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# Diagnostic Expectations and Stock Returns

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**ABSTRACT**

We revisit La Porta's finding that returns on stocks with the most optimistic analyst long-term earnings growth forecasts are lower than those on stocks with the most pessimistic forecasts. We document the joint dynamics of fundamentals, expectations, and returns of these portfolios, and explain the facts using a model of belief formation based on the representativeness heuristic. Analysts forecast fundamentals from observed earnings growth, but overreact to news by exaggerating the probability of states that have become more likely. We find support for the model's predictions. A quantitative estimation of the model accounts for the key patterns in the data.

La Porta (1996) shows that stock market analysts' expectations of the long-term earnings growth of the companies they cover have strong predictive power for these companies' future stock returns. Companies whose earnings growth analysts are most optimistic about earn poor returns relative to companies whose analysts are most pessimistic about. Betting against extreme analyst optimism has thus been a good idea on average. La Porta (1996) interprets this finding as evidence that analysts, as well as investors who follow them or think like them, are too optimistic about stocks with rapidly growing earnings and too pessimistic about stocks with deteriorating earnings. As a result, the former stocks are overvalued, the latter are undervalued, and the predictability of returns is driven by the correction of expectations.

In this paper, we revisit this puzzle using a model of Kahneman and Tversky's (1972) representativeness heuristic developed by Gennaioli and Shleifer (2010) and Bordalo et al. (2016). Relative to the existing work, we make three innovations. First, we rely on a portable and psychologically founded model that has been used to describe the dynamics of beliefs in a variety of settings, not only in the one at hand. Second, we assess the qualitative and quantitative performance of this model in explaining not only a cross-section of stock

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returns, but also the paths of fundamentals leading to overvaluation and of analyst expectations. Third, we test new predictions that distinguish our model from mechanical models of extrapolation such as adaptive expectations. These new predictions follow from the fact that in our model expectations contain a "kernel of truth" in that they exaggerate true features of reality. Beliefs thus move in the correct direction, but by more than the right amount.

As a first step, we look at the data (described in Section I). In Section II, we confirm with 20 additional years of data La Porta's (1996) finding that stocks with low long-term earnings growth forecasts (LLTG stocks) outperform stocks with high long-term earnings growth forecasts (HLTG stocks). We then present three additional facts. First, HLTG stocks exhibit fast past earnings growth, which slows down going forward. Second, forecasts of future earnings growth of HLTG stocks are too optimistic, and are systematically revised downward later. Third, HLTG stocks exhibit good past returns but their returns going forward are low. The opposite dynamics obtain for LLTG stocks, but in a less extreme form. We do not account for this asymmetry in our model.

The data suggest that analysts use a firm's past performance to infer its future performance, but overreact. This updating mechanism arises naturally from overweighting of representative types. Gennaioli and Shleifer (2010) and Bordalo et al. (2016) model a type $t$ as representative of a group $G$ when it occurs more frequently in that group than in a reference group $-G$. For instance, after a positive medical test, the representative patient is $t$ = "sick" because sick people are truly more prevalent among those who tested positive than in the overall population. After such a positive test, the representative sick type quickly comes to mind and the doctor inflates its probability too much, which may still be objectively low if the disease is rare (Casscells, Schoenberger, and Graboys (1978)). There is a kernel of truth in departures from rationality: the doctor overreacts to the objectively useful information from the test.

In Section III, we incorporate this model of representativeness into a problem of an analyst learning about a firm's unobserved fundamentals in light of a noisy signal such as current earnings. This approach yields a distorted Kalman filter, which we refer to as the "diagnostic Kalman filter," that overinflates the probability of future earnings growth realizations whose likelihood has objectively increased the most in light of recent news. After strong earnings growth, the probability that the firm is the next "Google" goes up. This type becomes representative and analysts inflate its probability excessively, even though Googles remain rare in absolute terms. As good news stops arriving, overoptimism cools off. Rapid earnings growth thus causes overvaluation and subsequent disappointment, leading to reversals of optimism and low returns.

In Section IV, we show that this model makes predictions consistent with the evidence in Section II. In Section V, we perform a first pass quantitative assessment of the model. We first show that expectations data point to overreaction to news, in the sense that revisions of the forecast of LTG negatively predict errors in that forecast. We then use the simulated method of moments (SMM) to estimate the parameter controlling the strength of representativeness by matching observed overreaction and the empirical moments of the earnings process. We find that forecasters react about twice as much to information as is objectively warranted. This estimate lies in the ballpark of those obtained using the expectations of professional forecasters about credit spreads (Bordalo, Gennaioli, and Shleifer (2018)) and many macroeconomic variables (Bordalo et al. (2018)). We also find that stocks overreact to news over a time scale of about three years. Notably, while the estimation uses expectations and fundamentals alone, it predicts the observed return spread between LLTG and HLTG stocks reasonably well.

In Section VI, we test three novel predictions of the model, all following from the kernel-of-truth property as applied to the present dynamic setting. First, we show that the HLTG group contains a fat right tail of future exceptional performers, and that analysts attach an excessively high probability that HLTG firms are in that tail. There are very few Googles, but they are concentrated in the HLTG group, and analysts exaggerate their frequency. This is precisely what the kernel-of-truth property of expectations predicts.

Second, we show that the return spread between LLTG and HLTG stocks widens among firms with more volatile or persistent fundamentals. This is also in line with the kernel of truth: in both cases, good news is even more informative about strong future performance, which renders Googles even more representative. Third, we show that expectations about HLTG (LLTG) stocks revert downward (upward) even in the absence of bad (good) news. This is again in line with the kernel of truth: analyst forecasts reflect the true mean reversion in earnings growth.

Mechanical models of beliefs, such as adaptive expectations, cannot yield these facts, which are due to the forward-looking but not fully rational learning that we see in the data.

Our paper follows extensive research on overreaction and volatility that begins with Shiller (1981), De Bondt and Thaler (1985, 1987), Cutler, Poterba, and Summers (1990, 1991), and De Long et al. (1990a, 1990b). This work often uses mechanical rules for belief updating such as adaptive expectations or adaptive learning (e.g., Barsky and De Long (1993), Barberis et al. (2015), Adam, Marcet, and Beutel (2017)).[1] Barberis, Shleifer, and Vishny (1998) is closest in spirit, though not in formulation, to our current work, since it is also motivated by representativeness. The authors present a model of Bayesian learning in which the decision maker is trying to distinguish models that are all incorrect. Because they do not model representativeness explicitly, their specification does not allow for a tight link between measurable reality and measurable beliefs that is central to our theory and evidence. Daniel, Hirshleifer, and Subramanyam (1998) and Odean (1998) model investor overconfidence, the tendency of

[1] Other papers include Barberis and Shleifer (2003), Glaeser and Nathanson (2015), Hong and Stein (1999), Lakonishok, Shleifer, and Vishny (1994), Marcet and Sargent (1989), and Adam, Marcet, and Nicolini (2016). In Adam, Marcet, and Beutel (2017), agents learn the mapping between fundamentals and prices, but they are rational about fundamentals. Pastor and Veronesi (2003, 2005, 2009) present rational learning models in which uncertainty about the fundamentals of some firms can yield predictability in aggregate stock returns. This approach does not analyze expectations data or cross-sectional differences in returns.

decision makers to exaggerate the precision of private information, which leads to inaccurate beliefs and excess trading. Unlike overconfidence, our model yields overreaction to not just private but also public information, such as earnings news.

Our model is based on a psychologically founded distortion that applies whenever agents make probability judgments, and is not specific to asset pricing. It allows us to unify several biases hitherto viewed as separate, such as extrapolation, overreaction to information, and neglect of tail risk. The model is portable to different domains (Rabin 2013), and has been used to shed light on fallacies in probabilistic judgments (Gennaioli and Shleifer (2010), social stereotypes (Bordalo et al. (2016)), beliefs about gender (Bordalo et al. (2019)), credit cycles (Bordalo, Gennaioli, and Shleifer (2018)), and macroeconomic forecasts (Bordalo et al. (2018)). Unification of biases, quantification, and cross-context applicability discipline the model, helping to identify realistic and robust alternatives to rational expectations.

We use expectations data not only to predict returns, but also as a central element of the model and its empirical estimation.[2] Such data are becoming increasingly common in recent work (Ben David, Graham, and Harvey (2013), Greenwood and Shleifer (2014), Gennaioli, Ma, and Shleifer (2016)). Bouchaud et al. (2018) use analyst expectations data to study the profitability anomaly and offer a model in which expectations underreact to news, in contrast with our focus on overreaction. As we show in Section V.A, in our data there is also some short-term underreaction, but at the long horizons of long-term growth (LTG) forecasts overreaction prevails. Daniel, Klos, and Rottke (2017) find that stocks featuring high dispersion in analyst expectations and high illiquidity earn high returns, but do not offer a theory of expectations and their dispersion.

We conclude in Section VII with a summary of our results and some observations on the usefulness of forward-looking but not fully rational models of expectations in finance.

## I. Data and Summary Statistics

### A. Data

We gather data on analysts' expectations from IBES, stock prices and returns from CRSP, and accounting information from CRSP/Compustat. Below we describe the measures used in the paper and, in parentheses, provide their mnemonics in the primary data sets.

From the IBES Unadjusted U.S. Summary Statistics file, we obtain mean analysts' forecasts for earnings per share (EPS) and their expected long-run growth rate (*meanest,* henceforth "LTG") for the period December 1981, when LTG becomes available, through December 2016. IBES defines LTG as the "expected annual increase in operating earnings over the company's next full

[2] Early work using beliefs data includes Dominguez (1986) and Frankel and Froot (1987, 1988). A large literature on analyst expectations shows that on average they are too optimistic (Easterwood and Nutt (1999), Michaely and Womack (1999), Dechow, Hutton, and Sloan (2000)).

business cycle," a period ranging from three to five years. From the IBES Detail History Tape file, we obtain analyst-level data on earnings forecasts. We use CRSP daily data on stock splits (*cfacshr*) to adjust IBES EPS figures. On December of each year between 1981 and 2015, we form LTG decile portfolios based on stocks that report earnings in U.S. dollars.[3]

The CRSP sample includes all domestic common stocks listed on a major U.S. stock exchange (i.e., NYSE, AMEX, and NASDAQ) except for closed-end funds and real estate investment trusts. Our sample starts in 1978 and ends in 2016. We present results for both buy-and-hold annual returns and daily cumulative abnormal returns for various earnings' announcement windows. We compute annual returns by compounding monthly equally weighted returns for LTG portfolios. If a stock is delisted, whenever a post-delisting price exists in CRSP we use it in the computations. When CRSP is unable to determine the value of a stock after delisting, we assume that the investor was able to trade at the last quoted price. Given that IBES surveys analysts around the middle of the month (on Thursday of the third week of the month), LTG is in the information set when we form portfolios. Daily cumulative abnormal returns are defined relative to CRSP's equally weighted index. We also gather data on market capitalization in December of year $t$ as well as the preformation three-year return ending on December of year $t$. Finally, we rank stocks into deciles based on market capitalization using breakpoints for NYSE stocks.

We collect from the CRSP/Compustat merged file data on assets (*at*), sales (*sale*), net income (*ni*), book equity, common shares used to calculate EPS (*cshpri*), the adjustment factor for stock splits (*adjex_f*), and *Wall Street Journal* dates for quarterly earnings releases (*rdq*). Our CRSP/Compustat data cover the period 1978 to 2016. We use annual and quarterly accounting data. We define book equity as stockholders' equity (depending on data availability *seq*, $ceq + pfd$, or *at-lt*) plus deferred taxes (depending on data availability *txditc* or $txdb + itcb$) minus preferred equity (depending on data availability *pstkr*, *pstkl*, or *pstk*). We define operating margin as the difference between sales and cost of goods sold (*cogs*) divided by assets, and return on equity as net income divided by book equity. We compute the annual growth rate in sales per share in the most recent three fiscal years. When merging IBES with CRSP/Compustat, we follow the literature and assume that data for fiscal periods ending after June become available during the next calendar year.

