{WTP}(q) > p_\gamma/2$, since the left hand side of (14) increases as WTP rises above $p_\gamma/2$.[4] As a consequence, WTP(q) increases with p_γ. To see this, suppose (14) is satisfied and then increase p_γ. Then the willingness to pay increases in order to compensate the reduction in the salience weighting.

Consider now the case where $p_\gamma/2 \cdot f(1) > q \cdot f(3/2)$. A reasoning similar to the above shows that now WTP(q) $< p_\gamma/2$. Note that in this regime a solution to (14) always exists since the LHS goes to zero with WTP(q) (as long as f is bounded, as assumed). Moreover, as p_γ increases, the salience weighting increases as well, causing WTP(q) to fall.

Sumarizing, the condition (14) defines WTP(q) as a function of the expected price p_γ, taking q as given. This function is inverse-U-shaped, increasing with expected price p_γ for p_γ up to $q \cdot K$ (where $K = 2f(3/2)/f(1) > 1$) and decreasing with expected price above that. At its maximum value, willingness to pay satisfies WTP(q) $= q \cdot f(3/2)/f(1) > q$.

C Price Shocks and Consumer Demand

Hastings and Shapiro (2013) show that consumers react to parallel increases in gas prices by switching to cheaper (and lower quality) gasoline, and to parallel decreases in gas prices by switching to more expensive (higher quality) gasoline. Here we show how this pattern emerges in our model when consumers hold rational expectations for gasoline prices at the time of choosing which gasoline to purchase.

There are two grades of gas, with qualities $q_h > q_l$ and prices p_{ht}, p_{lt} at time t. At each t, the consumer must buy one unit of gas and must decide which grade to buy. Here, we assume that gas prices follow a random walk, so that the consumer's expectation for gas prices for the current period t is simply the realisation of prices in the previous period $t-1$. This captures the intuition that when the consumer chooses gas, he recalls gas prices from

[4]There can also be a solution to (14) below $p_\gamma/2$ but by definition WTP is the largest solution satisfying (14).

the last time he bought gas. [5] As a result, his choice context is:

$$\mathbf{C}_t = \{(q_h, p_{ht}), (q_l, p_{lt}), (q_h, p_{h,t-1}), (q_l, p_{l,t-1})\}.$$

Following Hastings and Shapiro (2013), we focus on parallel price shifts $p_{ht} - p_{h\ t-1} = p_{lt} - p_{l\ t-1} = \Delta_t$.

In the choice context $\mathbf{C}_t$, the reference quality and price are given by

$$\overline{q}_t = \frac{q_h + q_l}{2}, \qquad \overline{p}_t = \frac{p_{h,t-1} + p_{l,t-1} + \Delta_t}{2}.$$

Suppose that the two grades yield the same intrinsic utility to the consumer, namely $q_h - p_{ht} = q_l - p_{lt}$. In this case, demand is fully determined by salience: the consumer chooses the high grade gas if and only if its quality is salient. The salience function $\sigma(\cdot, \cdot)$ satisfies the usual properties of diminishing sensitivity, ordering and symmetry, as well as homogeneity of degree zero. The salience of quality and price for the high quality gas are:

$$\sigma(q_h, \overline{q}) = \sigma\left(\frac{2}{1 + q_l/q_h}, 1\right), \qquad \sigma(p_{ht}, \overline{p}_t) = \sigma\left(\frac{2}{1 + \frac{p_{t-1,l}}{p_{th}}}, 1\right). \tag{15}$$

The most intuitive case is one in which, after the parallel price change Δ_t, the high grade gas is still more expensive than the reference price $\overline{p}_t$. This condition is equivalent to $\Delta_t + (p_{ht} - p_{lt}) > 0$. It is satisfied as long as the price shock is not too negative between two visits at the gas station. We later discuss what happens when $\Delta_t + (p_{ht} - p_{lt}) < 0$.

From Equation (15), q_h is salient (and thus the high grade gas is chosen) when:

$$\frac{q_h}{p_{h,t-1} + \Delta_t} > \frac{q_l}{p_{l,t-1}}. \tag{16}$$

which is satisfied provided Δ_t is sufficiently low (it is always fulfilled for $\Delta_t + (p_{ht} - p_{lt}) = 0$).

The demand for low quality gas decreases, namely Equation (16) is more likely to hold,

[5] An alternative specification would be to assume a static price distribution. In this case the expected price would be fixed for all t. If realised prices are above the expected price (e.g. due to a temporary oil shock), then salience of gas price increases with the realised price, from which the Hastings and Shapiro evidence follows. By assuming instead that prices follow a random walk, we show that this prediction is very robust to assumptions about price paths.

when there is a sufficiently large drop in gas prices (i.e., Δ_t is sufficiently negative). The demand for low quality gas increases, namely Equation (16) is less likely to hold, when there is a sufficiently large hike in gas prices (i.e., Δ_t is sufficiently positive). In particular, suppose that in the previous two visits at the gas station the price of gas was stable, namely $\Delta_{t-1} = 0$. Then, the change in the demand for the low grade gas between $t-1$ and t as a function of the price change Δ_t is plotted in Figure 1 (where W_{t-1} is a constant determined below).