### B. Summary Statistics

Table I reports statistics for LTG decile portfolios. The number of stocks with CRSP data on stock returns and IBES data on LTG varies by year, ranging from 1,310 in 1981 to 3,849 in 1997. On average, each LTG portfolio contains 236 stocks. The forecasted growth rate in EPS ranges from 4% for the lowest

[3] We form portfolios in December of each year because IBES data on LTG start in December of 1981. Unlike Fama and French (1993), we know exactly when the information required for an investable strategy is public.

**Table I**

**Descriptive Statistics for Portfolios Formed on *LTG***

We form decile portfolios based on analysts' expected growth in EPS (*LTG*) in December of each year between 1981 and 2015. The table reports time-series means of the variables described below for equally-weighted *LTG* portfolios. Unless otherwise noted, accounting variables pertain to the most recently available fiscal year, where we follow the standard assumption that data for fiscal periods ending after June become available during the next calendar year. *Assets* is book value of total assets (in millions). *Market capitalization* is the value of common stock on the last trading day of year $t$ (in millions). *Size decile* refers to deciles of market capitalization with breakpoints computed using only NYSE stocks. *Years publicly traded* is the number of years since the first available stock price on CRSP. *%Listed after 5 years* is the percentage of firms that remain publicly traded five years after the formation period. *Operating margin to assets* is the difference between sales and cost of goods sold divided by assets. *Return on equity* is net income divided by book equity. *Percent EPS positive* is the fraction of firms with positive earnings. *Observations* is the number of observations in a year. All variables are capped at the 1% and 99% levels.

| | *LTG* Decile | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| Expected growth EPS (*LTG*) | 4% | 9% | 11% | 12% | 14% | 16% | 18% | 20% | 25% | 38% |
| *Assets* ($M) | 7,123 | 9,387 | 9,398 | 7,508 | 5,008 | 3,591 | 2,340 | 1,974 | 1,373 | 1,027 |
| *Market capitalization* ($M) | 3,862 | 4,692 | 5,316 | 4,552 | 3,967 | 3,270 | 2,749 | 2,431 | 1,679 | 1,641 |
| *Size decile* | 5.0 | 5.0 | 5.4 | 5.1 | 5.0 | 4.5 | 4.4 | 3.8 | 3.5 | 3.4 |
| *Years publicly traded* | 27.5 | 24.4 | 23.0 | 21.3 | 18.9 | 15.9 | 14.0 | 11.0 | 8.5 | 6.8 |
| *%Listed after 5 years* | 70% | 66% | 68% | 69% | 68% | 67% | 67% | 65% | 62% | 60% |
| *Operating margin to assets* | 22% | 27% | 32% | 35% | 39% | 41% | 44% | 44% | 43% | 39% |
| *Return on equity* | 5% | 7% | 9% | 10% | 10% | 8% | 10% | 7% | 2% | -7% |
| *Percent EPS positive* | 88% | 90% | 93% | 94% | 93% | 91% | 92% | 88% | 82% | 69% |
| *Observations* | 246 | 246 | 230 | 244 | 237 | 249 | 214 | 236 | 235 | 222 |

LTG decile (LLTG) to 38% for the highest decile (HLTG), an enormous difference. LLTG stocks are larger than HLTG stocks in terms of both total assets (7,123 million vs. 1,027 million) and market capitalization (3,862 million vs. 1,641 million). However, differences in size are not extreme: the average size decile is 5.0 for LLTG and 3.4 for HLTG. Finally, LLTG firms are older than HLTG firms (27.5 years vs. 6.8 years) and more likely to remain publicly traded during the five years following the formation period (70% vs. 60%).

LLTG stocks have lower operating margins to asset ratios than HLTG stocks but higher return on equity (5% vs. –7%). In fact, 31% of HLTG firms have negative EPS while the same is true for only 12% of LLTG stocks. The high

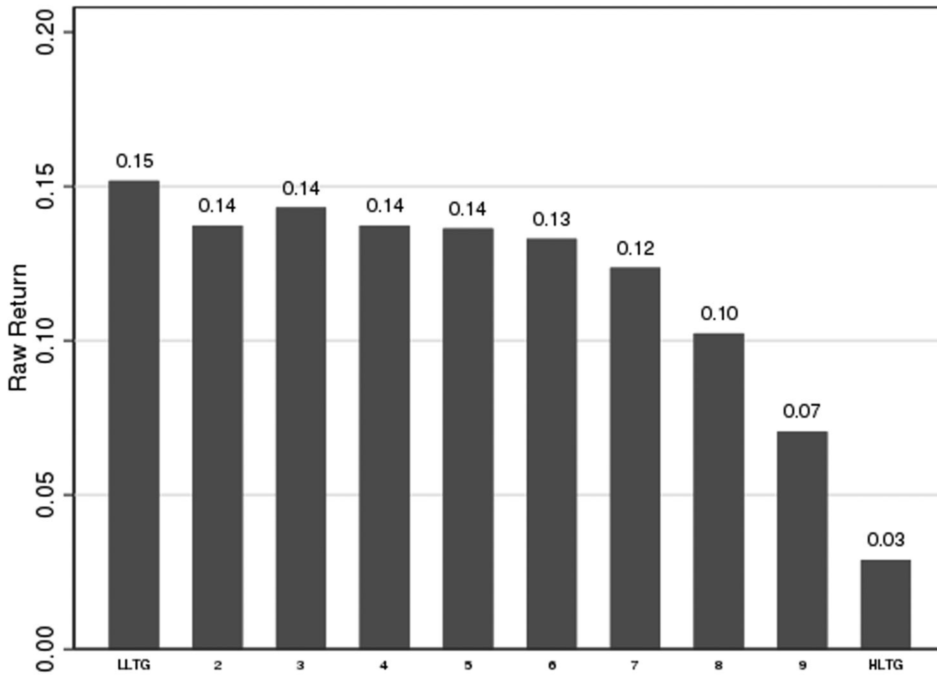


**Figure 1. Annual returns for portfolios formed on LTG.** In December of each year between 1981 and 2015, we form decile portfolios based on ranked analysts' expected growth in long-term EPS and display the geometric average one-year return over the subsequent calendar year for equally weighted portfolios with monthly rebalancing. A portfolio that is long LLTG stocks and short HLTG earns an average yearly (log) return of 13.6%, with a $t$-statistic of 2.22.

incidence of negative EPS companies in the HLTG portfolio underscores the importance of the definition of LTG in terms of annual earnings growth over a full business cycle. Current negative earnings do not hinder these firms' future prospects.

## II. A New Look at the Data

To revisit the La Porta (1996) finding, we sort stocks by analysts' forecast of their long-term growth in EPS (LTG). The LLTG portfolio is the 10% of stocks with the most pessimistic forecasts, and the HLTG portfolio is the 10% of stocks with the most optimistic forecasts. Figure 1 displays geometric averages of one-year returns on equally weighted portfolios.

Consistent with La Porta (1996), the LLTG portfolio earns a compounded average return of 15% in the year after formation, while the HLTG portfolio earns only 3%.[4] Adjusting for systematic risk only deepens the puzzle: the

[4] The spread in Figure 1 is in line with, although smaller than, previous findings. La Porta (1996) finds an average yearly spread of 20% but employs a shorter sample (1982 to 1991). Dechow

HLTG portfolio has higher market beta than the LLTG portfolio, and performs worse in market downturns (see Table IA.I in the Internet Appendix). A conventional factor analysis reveals that the excess return of LLTG stocks survives controlling for three Fama-French factors, although it is positively correlated with the value premium and becomes insignificant after additionally controlling for profitability or for betting against beta (Table IA.III in the Internet Appendix). This is unsurprising: LLTG stocks have low market-to-book ratios, low betas, and good profitability despite their low earnings growth.[5] Kozak, Nagel, and Santosh (2018) show that it is generally incorrect to interpret these factor controls as adjustments for fundamental risk.

So why do LLTG stocks appear undervalued and HLTG overvalued? To assess the expectations-based hypothesis that analysts and the stock market may be too bullish on firms they are optimistic about and too bearish on firms they are pessimistic about, we document some basic facts connecting firms' performance, expectations, and returns.

Figure 2 displays average EPS of HLTG and LLTG portfolios in years $t-3$ to $t+3$, where $t=0$ corresponds to portfolio formation. We normalize year $t-3$ EPS of both portfolios to \$1. EPS for HLTG stocks exhibits explosive growth during the preformation period, rising from \$1 in year –3 to \$1.56 in year 0. Earnings of LLTG firms decline to \$0.92 during the corresponding period. But the past does not repeat itself after portfolio formation: earnings growth of HLTG firms slows down, while earnings of LLTG firms recover during the postformation period. HLTG firms remain more profitable on average than LLTG firms three years after formation, but actual growth rates are much closer than the LTG differences in Table I.

Figure 3 displays the average LTG for the HLTG and LLTG portfolios over the same time window. Prior to portfolio formation, expectations of LTG for HLTG firms rise dramatically in response to strong earnings growth (compare with Figure 2), while expectations for LLTG drop. After formation, expectations for HLTG firms are revised downward, particularly during the first year, whereas those for LLTG firms are revised up. Three years after portfolio formation, earnings of HLTG firms are still expected to grow faster than those of LLTG firms, but the spread in expected growth rates of earnings has narrowed considerably. Data attrition is not responsible for this or other findings in this section: the level of attrition is similar across HLTG and LLTG portfolios (40% for HLTG and 30% for LLTG five years postformation). The findings of the figures also hold when we restrict attention to firms that survive for five years after formation.

Expectations of LTG follow the pattern of actual EPS of HLTG and LLTG portfolios displayed in Figure 2. Analysts seem to be learning about firms'

and Sloan (1997) use a similar sample to La Porta (1996) and find a 15% spread. Figure IA.1 in the Internet Appendix shows that the spread also holds in sample subperiods. The Internet Appendix is available in the online version of this article on the *Journal of Finance* website.

[5] The LLTG-HLTG spread stays roughly constant across momentum terciles and size terciles; see Tables IA.II and IA.III in the Internet Appendix (because the underperforming HLTG stocks tend to be small, the usual rationale for value-weighting does not apply).

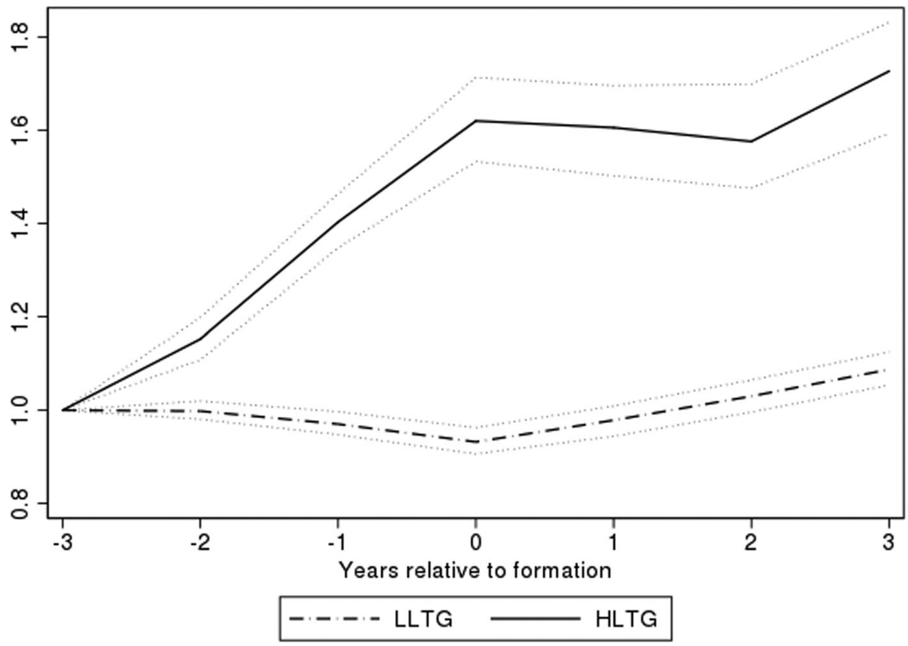


**Figure 2. Evolution of EPS.** In December of years ($t$) 1981, 1984, ... 2011, and 2014, we form decile portfolios based on ranked analysts' expected growth in long-term EPS (LTG). We report the (bootstrapped) mean value of EPS for the highest (HLTG) and lowest (LLTG) LTG deciles for each year between $t-3$ and $t+3$. We exclude firms with negative earnings in $t-3$ and normalize to 1 the value of EPS in $t-3$. To do so, we restrict the computation to firms that have positive earnings at $t-3$. The dotted lines indicate 5th and 95th confidence levels determined via nonparametric bootstrapping using 1,000 samples (the results are robust to changing the size of sample draws).

earnings growth from their past performance. Fast preformation growth leads analysts to place a firm in the HLTG category. Postformation growth slowdown triggers a downward revision of forecasts.

Mean reversion in forecasts may be caused by mean reversion in fundamentals, which is evident in Figure 2, or by the correction of analysts' expectations errors. To examine the role of expectations errors, Figure 4 displays the difference between realized earnings growth and analysts' LTG expectations in each portfolio, from formation to year $t+3$. We find strong overoptimism, that is, very negative forecast errors, for HLTG firms. We also find overoptimism for LLTG firms, consistent with previous studies and usually explained by distorted analyst incentives (Dechow, Hutton, and Sloan (2000), Easterwood and Nutt (1999), Michaely and Womack (1999)).

The overestimation of earnings growth for HLTG firms is economically large. By year 3, actual earnings are a small fraction of what analysts forecast: EPS grows from 0.16 at formation to 0.21 in $t+3$, compared to the prediction of 0.70 based on LTG at formation.

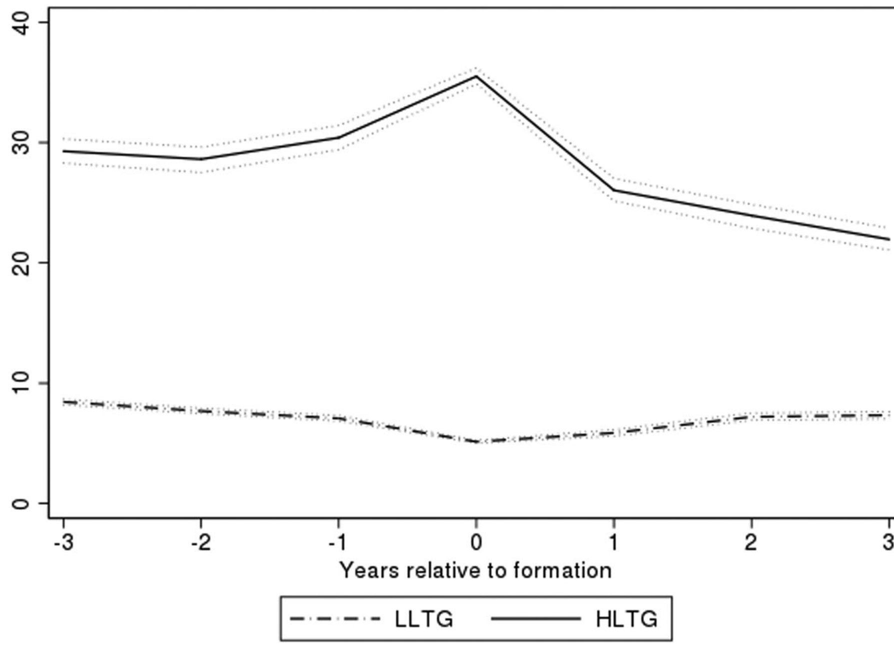


**Figure 3. Evolution of LTG.** In December of each year $t$ between 1984 and 2013, we form decile portfolios based on ranked analysts' expected growth in long-term EPS (LTG) and report the (bootstrapped) mean value of LTG on December of years $t-3$ to $t+3$ for the highest (HLTG) and lowest (LLTG) LTG deciles. We include in the sample stocks with LTG forecasts in year $t-3$. Values for $t+1$, $t+2$, and $t+3$ are based on stocks with IBES coverage for those periods. The dotted lines indicate $5^{\text{th}}$ and $95^{\text{th}}$ confidence levels determined via bootstrapping.

There are two concerns with the above evidence, both of which relate to the interpretation and reliability of our expectations data. The first is whether Figure 4 reflects genuine errors in analyst beliefs or alternatively distorted analyst incentives. For instance, analysts may blindly follow the market, reporting high LTG for firms that investors are excited about. This possibility does not undermine our analysis. To the extent that analysts' forecasts reflect investor beliefs, they are informative about the expectations shaping market prices. A more radical concern is that analysts' beliefs are unrelated to investor beliefs. To assess this possibility, Figure 5 plots stock returns around earnings announcements, both pre- and postformation. For every stock in the HLTG and LLTG portfolios, we compute the 12-day cumulative return during the four quarterly earnings announcement days, in years $t-3$ to $t+3$, following the methodology of La Porta et al. (1997).

HLTG stocks positively surprise investors with their earnings announcements in the years prior to portfolio formation, and their LTG is revised up.[6]

[6] The persistence of positive surprises and high returns exhibited by the HLTG portfolio preformation in Figure 5 should not be confused with time series momentum. On average, firms require a sequence of positive shocks to be classified as HLTG.

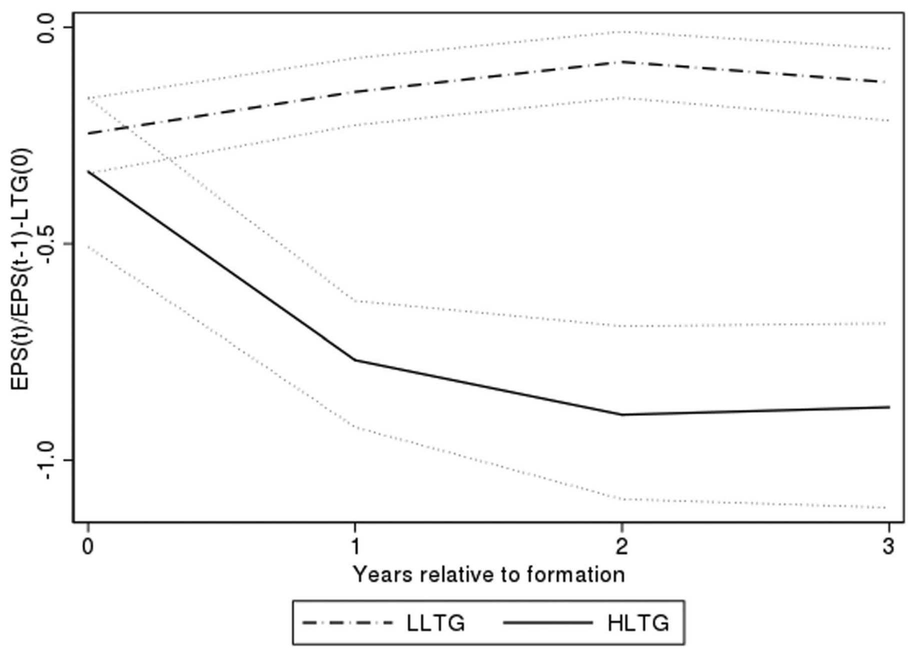


**Figure 4. Realized EPS versus expected growth in long-term earnings.** In December of each year $t$ between 1981 and 2013, we form decile portfolios based on ranked analysts' expected growth in long-term EPS. We plot the (bootstrapped) mean difference between the annual growth rate in EPS in each year between $t$ and $t+3$ and the forecast for long-term growth in earnings made in year $t$. To compute earnings growth, here we restrict the sample to firms with positive earnings at $t = -1$. The dotted lines indicate $5^{\text{th}}$ and $95^{\text{th}}$ confidence levels determined via bootstrapping.

Returns are low afterward, especially in year 1, consistent with the sharp decline in LTG in this period. Analysts' overoptimism thus seems to be shared by investors, so that HLTG stocks consistently disappoint in the postformation period. The converse holds for LLTG stocks, but in a milder form. Analysts' expectations are not noise: they correlate with both actual earnings and stock returns.

In summary, the data point to three key findings. First, optimism about HLTG firms follows observation of fast earnings growth, and is reversed when growth slows down. Analysts appear to be learning about the firms' fundamentals based on past performance. Second, one should be skeptical of analyst rationality, as shown by the evidence on systematic forecast errors in the HLTG group (which includes more than 200 firms per year). Third, the dynamics of returns match the dynamics of expectations and their errors, both pre- and postformation.

We next present a model derived from psychological first principles that sheds light on this evidence. The model assumes that the consensus analyst

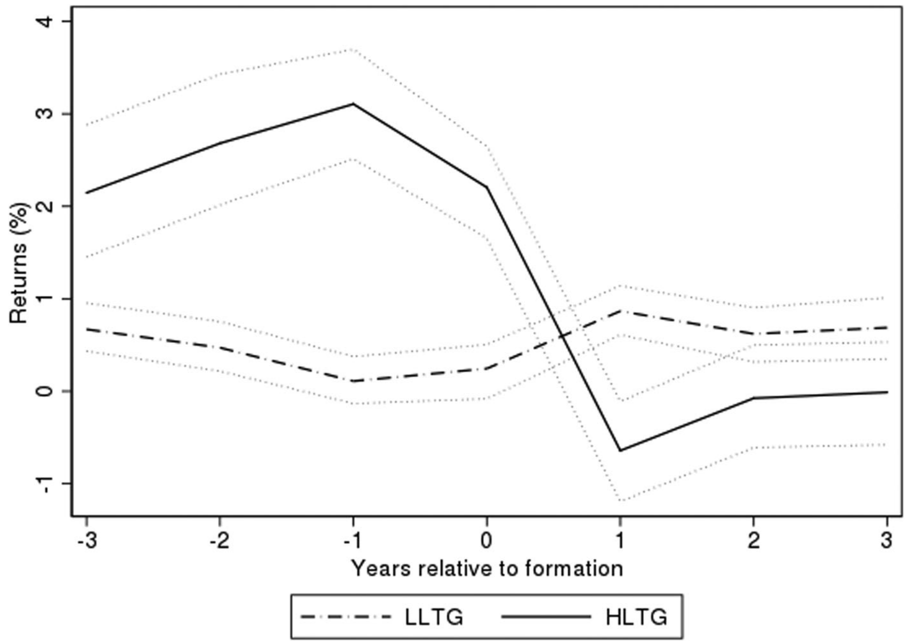


**Figure 5. Twelve-day returns on earnings announcements for LTG portfolios.** In December of each year $t$ between 1981 and 2013, we form decile portfolios based on ranked analysts' expected growth in long-term EPS. Next, for each stock, we compute the three-day market-adjusted return centered on earnings announcements in years $t-3, \ldots, t+3$. Next, we compute the annual return that accrues over earnings announcements by compounding all three-day stock returns in each year. We report the equally weighted (bootstrapped) average annual return during earnings announcements for the highest (HLTG) and lowest (LLTG) LTG deciles. Excess returns are defined relative to the equally weighted CRSP market portfolio. The dotted lines indicate $5^{\text{th}}$ and $95^{\text{th}}$ confidence levels determined via bootstrapping.

forecasts are the relevant expectations that shape prices and seeks to jointly explain these forecasts, their errors, and return predictability.[7]

## III. A Model of Learning with Representativeness

### A. The Setup

At time $t$, the natural logarithm of the EPS $x_{i,t}$ of firm $i$ is given by

$$x_{i,t} = bx_{i,t-1} + f_{i,t} + \epsilon_{i,t}, \tag{1}$$

where $b \in [0, 1]$ captures mean-reversion in EPS, and $\epsilon_{i,t}$ denotes a transitory i.i.d. normally distributed shock to EPS, $\epsilon_{i,t} \sim \mathcal{N}(0, \sigma_\epsilon^2)$. The term $f_{i,t}$, which

[7] As suggested by Pietro Veronesi, predictable returns (but not forecast errors) can arise under full rationality if analysts jointly learn a firm-specific required return and earnings. We return to this point in Section V.

we refer to as the firm's "fundamental," captures the firm's persistent earnings capacity. It follows the law of motion

$$f_{i,t} = a \cdot f_{i,t-1} + \eta_{i,t}, \tag{2}$$

where $a \in [0, 1]$ is persistence and $\eta_{i,t} \sim \mathcal{N}(0, \sigma_\eta^2)$ is an i.i.d. normally distributed shock. We can think of firms with exceptionally high $f_{i,t}$ as "Googles" that will produce very high earnings in the future, and firms with low $f_{i,t}$ as "lemons" that will produce low earnings in the future. We assume stationarity of earnings by imposing $b \leq a$.