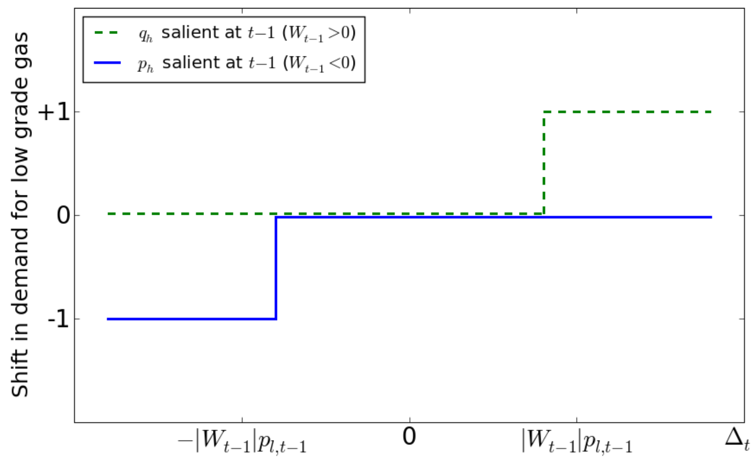

Figure 2: Price shocks and shifts in demand for the low grade gas.

Three features stand out:

- The demand for low grade gas tracks price changes. A sufficiently large price hike ($\Delta_t > 0$) increases the demand for low grade gas, while a sufficiently large price drop ($\Delta_t < 0$) decreases it. The intuition is that when the price of gas increases, the consumer views the current high grade price as a bad deal relative to yesterday. This renders its price salient. When the price of gas drops, the consumer sees the current high grade as a good deal relative to yesterday. This renders its quality salient. Thus, salience predicts history dependence in the demand for gas at given price levels.

- Demand changes only if the price change is sufficiently large. This is because small price changes do not affect salience.

- Demand is more sensitive to a given price change Δ_t when the price level $p_{l,t-1}$ is low.

This is because at lower price levels a given price change is more noticeable, due to diminishing sensitivity. Thus, salience predicts history dependence in the reaction of demand for gas to a given price change, even with linear utility.

Two further comments. First, consider large price drops such that $\Delta_t + (p_{ht} - p_{lt}) < 0$. In this case, it is still true that demand for the low grade gas decreases, but only up to a threshold drop $\widehat{\Delta} < 0$. For $\Delta_t < \widehat{\Delta}$ price becomes salient and thus the consumer again chooses the low grade gas. We can ignore this case, however, as for a reasonable difference of grade qualities q_h, q_l the required price drop $\widehat{\Delta}$ is of the order of the price level $p_{l,t-1}$ itself.[6]

Second, to fully appreciate the implications of history dependence, the model should be studied for all possible past price changes Δ_{t-1} (remember that here we restricted to the case $\Delta_{t-1} = 0$ for simplicity).

Let us go back to the determination of the threshold level W_{t-1}. To study the change in demand between $t-1$ and t we need to determine demand at $t-1$ when $\Delta_{t-1} = 0$. Iterating Equation (16) backward, the consumer picks the high grade gas at $t-1$ if and only if:

$$\frac{q_h}{p_{h,t-1}} > \frac{q_l}{p_{l,t-1}}. \tag{17}$$

According to Equations (16,17), the demand for high grade gas increases fom 0 to 1 when

$$\frac{p_{h,t-1}}{p_{l,t-1}} + \frac{\Delta_t}{p_{l,t-1}} < \frac{q_h}{q_l} < \frac{p_{h,t-1}}{p_{l,t-1}}.$$

This requires a sufficiently large price drop $\Delta_t < 0$. In contrast, the demand for high grade gas decreases from 1 to 0 when

$$\frac{p_{h,t-1}}{p_{l,t-1}} < \frac{q_h}{q_l} < \frac{p_{h,t-1}}{p_{l,t-1}} + \frac{\Delta_t}{p_{l,t-1}},$$

which requires a sufficiently large price hike Δ_t. To construct Figure 1, denote $W_{t-1} = \frac{q_h}{q_l} - \frac{p_{h,t-1}}{p_{l,t-1}}$. Condition (17) becomes $W_{t-1} > 0$, while condition (16) reads $\Delta_t < W_{t-1} \cdot p_{l,t-1}$. Note that the thresholds $|W_{t-1}| \cdot p_{l,t-1}$ increase in absolute value with the price level $p_{l,t-1}$.

[6]The precise threshold is $\widehat{\Delta}_t = \frac{1+\lambda}{3\lambda-1} p_{l,t-1} - p_{h,t-1}$, where $\lambda = q_h/q_l$. In particular, $\widehat{\Delta}_t = -p_{l,t-1}$ when $p_{h,t-1}/p_{l,t-1} = 4\lambda/(3\lambda - 1)$.