The analyst observes EPS $x_{i,t}$ but not the fundamental $f_{i,t}$. The Kalman filter characterizes the forecasted distribution of $f_{i,t}$ at any time $t$ conditional on the firm's past and current earnings $(x_{i,u})_{u \leq t}$. Given the mean forecasted fundamental $\hat{f}_{i,t-1}$ for firm $i$ at $t-1$ and its current earnings $x_{i,t}$, the firm's current forecasted fundamental is normally distributed with variance $\sigma_f^2$ and mean[8]

$$\hat{f}_{i,t} = a\hat{f}_{i,t-1} + K\left(x_{i,t} - bx_{i,t-1} - a\hat{f}_{i,t-1}\right), \tag{3}$$

where $K \equiv \frac{a^2\sigma_f^2 + \sigma_\eta^2}{a^2\sigma_f^2 + \sigma_\eta^2 + \sigma_\epsilon^2}$ is the signal-to-noise ratio.

The new forecast of fundamentals starts from the history-based value $a\hat{f}_{i,t-1}$ but adjusts it in the direction of the current surprise $x_{i,t} - bx_{i,t-1} - a\hat{f}_{i,t-1}$. The extent of adjustment increases in $K$. Absent transitory shocks ($\sigma_\epsilon^2 = 0$), earnings are so informative about fundamentals that the adjustment is full (i.e., $\hat{f}_{i,t} = x_{i,t} - bx_{i,t-1}$). As transitory shocks get more frequent, earnings becomes a noisier signal, which gets discounted in assessing fundamentals ($K < 1$). The signal-to-noise ratio thus separates the transitory from the persistent earnings shock (i.e., $\eta_{i,t}$ from $\epsilon_{i,t}$).

We next describe how the representativeness heuristic distorts this learning process.

### *B. Representativeness and the Diagnostic Kalman Filter*

Kahneman and Tversky (1972) argue that the automatic use of the representativeness heuristic causes individuals to estimate a type as likely in a group when it is merely representative. The authors define representativeness as follows: "an attribute is representative of a class if it is very diagnostic; that is, the relative frequency of this attribute is much higher in that class than in a relevant reference class" (Tversky and Kahneman (1983)). Starting with Kahneman and Tversky (1972), experimental evidence has found ample support for the role of representativeness.

[8] In the presence of fundamental shocks $\eta_{i,t}$, a firm's fundamental is never learned with certainty. Equation (3) obtains when the variance of fundamentals converges to its steady state $\sigma_f^2$, which is given as the solution to $a^2\sigma_f^4 + \sigma_f^2[\sigma_\eta^2 + (1-a^2)\sigma_\epsilon^2] - \sigma_\eta^2\sigma_\epsilon^2 = 0$.

Consider a decision maker assessing the distribution $h(T=\tau|G)$ of a variable $T$ in a group $G$. Gennaioli and Shleifer (2010) define the representativeness of type $\tau$ for $G$ as:

$$R(\tau, G) \equiv \frac{h(T=\tau|G)}{h(T=\tau|-G)}. \tag{4}$$

As in Tversky and Kahneman (1983), a type is more representative if it is *relatively* more frequent in $G$ than in the comparison group $-G$. To capture overestimation of representative types, Bordalo et al. (2016) assume that probability judgments are formed using the representativeness-distorted density

$$h^{\theta}(T=\tau|G) = h(T=\tau|G)\left[\frac{h(T=\tau|G)}{h(T=\tau|-G)}\right]^{\theta} Z, \tag{5}$$

where $\theta \geq 0$ and $Z$ is a constant ensuring that the distorted density $h^{\theta}(T=\tau|G)$ integrates to 1. The extent of probability distortions increases in $\theta$, with $\theta = 0$ capturing the rational benchmark.

In both Tversky and Kahneman's (1983) quote and equation (4), the representativeness of a type depends on its *true* relative frequency in group $G$. Gennaioli and Shleifer (2010) see this as coming from limited and selective memory. True information $h(T=\tau|G)$ and $h(T=\tau|-G)$ about a group is stored in a decision maker's long-term memory. Representative types, being distinctive of the group under consideration, are more readily recalled than other types and thus overweighted.

This setup can be applied to prediction and inference problems (as in Bordalo et al. (2016) and Bordalo, Gennaioli, and Shleifer (2018)). Consider, for example, the doctor assessing the health status of a patient, $T = \{healthy, sick\}$, in light of a positive medical test, $G = positive$. The positive test is assessed in the context of untested patients ($-G = untested$). Applying the previous definition, being sick is representative of patients who tested positive if and only if

$$\frac{\Pr(T=sick\,|\,G=positive)}{\Pr(T=sick\,|\,-G=untested)} > \frac{\Pr(T=healthy\,|\,G=positive)}{\Pr(T=healthy\,|\,-G=untested)},$$

that is, when $\Pr(G=positive\,|\,T=sick) > \Pr(G=positive\,|\,T=healthy)$. The condition holds if the test is even minimally informative of the health status. A positive test brings "sick" to mind because the true probability of this type has increased the most after the positive test. The doctor may then deem the sick state likely even if the disease is rare (Casscells, Schoenberger, and Graboys (1978)), committing a form of base rate neglect (Tversky and Kahneman (1974)).

We apply this logic to the problem of forecasting a firm's earnings. The analyst must infer the firm's type $f_{i,t}$ after observing the current earnings surprise $x_{i,t} - bx_{i,t-1} - a\hat{f}_{i,t-1}$. Such a surprise is akin to seeing the medical test. As we saw previously, the *true* conditional distribution of firm fundamentals $f_{i,t}$ is normal, with variance $\sigma_f^2$ and the mean given by equation (3). This is our target distribution $h(T=\tau|G)$. As in the medical example, the information content of the earnings $x_{i,t}$ for fundamentals $f_{i,t}$ is assessed relative to the background information set in which no news is received, that is, if the earnings surprise is zero $x_{i,t} - bx_{i,t-1} = a\hat{f}_{i,t-1}$. The comparison distribution $h(T = \tau | -G)$ is then also normal with mean $a\hat{f}_{i,t-1}$ and variance $\sigma_f^2$.

The assumption of normality implies the monotone likelihood ratio property of posteriors relative to priors, which makes representativeness easy to characterize. After good news, the most representative firm types are in the right tail. These firms are overweighted in beliefs while firms in the left tail are neglected. After bad news, the reverse is true. In the Internet Appendix, we prove that these distortions generate diagnostic beliefs as in the following proposition.

PROPOSITION 1: (Diagnostic Kalman filter). *In the long run, upon seeing $x_{i,t} - bx_{i,t-1}$, the analyst's posterior about the firm's fundamentals is normally distributed with variance $\sigma_f^2$ and mean*

$$\hat{f}^{\theta}_{i,t} = a\hat{f}_{i,t-1} + K(1+\theta)\left(x_{i,t} - bx_{i,t-1} - a\hat{f}_{i,t-1}\right). \tag{6}$$

When analysts overweight representative types, their beliefs exaggerate the signal-to-noise ratio relative to the standard Kalman filter, inflating the fundamentals of firms receiving good news and deflating those of firms receiving bad news. Exaggeration of the signal-to-noise ratio is reminiscent of overconfidence, but here overreaction occurs with respect to public as well as private news.[9] The psychology is, however, very different from overconfidence: in our model, as in the medical test example, overreaction is caused by the neglect of base rates. After good news, the most representative firms are Googles, those with high earnings capacity $f_{i,t}$. This firm type readily comes to mind and its probability is inflated, despite the fact that Googles are rare. After bad news, the most representative firms are lemons. The analyst exaggerates the probability of this type, despite the fact that lemons are also quite rare. Overreaction to news increases in $\theta$. At $\theta = 0$, the model reduces to rational learning.

The key property of diagnostic expectations is the kernel of truth, whereby distortions in beliefs exaggerate true patterns in the data. In equation (6), departures from rationality are a function of the true persistence $a$ of fundamentals and of the true signal-to-noise ratio $K$, which depends on the volatility of fundamentals. The kernel of truth distinguishes our approach from alternative theories of extrapolation such as adaptive expectations or Barberis, Shleifer, and Vishny (1998). In Section VI, we present empirical tests comparing alternative models.

To streamline the analysis, we defined representativeness with respect to the recent news $x_{i,t} - bx_{i,t-1} - a\hat{f}_{i,t-1}$. As we discuss in Bordalo, Gennaioli, and Shleifer (2018), two alternative specifications may be relevant. First, memory can cause the reference group $-G$ to move slowly, leading the news to be compared to conditions further in the past. Formally, representativeness at

[9] In fact, overconfidence predicts underreaction to public news such as earnings releases (see Daniel, Hirshleifer, and Subramanyam (1998)).

$t$ can be defined with respect to the information available at $t-s$. In this case, the comparison distribution $h(t|-G)$ is the true distribution of fundamentals conditional on the state $a^s \hat{f}_{i,t-s}$ that would arise if no news is attended to for the last $s$ periods. Slow moving $-G$ causes more persistent departures from rationality, which last for $s$ periods in our model. In Section V, we show that this is important for calibrating the model, although it does not affect its qualitative properties.

Second, the reference group may alternatively be formed by lagged *distorted* expectations, as opposed to unbiased beliefs based on past information. This may occur because news brings to mind the recently held expectations (rather than the full distribution stored in memory), so that the comparison distribution $h(t|-G)$ is the diagnostic forecast at $t-s$ of fundamentals at $t$, which is normal with mean $a^s \hat{f}^{\theta}_{i,t-s}$. This specification loses the key predictive feature of the model, namely, the kernel-of-truth hypothesis that beliefs overreact to objective information. It also is not portable across different domains (e.g., it cannot be applied to static problems such as group stereotypes).[10]

## IV. The Model and the Facts

To link our model to the data, we shift attention from the level to the growth rate of earnings, which directly maps into LTG. Denote by $h$ the horizon over which the growth forecast applies, which is about four years for LTG. The LTG of firm $i$ at time $t$ is the firm's expected earnings growth over this horizon, $LTG_{i,t} = \mathbb{E}^{\theta}_{i,t}(x_{t+h} - x_t)$. By equations (1) and (6), this boils down to:

$$LTG_{i,t} = -\left(1-b^h\right)x_t + a^h \frac{1-(b/a)^h}{1-(b/a)}\hat{f}^{\theta}_{i,t}.$$

Expectations of LTG are shaped by mean reversion in EPS and by fundamentals. LTG is high when firms have experienced positive news, so $\hat{f}^{\theta}_{i,t}$ is high, and/or when current earnings $x_t$ are low, which also raises future growth. Both conditions line up with the evidence, which shows that HLTG firms have experienced fast growth (Figure 2) and have low EPS (Table I).

We next show that the model accounts for the previously documented facts regarding the long-run distribution of fundamentals $f_{i,t}$ (which has zero mean and variance $\frac{\sigma^2_\eta}{1-a^2}$) and analysts' mean beliefs $\hat{f}^{\theta}_{i,t}$ (which has zero mean and variance $\sigma^2_{\hat{f}^\theta}$).[11] At time $t$, we identify the high LTG group, $\text{HLTG}_t$, as the 10% of firms with the highest assessed future earnings growth, and the low LTG

[10] In Bordalo, Gennaioli, and Shleifer (2018), we work out this specification and show that, over long time series, it also exhibits overreaction to objective news, but at each time $t$ the distortion of expectations propagates arbitrarily far into the future. In Section V, we show that, at the relevant time scales, the two specifications perform similarly.

[11] There is no distortion in the average diagnostic expectation across firms because in steady state the average earning surprise is zero. Thus, the average diagnostic expectation coincides with the average rational expectation. However, diagnostic beliefs are fatter-tailed than rational ones because they exaggerate the frequency of Googles and Lemons.

group, $LLTG_t$, as the 10% of firms with the lowest assessed future earnings growth.

### *A. Representativeness and the Features of Expectations*

We first consider the patterns of fundamentals and expectations documented in Figures 2 to 4. In Section B, we review the patterns of returns documented in Figures 1 and 5.

We start from Figure 2, which shows that HLTG firms experience a period of pronounced growth before portfolio formation, while LLTG firms experience a period of decline.

PROPOSITION 2: *Provided that $a$, $b$, $K$, and $\theta$ satisfy*

$$b^h + a^h \frac{1-(b/a)^h}{1-(b/a)} \left[ \frac{K(1+\theta)-a}{1-a} \right] > 1, \tag{7}$$

*the average $HLTG_t$ ($LLTG_t$) firm experiences positive (negative) earnings growth preformation.*

Positive earnings news has two conflicting effects on LTG. On the one hand, it increases estimated fundamentals $\hat{f}^{\theta}_{i,t}$, which raise future growth forecasts. On the other hand, it lowers future growth via mean reversion. Condition (7) ensures that the former effect dominates, so that HLTG firms are selected from the recent good performers, while LLTG firms are selected from recent bad performers. Condition (7) is more likely to hold the less severe is mean reversion (i.e., when $b$ is close to 1) and the higher is the signal-to-noise ratio $K$, as well as for high $\theta$.

Combined with mean reversion of earnings, Proposition 2 accounts for Figure 2, in which HLTG firms experience positive growth preformation that subsequently cools off, while LLTG firms go through the opposite pattern. But the model can also account for the fact documented in Figure 4 that expectations for the LTG of HLTG firms are excessively optimistic.

PROPOSITION 3: *If analysts are rational, they make no systematic error in predicting the log growth of earnings of* $\mathrm{HLTG}_t$ *and* $\mathrm{LLTG}_t$ *portfolios:*

$$\mathbb{E}\left(x_{t+h} - x_t - LTG^{\theta}_t | \mathrm{HLTG}_t\right) = \mathbb{E}\left(x_{t+h} - x_t - LTG^{\theta}_t | \mathrm{LLTG}_t\right) = 0 \qquad for\ \theta = 0.$$

*Under diagnostic expectations, in contrast, analysts systematically overestimate growth of* $\mathrm{HLTG}_t$ *firms and underestimate growth of* $\mathrm{LLTG}_t$ *firms*:

$$\mathbb{E}\left(x_{t+h} - x_t - LTG^{\theta}_t | \mathrm{HLTG}_t\right) < 0 < \mathbb{E}\left(x_{t+h} - x_t - LTG^{\theta}_t | \mathrm{LLTG}_t\right) \qquad for\ \theta > 0.$$

Under rational expectations, no systematic forecast error can be detected by an econometrician looking at the data (expectations are computed using the true steady-state probability measure). Indeed, when $\theta = 0$, the average forecast within the many firms of $\mathrm{HLTG}_t$ and $\mathrm{LLTG}_t$ portfolios is well calibrated to the respective means. Diagnostic expectations, in contrast, cause systematic errors. Firms in the $\mathrm{HLTG}_t$ group are systematically overvalued: analysts overreact to their preformation positive surprises, forming excessively optimistic forecasts of fundamentals.[12] As a consequence, the realized earnings growth is on average below the forecast. Firms in the $\mathrm{LLTG}_t$ group are systematically undervalued, and as a consequence their realized earnings growth is on average above the forecast. As noted in Section II, systematic pessimism about LLTG firms is not borne out in the data.

Finally, our model yields the boom-bust LTG pattern in the HLTG group (and a reverse pattern in the LLTG group) documented in Figure 3. By Proposition 2, the improving preformation forecasts of HLTG firms are due to positive earnings surprises, while the deteriorating preformation forecasts of LLTG firms are due to negative surprises. But the model also predicts postformation reversals in LTG for both groups of firms. To see this, we compare forecasts made at $t$, $LTG_{i,t}$,with forecasts made for the same firm at $t+s$, $LTG_{i,t+s}$.

PROPOSITION 4: *Under rational expectations ($\theta = 0$), we have that*

$$\mathbb{E}\left(\mathrm{LTG}^{\theta}_{t+s}|HLTG_t\right) - \mathbb{E}\left(\mathrm{LTG}^{\theta}_{t}|HLTG_t\right) < 0.$$

*Under diagnostic expectations* ($\theta > 0$), *we have that*

$$\mathbb{E}\left(\mathrm{LTG}^{\theta}_{t+s}|HLTG_t\right) - \mathbb{E}\left(\mathrm{LTG}^{\theta}_{t}|HLTG_t\right) = \mathbb{E}\left(\mathrm{LTG}^{\theta=0}_{t+s}|HLTG_t\right) - \mathbb{E}\left(\mathrm{LTG}^{\theta=0}_{t}|HLTG_t\right) - \theta\Psi$$

*for some $\Psi > 0$. The opposite pattern, with reversed inequality and $\Psi < 0$, occurs for $LLTG_t$.*

Mean reversion in LTG obtains under rational expectations due to mean reversion in fundamentals. Under diagnostic expectations, however, mean reversion is amplified by the correction of initial forecast errors. Postformation, the excess optimism of HLTG firms dissipates on average, causing a cooling off in expectations $\theta\Psi$ that is more abrupt than what would be implied by mean reversion alone. The cooling off of excess optimism arises because there is no news on average in the HLTG portfolio, which leads to no overreaction. Likewise, the excess pessimism of LLTG firms dissipates, strengthening the reversal in that portfolio.

The assumption that the comparison group $-G$ immediately adapts to the recent state implies that forecast errors are corrected on average in one period. If $-G$ moves slowly, it takes longer for forecast errors to be corrected. As we show in Section V, the horizon at which systematic forecast errors are corrected allows us to estimate the speed of adjustment of $-G$, which has important implications for the quantitative fit of the model, including return dynamics.

[12] In general, forecasters err not only by overestimating growth for HLTG firms but also by misclassifying firms as HLTG. Analysts miss out on firms that have high growth potential but whose recent performance is poor.

*B. The Diagnostic Kalman Filter and Returns*

To study the pattern of returns, we take the required return $R > 1$ as given. The pricing condition for a firm $i$ at date $t$ is then given by:

$$\mathbb{E}_t^\theta \left( \frac{P_{i,t+1} + D_{i,t+1}}{P_{i,t}} \right) = R, \tag{8}$$

so that the stock price of firm $i$ is the discounted stream of expected future dividends as of $t$. We assume that $R$ is high enough that the discounted sum converges. By equation (8), the equilibrium price at $t$ is $P_{i,t} = \mathbb{E}_t^\theta(P_{i,t+1} + D_{i,t+1})/R$. Using this formula and Proposition 2, we establish the following result.

PROPOSITION 5: *Denote by $R_{t,P}$ the realized return of portfolio $P = HLTG, LLTG$ at formation date t. Then, under the condition of Proposition 2, we have*

$$R_{t,HLTG} > R > R_{t,LLTG}.$$

Because firms in the HLTG portfolio receive positive news before formation (Proposition 2), they earn higher returns than required. Our model thus yields positive abnormal preformation returns for HLTG stocks, as well as the low preformation returns of LLTG stocks, as in Figure 5. This effect also arises under rationality, $\theta = 0$. The key implication of representativeness is predictability of postformation returns (Figures 1 and 5).

To see this, note that the average realized return going forward (according to the true probability measure) for a given firm $i$ is given by

$$\mathbb{E}_t \left( \frac{P_{i,t+1} + D_{i,t+1}}{P_{i,t}} \right) = \frac{\mathbb{E}_t \left( P_{i,t+1} + D_{i,t+1} \right)}{\mathbb{E}_t^\theta \left( P_{i,t+1} + D_{i,t+1} \right)} R, \tag{9}$$

which is below the required return $R$ when at $t$ investors overvalue the future expected price and dividend of firm $i$. Conversely, the average realized return at $t+1$ is higher than the required return $R$ when at $t$ investors undervalue the firm's future expected price and dividend.

Diagnostic expectations thus yield the return predictability patterns of Figure 5.

PROPOSITION 6: *(Predictable returns). Denote by $\mathbb{E}_t(R_{t+1}^\theta|P)$ the average future return of portfolio $P = HLTG,\ LLTG$ at $t+1$. Under rationality, excess returns are not predictable:*

$$\mathbb{E}_t \left( R_{t+1}^\theta | HLTG \right) = R = \mathbb{E}_t \left( R_{t+1}^\theta | LLTG \right) \quad for\ \theta = 0.$$

*Diagnostic expectations generate predictable excess returns:*

$$\mathbb{E}_t \left( R_{t+1}^\theta | HLTG \right) < R < \mathbb{E}_t \left( R_{t+1}^\theta | LLTG \right) \quad for\ \theta > 0$$

Under rational expectations (i.e., for $\theta = 0$), realized returns may differ from the required return $R$ for particular firms. However, the rational model cannot account for systematic return predictability in a large portfolio of firms sharing a certain forecast. Conditional on current information, rational forecasts are on average (across firms) correct and returns are unpredictable.

Under the diagnostic filter ($\theta > 0$), in contrast, the HLTG portfolio exhibits abnormally high returns up to portfolio formation and abnormally low returns after formation. The converse holds for the LLTG portfolio, just as we saw in Table I and Figure 1. Indeed, postformation expectations systematically revert to fundamentals, and in particular investors are systematically disappointed in HLTG firms and their returns are abnormally low. The speed with which postformation returns go back to $R$ depends on the sluggishness of the comparison group $-G$.

## V. Overreaction to News and Quantification of Diagnostic Expectations

Diagnostic expectations yield a coherent account of the dynamics of news, analyst expectations, and returns documented in Section II. The predictability of returns, and especially of expectations errors, cannot be accounted for by Bayesian learning. Departures from rationality are summarized by the parameter $\theta$, which controls the extent of overreaction to information. This raises two questions. First, can overreaction be detected in expectations data? Second, can we quantitatively assess the explanatory power of our model? We address these questions next.

### *A. Overreaction to News*

To assess overreaction, we need a measure of news at each $t$. Coibion and Gorodnichenko (2015) propose analysts' forecast revision at $t$, which summarizes all information received by the forecaster in this period. Over- or underreaction to information can then be assessed by correlating the forecast revision with the subsequent forecast error. When expectations overreact, a positive forecast revision indicates excessive upward adjustment. As a consequence, it should predict negative forecast errors (i.e., realizations below the forecast). Likewise, when expectations underreact, the correlation between revisions and forecast errors should be positive. Bouchaud et al. (2018) use this method to diagnose underreaction to news about firms' profitability.

PROPOSITION 7: *Assume condition (7) of Proposition 2. Consider the firm-level regression*

$$x_{i,t+h} - x_{i,t} - LTG_{i,t} = \alpha + \gamma \left(LTG_{i,t} - LTG_{i,t-k}\right) + v_{i,t+h}. \tag{10}$$

*Under rationality, $\theta = 0$, the estimated $\gamma$ is zero, while for $\theta > 0$ it is negative.*

Table II reports estimates from the univariate regression of forecast error, defined as the difference between average growth over $h = 3,\ 4,$ and 5 years and current LTG, and the revision of LTG over the past $k = 1,\ 2,$ and 3 years.

**Table II**

**Coibion-Gorodnichenko Regressions for EPS**

Each entry in the table corresponds to the estimated coefficient of regressing the forecast errors $(EPS_{t+n}/EPS_t)^{1/n} - LTG_t$ for $n = 3$, 4, and 5 on the variables listed in the first column of the table and year fixed effects (not shown).[a] indicates significance at the 1% level.

| | Dependent Variable | | |
|---|---|---|---|
| | $(EPS_{t+3}/EPS_t)^{1/3} - LTG_t$ | $(EPS_{t+4}/EPS_t)^{1/4} - LTG_t$ | $(EPS_{t+5}/EPS_t)^{1/5} - LTG_t$ |
| $LTG_t - LTG_{t-1}$ | −0.0351 | −0.1253[a] | −0.1974[a] |
| | (0.0734) | (0.0642) | (0.0516) |
| $LTG_t - LTG_{t-2}$ | −0.2335[a] | −0.2687[a] | −0.2930[a] |
| | (0.0625) | (0.0602) | (0.0452) |
| $LTG_t - LTG_{t-3}$ | −0.2897[a] | −0.2757[a] | −0.3127[a] |
| | (0.0580) | (0.0565) | (0.0437) |

To estimate $\gamma$, we use consensus forecasts rather than individual analyst estimates because many analysts drop out of the sample.[13]

Consistent with diagnostic expectations, an upward LTG revision predicts excess optimism, pointing to overreaction to news. The estimated $\gamma$ tends to become more negative and more statistically significant at longer forecast horizons $h = 3,\ 4,$ and $5$ (moving from left to right in Table II), perhaps reflecting the difficulty of projecting growth into the future.[14]

The estimated $\gamma$ also gets higher in magnitude and more statistically significant as we lengthen the revision period $k = 1, 2,$ and $3$ (moving from top to bottom in Table II). These patterns are informative about $-G$. Under diagnostic expectations, the correlation between forecast revisions and forecast errors is maximized when the lag $k$ at which the revision is computed is equal to the lag of $-G$.[15] Table II shows that the correlation is highest at $k = 3$, which roughly indicates that $-G$ should be defined by a three-year lag.

### *B. Model Estimation with SMM*

To quantify the model's explanatory power, we use the magnitude of overreaction in Table II to estimate the diagnostic parameter $\theta$ using SMM. We then assess how well the estimated model can account for the observed spread in average returns.

[13] Estimating (10) on the consensus LTG may misleadingly indicate underreaction if individual analysts observe noisy signals, so that there is dispersion in their forecasts (see Coibion and Gorodnichenko (2015)).

[14] Bouchaud et al. (2018) find underreaction of forecasts for the level of EPS over one- or two-year horizons. In Table IA.V of the Internet Appendix, we show that at these horizons there is some evidence of underreaction in our data also. Over- and underreaction may be reconciled by combining diagnostic expectation with short-run rigidity in analyst forecasts.

[15] At sluggishness $s$, the diagnostic expectation is $\mathbb{E}_t^\theta(x_{t+h}) = \mathbb{E}_t(x_{t+h}) + \theta[\mathbb{E}_t(x_{t+h}) - \mathbb{E}_{t-s}(x_{t+h})]$. It can be shown that the correlation between $x_{t+h} - \mathbb{E}_t^\theta(x_{t+h})$ and $\mathbb{E}_t^\theta(x_{t+h}) - \mathbb{E}_{t-k}^\theta(x_{t+h})$ reaches its maximum at $k = s$.

The dynamics of earnings and expectations of a representative firm[16] depend on six parameters: the persistence and conditional variance of log EPS ($b$ and $\sigma_\epsilon$ from equation (1)), those of fundamentals $f$ ($a$ and $\sigma_\eta$ from equation (2)), the strength of representativeness $\theta$, and the sluggishness $s$ of the lagged expectations $-G$. Physical parameters $a,\ b,\ \sigma_\eta,$ and $\sigma_\epsilon$ are defined at the quarterly time scale, the natural scale of the data-generating process.

We set the six parameters $(a,\ b, \sigma_\eta,\ \sigma_\epsilon, \theta, s)$ to match the autocorrelation of log EPS $\rho_l$ at lags $l = 1,\ 2,\ 3,$ and 4 years, and the two coefficients $\gamma_k$ estimated in Table II linking forecast error $(\frac{eps_{t+4}}{eps_t})^{1/4} - LTG_t$ to forecast revision $LTG_t - LTG_{t-k}$ for $k = 1,\ 3$ years.

For each parameter combination $(a,\ b, \sigma_\eta,\ \sigma_\epsilon, \theta, s)$, we first simulate a time series of log earnings $x_t$ and compute the associated diagnostic expectations $\hat{f}_t^\theta$ about fundamentals.[17] From the earnings time series, we then compute the autocorrelations $\hat{\rho}_l = \frac{\text{cov}(x_t, x_{t-4l})}{\text{var}(x_t)}$ at lags $l = 1$ through 4 years. To compute the model-implied coefficients $\hat{\gamma}_1, \hat{\gamma}_3$, we first generate the expectations for long-term growth $LTG_t$ at a horizon of $h = 16$ quarters, and then regress the forecast error $x_{t+16} - x_t -\ LTG_t$ on the forecast revision over one year, $LTG_t - LTG_{t-4}$, or over three years, $LTG_t - LTG_{t-12}$ (equation (10)). This yields the vector

$$v\left(a,\ b, \sigma_\eta,\ \sigma_\epsilon, \theta\right) = (\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4, \hat{\gamma}_1, \hat{\gamma}_3).$$

We repeat this exercise for each parameter combination on a grid defined by $a,\ \ b \in [0, 1],\ \sigma_\eta,\ \ \sigma_\epsilon \in [0, 0.5],\ \theta \in [0,\ 3],$ and $s \in \{1, 2, \ldots,\ 20\}$ quarters.[18] We estimate the parameters by picking the combination that minimizes the Euclidean distance loss function

$$\ell(v) = \|v - \bar{v}\|,$$

where $\bar{v}$ is the vector of target moments estimated from the pooled data of all firms, given by

$$\bar{v} = (0.82,\ 0.75,\ 0.70,\ 0.65,\ -0.276, -0.126).$$

Table III reports the average and standard deviation of the parameter combinations that yield the lowest value of the Euclidean loss function across 30 independent runs.

Fundamentals (and, to a lesser extent, log earnings) are estimated to be very persistent. The variance of fundamentals $\sigma_\eta$ is estimated to be about 60% higher than that of transitory earnings $\sigma_\epsilon$, leading to a Kalman gain of about 0.76.

[16] In Section V.D, we look at the cross-section of firms to assess the comparative statics predicted by the kernel of truth. We leave to future work performing a fuller estimation with heterogeneous firms. One problem is that, for many firms, time series of annual data are short, and can have negative earnings (which are not considered in the model).

[17] We use one time series because the empirical estimates $\rho_l$ and $\gamma_k$ were both computed using the pooled data.

[18] The grid was defined in steps of 0.025 to 0.1 (depending on the parameter and the parameter region) for all parameters other than the timescale.

**Table III**

**Estimates of Model Parameters**

The model parameters are the persistence and conditional variance of log *EPS* ($b$ and $\sigma_\epsilon$ from equation (1)), those of fundamentals $f$ ($a$ and $\sigma_\eta$ from equation (2)), the strength of representativeness $\theta$, and the sluggishness $s$ of the lagged expectations $-G$. Physical parameters $a$, $b$, $\sigma_\eta$, and $\sigma_\epsilon$ are defined at the quarterly time scale. We pick values for the vector $v(a,\ b, \sigma_\eta,\ \sigma_\epsilon, \theta) = (\hat{\rho}_1, \hat{\rho}_2, \hat{\rho}_3, \hat{\rho}_4, \hat{\gamma}_1, \hat{\gamma}_3)$ to minimizes the Euclidean distance loss function $\ell(v) = \|v - \bar{v}\|$, where $\bar{v}$ is the vector of target moments (i.e., $\bar{v} = (0.82,\ 0.75,\ 0.70,\ 0.65,\ -0.276, -0.126)$). See the text for other details.

| $a$ | $b$ | $\sigma_\eta$ | $\sigma_\epsilon$ | $\theta$ | $s$ |
|---|---|---|---|---|---|
| 0.97 | 0.56 | 0.14 | 0.08 | 0.90 | 11 |
| (0.00) | (0.01) | (0.03) | (0.03) | (0.07) | (1.9) |

Turning to the diagnostic parameter, our estimation yields a significantly positive $\theta$. The value of 0.90 coincides with the estimate for the same parameter obtained by Bordalo, Gennaioli, and Shleifer (2018) in the context of credit spreads ($\theta = 0.91$), and is close to the average of 0.6 found in the context of forecasts of macroeconomic variables (Bordalo et al. (2018)). A $\theta$ of one intuitively implies that the magnitude of forecast errors is comparable to the magnitude of news. In the current context, it implies a doubling of the signal-to-noise ratio to $(1 + \theta)K \sim 1.44$. A diagnostic Kalman gain above one captures the key finding of overreaction, and systematic reversal, of the consensus LTG forecasts to both private and public news.[19]

Finally, consistent with the intuition from Table II, we estimate the time scale of diagnostic distortions $s$ to be about three years. As we show below, this is consistent with the boom-bust pattern in expectations, and in returns, to occur over a time scale of three years. At this time scale, the data do not strongly differentiate between the specification in which news is assessed relative to past conditions or relative to lagged distorted beliefs.[20]

Using these estimated parameter values, we generate and report in Figure 6 the simulated versions of Figures 1 through 5 (Panels A through E). To do so,

[19] To test the robustness of our estimate of $\theta$, we evaluate the matching between the empirical moments $\bar{v}$ and the predicted counterpart when $\theta$ varies in the interval $[0, 2]$ (keeping the other parameters constant). Figure IA.5 in the Internet Appendix shows that the loss function is minimized for $\theta = 0.9$ and that it increases fast as $\theta$ departs from this value. As expected, this is driven entirely by an increasingly worse match of the expectations moments $\hat{\gamma}_1, \hat{\gamma}_3$.

[20] Our benchmark specification achieves a loss function minimum that is about 2% lower than that of the alternative specification. To see why the specifications yield similar results, examine the role of the timescale s on the distortions, which under the benchmark are proportional to $\theta(x_{i,t} - bx_{i,t-1} - a^s \hat{f}_{i,t-s})$. The longer the timescale, the smaller the difference in lagged forecasts, $a^s\theta(\hat{f}_{i,t-s} - \hat{f}^{\theta}_{i,t-s})$, and the greater the variance in the actual shocks $x_{i,t} - bx_{i,t-1} - a^s \hat{f}_{i,t-s}$. Thus, for long timescales, the distortions entailed by the two specifications are similar. To check this intuition, in our simulation we regress diagnostic expectations relative to distorted beliefs on those under our benchmark specification, setting all parameters but the timescale to the values in Table III. The explanatory power ($R^2$) of diagnostic expectations under the benchmark specification increases monotonically from 0.87 to 0.97 as the timescale increases from 1 to 11 quarters.

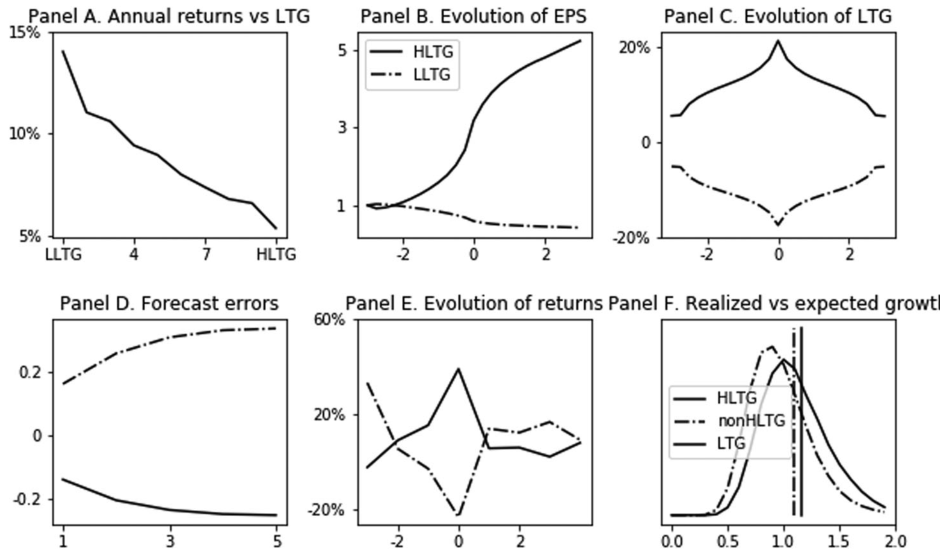


**Figure 6. Simulation of the estimated model.** Using the parameters in Table III, we simulate 4,000 firms over 140 time periods (quarters), generating time series of fundamentals, earnings, and growth expectations. The required rate of return $R$ is set so that the average return matches the historical value-weighted market return of 9.7%. Every year (four periods), we sort firms on LTG forecasts. Panel A shows the average return spread one-year postformation across LTG deciles (compounding over the four quarters, then averaging over the portfolio). Finally, we compute the geometric average of portfolio returns over time. Panels B, C, and E show the average EPS, LTG, and returns of the HLTG and LLTG portfolios from year $t-3$ to year $t+3$. Panel D shows the forecast error on HLTG and LLTG portfolios in five years postformation. Panel F shows the distribution of realized earnings growth after five years (annualized) for HLTG and non-HLTG firms, together with the average long-term growth for HLTG firms (dotted vertical line) and the LTG forecast for HLTG firms (solid vertical line, see Figure 9).

we simulate earnings paths $x_{i,t}$ and expectations of fundamentals $\hat{f}^{\theta}_{i,t}$ for 4,000 firms (indexed by $i$) described by the parameters in Table III, over a period of 35 years. Each year, we sort firms into deciles of LTG expectations, $LTG_{i,t}$. We then compute, at the portfolio level, one-year-ahead returns as well as the dynamics of earnings, LTG, forecast errors, and earnings announcement returns in the years around portfolio formation. To compute returns, we set the required rate of return $R$ such that the average return matches the historical value-weighted market return of 9.7%.[21]

The model reproduces the main qualitative features of the data. In Panel A, it reproduces the return spread between HLTG and LLTG stocks. In Panel B, it reproduces the pattern of Figure 2 whereby preformation HLTG firms have fast growth, which then declines postformation. In Panel C, the model reproduces the boom-bust dynamics of LTG, with analysts' expectations becoming more

[21] In the simulation, average returns are higher than the required level of return, reflecting the fact that expectations are too volatile relative to fundamentals.

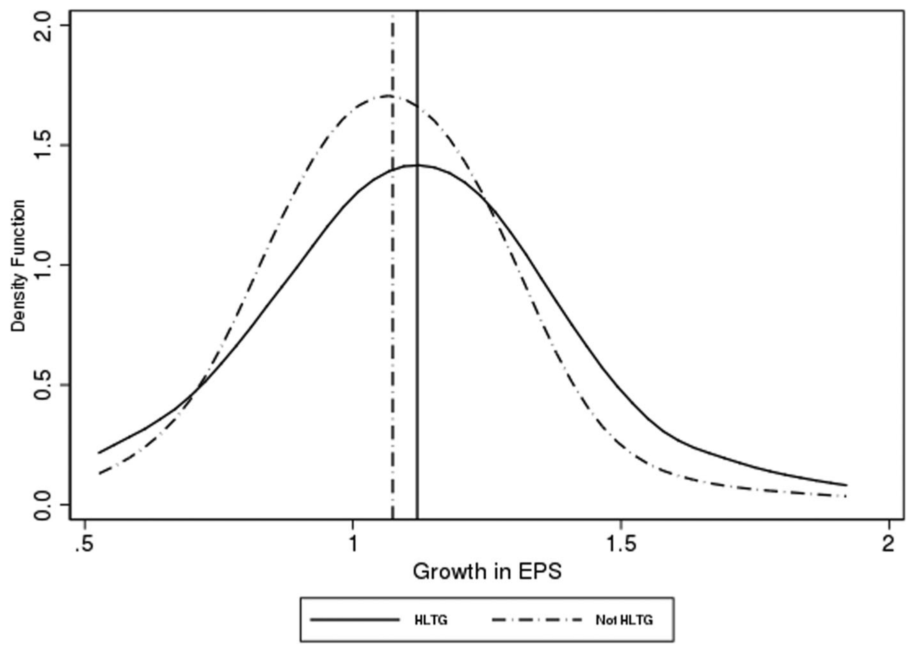


**Figure 7. Kernel density estimates of growth in EPS for LTG portfolios.** In December of years ($t$) 1981, 1986, ..., and 2011, we form decile portfolios based on ranked analysts' expected growth in long-term EPS (*LTG*). For each stock, we compute the gross annual growth rate of EPS between $t$ and $t + 5$. We exclude stocks with negative earnings in year $t$ and we estimate the kernel densities for stocks in the highest (*HLTG*) decile and for all other firms with LTG data. The graph shows the estimated density kernels of growth in EPS for stocks in the *HLTG* (solid line) and all other firms (dashed line). The vertical lines indicate the means of each distribution (1.11 vs. 1.08, respectively). To compute earnings growth between $t$ and $t + 5$, we restrict the sample to firms having positive EPS on these two dates.

optimistic preformation and then reverting postformation. Panel D reproduces the finding of Figure 4 of large forecast errors (excess optimism) for HLTG stocks. Panel E reproduces the boom-bust pattern in returns around portfolio formation. Returns for HLTG stocks are very high preformation, collapse below the required return $R$ in the immediate postformation period, and eventually revert back to their unconditional, long-term value. The opposite happens with the return of LLTG stocks.

Panel F illustrates a central implication of the kernel-of-truth property, namely, that HLTG firms have a fatter right tail of growth (and higher growth on average) than non-HLTG firms, but diagnostic expectations overestimate the prevalence of high growth firms in the HLTG portfolio. We examine this prediction empirically in Section VI.A (Figure 7).

The model fails to capture some qualitative features of the data, such as the negative forecast error for LLTG firms and the flat returns for firms with lower

LTG expectations. We return to these issues when we summarize the results of the estimation.

We next assess the model's quantitative performance. For the cross-sectional predictability of returns, we obtain an average LLTG-HLTG yearly return spread of 8.6% in year $t + 1$ (see Panel A), with the LLTG and HLTG portfolios yielding 14.0% and 5.4% average returns, respectively. The empirical counterparts to these numbers are 15% and 3%, for a spread of 12% (Figure 1). This is a supportive and nontrivial finding, since an estimation strategy using only earnings and expectations data provides a good fit for the evidence on spreads in returns.

The annualized levels of earnings growth (and expectations thereof) over a four-year horizon from the estimated model also provide a reasonable match to the data. The average yearly earnings growth for HLTG firms postformation is 11% (see Figure 7) in the data while in our model it is 9.8%. In turn, LTG for HLTG firms at formation is 39% while in the estimated model it is 22.5%.[22]

We can also assess model performance with respect to the dynamics of earnings, expectations, and returns around portfolio formation. As in Figure 3, growth expectations increase for three years preformation and decrease three years postformation, with most of the action happening in years $t - 1$ to $t + 1$. Crucially, event returns also exhibit a boom-bust pattern with an appropriate time scale: HLTG has strong returns for two years preformation, while LLTG has low returns during this period. Postformation, HLTG returns are lower than LLTG returns for up to three years (though the trough happens in year $t + 1$ in the data and in year $t + 3$ in the simulation). The fact that the time scale at which forecast errors are most predictable matches the time scale of abnormal returns suggests that predictability of returns is driven by expectation distortions.

Our overall assessment is that the model is both qualitatively and quantitatively able to account for several key features of the data, including the predictable return spread between HLTG and LLTG portfolios and the dynamics of expectations relative to the earnings process. At the same time, the model is very stylized, abstracting from both firm and investor heterogeneity. These assumptions can be relaxed without compromising the tractability of the diagnostic Kalman filter. An appropriate treatment of firm heterogeneity and variation in beliefs would likely help account for the features of the data that our model does not capture.

For example, the model exhibits too strong growth of EPS relative to Figure 2. Allowing for firm heterogeneity would improve the fit because HLTG firms are disproportionately younger and smaller. Allowing for variation in size would lead to more reasonable estimates of growth, and would also capture asymmetries in performance between HLTG and LLTG firms. In turn,

[22] Expectations in our model are less exaggerated than those of equity analysts. This is consistent with the fact that analysts' expectations are too optimistic on average across the board, while in our model the average expectations error is zero. As noted before, the model predicts a positive forecast error for the LLTG portfolio, which is not the case in the data.

accounting for heterogeneity in investors' beliefs would capture the dispersion in LTG forecasts, which we abstract from here.

## VI. Additional Predictions from Kernel of Truth

We next test some new cross-sectional predictions of the kernel-of-truth property of diagnostic expectations, the idea that expectations exaggerate true patterns in the data. This property allows us to further distinguish diagnostic expectations from mechanical models of extrapolation such as adaptive expectations, according to which individuals follow the constant gain adaptive rule

$$x^{ad}_{t+h|t} = x^{ad}_{t+h|t-1} + \mu \left( x_t - x^{ad}_{t|t-1} \right), \tag{11}$$

where $x^{ad}_{t+h|t}$ is the expectation held at $t$ about the level of EPS at $t + h$, $x_t$ is the current realized level of EPS, and $\mu \in [0, 1]$ is a fixed coefficient.

Under adaptive expectations, beliefs depend only on earnings' surprises $(x_t - x^{ad}_{t|t-1})$, not on the forward-looking data such as news, persistence, or volatility of the data-generating process. We test the implications of the kernel of truth along three directions: (i) the representativeness of future Googles in the HLTG group, (ii) how the persistence and volatility of earnings influence the LLTG-HLTG spread across firms, and (iii) the mechanics of expectations revisions.

### *A. Fat Tails and Representative Googles*

Diagnostic expectations account for excess optimism about HLTG firms as overreaction to the incidence of exceptional future performers, which are rare but representative. To assess this forward-looking mechanism, Figure 7 plots the true distribution of future EPS growth for the HLTG portfolio (solid line) and the distribution of future EPS growth of all other firms (dashed line).[23]

Two findings stand out. First, HLTG firms have higher average future EPS growth than all other firms, as we saw in a somewhat different format in Figure 2. Second, and critically, HLTG firms display a fatter right tail of exceptional performers. Googles are representative for HLTG in the sense of definition (4). In fact, based on the densities in Figure 7, the most representative future growth realizations for HLTG firms are above 40% annual growth (see Figures IA.2 and IA.3 in the Internet Appendix).[24]

[23] The logic behind this cross-sectional test is that growth preformation positively predicts growth postformation, so non-HLTG firms proxy for the performance of the HLTG portfolio under the counterfactual scenario of lower growth.

[24] Although HLTG firms tend to also have a slightly higher share of low performers, it is true that, as in our model, higher growth rates are more representative for HLTG firms. To compute earnings growth, Figure 7 is constructed using the subset of firms that have positive earnings at formation and five years after. The results are robust to alternative assumptions (see the Internet Appendix).

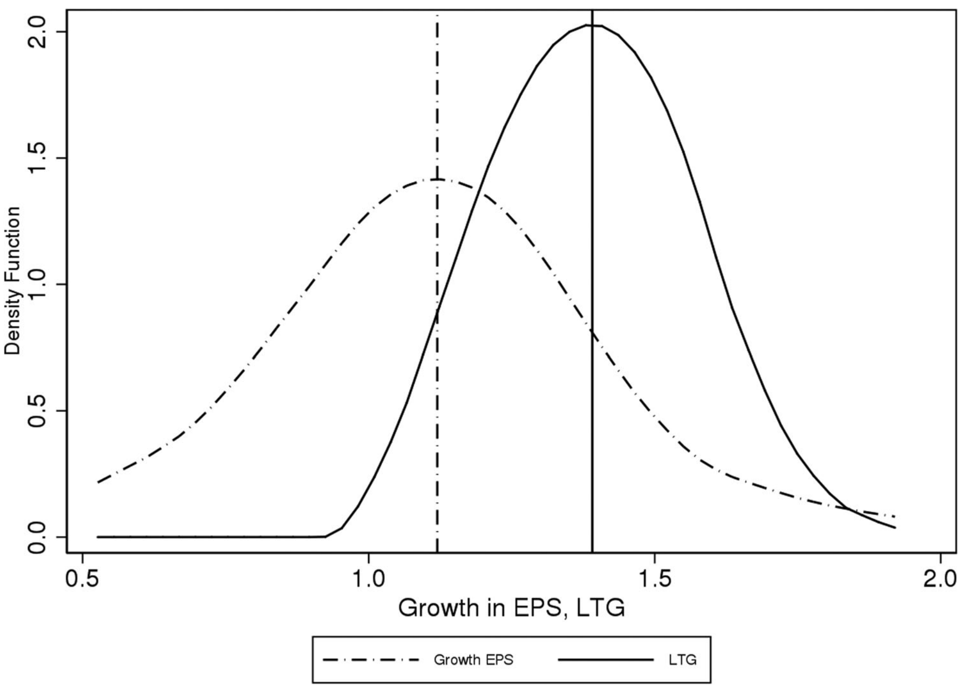


**Figure 8. Realized versus expected growth in EPS.** In December of years ($t$) 1981, 1986, ..., and 2011, we form decile portfolios based on ranked analysts' expected growth in long-term EPS (LTG). We plot two series. First, we plot the kernel distribution of the gross annual growth rate in EPS between $t$ and $t + 5$. Second, we plot the kernel distribution of the expected growth in long-term earnings at time $t$. The graph shows results for stocks in the highest decile of expected growth in long-term earnings at time $t$. The vertical lines indicate the means of each distribution (1.11 vs. 1.39, respectively).

Diagnostic expectations then predict that analysts should overestimate the number of right-tail performers in the HLTG group. Figure 8 compares the distribution of future performance of HLTG firms (dashed line) with the predicted performance for the same firms (solid line).[25] It is indeed the case that analysts vastly exaggerate the share of exceptional performers, which are most representative of the HLTG group according to the true distribution of future EPS growth. The kernel of truth can also shed light on the asymmetry between HLTG and LLTG firms. In the Internet Appendix, we show that future performance of LLTG firms tends to be concentrated in the middle, with a most representative growth rate of 0%. It is thus constant, rather than bad,

[25] In making this comparison, bear in mind that analysts report point estimates of a firm's future earnings growth and not its full distribution (in our model, they report only the mean $f^{\theta}_{i,t}$, not the variance $\sigma^2_f$). Thus, under rationality the LTG distribution would have the same mean but lower variance than realized EPS growth.

performance that is representative of LLTG firms.[26] As a robustness check, in the Internet Appendix we reproduce Figures 7 and 8 using two alternative measures of fundamentals (the change in earnings between $t$ and $t+5$ normalizing by the initial level of sales per share, and revenues minus cost of goods sold, which may be less noisy that EPS). The evidence again supports the kernel-of-truth hypothesis.

### *B. Volatility and Persistence of Earnings*

Another prediction of the kernel-of-truth property is that the predictability of expectations errors and hence of returns should depend on true features of the data, and in particular on the volatility and persistence of fundamentals. More volatile and more persistent fundamentals imply a higher signal-to-noise ratio of earnings, causing a larger overreaction to news. The LLTG-HLTG spread should then be higher for firms with more volatile or persistent fundamentals. In Figure IA.6 of the Internet Appendix, we illustrate these comparative statics in the context of a simulation.

We cannot estimate volatility and persistence firm by firm because some firms are too short-lived to allow for a reliable computation. This is particularly true for HLTG firms, which tend to be young. To circumvent this problem, for each year's HLTG and LLTG portfolios we match each firm to the group of firms in the same industry that are of comparable age. We require that each such group have at least five firms with at least 20 quarters of earnings data, which allows to estimate an average AR(1) coefficient for the earnings processes.[27] We then impute to each firm in the HLTG and LLTG portfolios the average estimated persistence of the matched group of firms and the average volatility of residuals.[28] We sort HLTG and LLTG firms into portfolios with firms in the top or bottom 30% of estimated persistence or volatility estimated in this way. We compute the postformation returns for each portfolio, as well as the returns of the portfolio that is long LLTG and short HLTG in each of the persistence or volatility buckets. Table IV reports the results.

The average spread between LLTG and HLTG portfolios exhibits a significant increase from 7.2% (i.e., 15% to 7.8%) to 11.9% (i.e., 12.8% to 0.9%) as firms move from the bottom 30% to the top 30% of estimated persistence of earnings growth. The return of a portfolio that is long LLTG and short HLTG stocks exhibits a similar increase, from 7.2% to 11.5%. A very similar pattern (10% vs. 16.6%) is present when sorting on volatility of earnings growth. Consistent

[26] This can explain why expectations about LLTG firms and their market values are not overly depressed. One could capture this difference between HLTG firms (representative high growth) and LLTG firms (representative 0% growth) by relaxing the assumption of normality, or alternatively by allowing lower volatility for firms in the LLTG group.

[27] For some groups this condition is not satisfied, in which case we can either group based on year and firm age alone, or drop the requirement of five observations. Here, we use the first method, but both give similar results.

[28] Data limitations do not allow us separate fundamentals and noise for individual firms in a Kalman filter estimation.

**Table IV**

**Return Spreads for Portfolios with Different Earnings Fundamentals**

In December of each year between 1981 and 2015, we form decile portfolios based on ranked analysts' expected growth in long-term EPS and compute equally weighted monthly returns for decile portfolios. The table reports annual average log returns for two-way sorts of extreme LTG portfolios (i.e., HLTG and LLTG), the portfolio that is long HLTG and short LLTG, and various portfolios formed on the basis of (1) the average persistence of growth in EPS for firms in the same industry and year, and (2) the average variance of the prediction errors from regressing $\text{EPS}_t$ on its first lag for firms in the same industry and year. We measure persistence based on the AR(1) coefficient of EPS on its lagged value in rolling regressions using 20 quarterly observations for years $t+5$ through $t$. Prediction errors are based on that regression. Finally, we form persistence and volatility portfolios based on the average persistence and volatility for stocks in the same industry and year. The "bottom 30%" is the portfolio of stocks in the bottom three deciles of persistence or volatility, as the case may be. Similarly, the "top 30%" is the portfolio of stocks in the top three deciles of persistence or volatility. The table also reports $t$-statistics for the portfolio that is long stocks in the top 30% of persistence and short stocks in the bottom 30% of persistence.

| | Persistence of Growth in EPS | | | Volatility of Prediction Error of Growth in EPS | | |
|---|---|---|---|---|---|---|
| | Bottom 30% | Top 30% | $t$-Statistics Top-Bottom | Bottom 30% | Top 30% | $t$-Statistics Top-Bottom |
| LLTG | 15.0% | 12.8% | −0.73 | 14.9% | 17.3% | 0.80 |
| HLTG | 7.8% | 0.9% | −2.46 | 4.9% | 0.6% | −1.82 |
| HLTG – LLTG | −7.2% | −11.5% | −2.05 | −9.8% | −16.6% | −2.42 |

with the kernel of truth, return spreads are predictable in terms of the firms' underlying fundamental earnings process.

In sum, the evidence is in line with the key prediction that patterns in returns are predictable from fundamentals. Expectations link fundamentals to predictable returns. Together with our quantification, these results further support the claim that predictability of returns is driven by forecast errors that exhibit overreaction.

### C. Revisions of Expectations

The last kernel-of-truth prediction that we test concerns revisions of expectations. Under diagnostic expectations, believed fundamentals are revised from one period to the next by

$$\begin{aligned}\hat{f}^{\theta}_{i,t+1} - \hat{f}^{\theta}_{i,t} = {} & K(1+\theta)\left(x_{i,t+1} - bx_{i,t} - a\hat{f}^{\theta}_{i,t}\right) - (1-a)\,\hat{f}_{i,t} \\ & -K\theta\,(1 - aK(1+\theta))\left(x_{i,t} - bx_{i,t-1} - a\hat{f}_{i,t-1}\right).\end{aligned}$$

Leaving aside current news (the term in the first line), beliefs about fundamentals are updated due to mean reversion of fundamentals (i.e., $-(1-a)\hat{f}_{i,t}$), and also due to the waning of the overreaction to previous shocks (i.e., $x_{i,t} - bx_{i,t-1} - a\hat{f}_{i,t-1}$). For HLTG stocks, both forces point to a downward revision of beliefs regardless of the current news received, while for LLTG stocks the opposite holds, leading to systematic mean reversion of LTG forecasts.

Under adaptive expectations, in contrast, analysts revise growth expectations downward if and only if bad news arrives, that is, $(x_t - x^{ad}_{t|t-1}) < 0$. This mechanical rule of expectations formation does not take mean reversion of fundamentals into account.

To see which theory better describes the data, we assess how LTG changes around earnings announcement dates as a function of the surprise in earnings. Take a firm with *Wall Street Journal* announcement dates $wsj_{t-1}$ and $wsj_t$ for earnings in fiscal years $t-1$ and $t$, respectively. We compute the earnings surprise at $wsj_t$ as the difference between the realized EPS in (fiscal) year $t$ announced at $wsj_t$ and the EPS forecast for time $t$ made soon after the previous announcement, specifically, on day $wsj_{t-1}$+45. We then compute the change in LTG between $wsj_{t-1}$+45 and $wsj_t$+45, which already incorporates the revision in response to the announcement at $wsj_t$.

To visualize the results, we normalize earnings surprise by the stock price when the forecast was made, and then rank observations into surprise deciles. The results are plotted in Figure 9.

The data show strong evidence of systematic mean reversion. Regardless of the actual earnings surprise, LTG expectations about HLTG firms (solid line) deteriorate on average by 2.6% while those about LLTG ones (dashed line) improve on average by 0.6%. The reversal in Figure 9 is not simply due to the fact that on average HLTG firms receive bad surprises and LLTG firms on average receive good surprises. Rather, even HLTG firms that experience positive earnings surprises are downgraded, and even LLTG firms that experience negative earnings surprises are upgraded. These findings are puzzling from the perspective of adaptive expectations, but are consistent with the forward-looking nature of diagnostic expectations.[29] In the Internet Appendix, we show that the same pattern emerges in our estimated model.[30]

Overall, the data offer support for the kernel-of-truth prediction of diagnostic expectations, but are difficult to reconcile with a mechanical model of adaptive expectations. Another significant alternative to diagnostic expectations is the

[29] In the Internet Appendix (Section IV), we show that adaptive expectations predict no overreaction to news after the persistence of the earnings process is accounted for. After controlling for current levels $x_t$, the adaptive forecast revision $(x^a_{t+1} - x^a_t)$ should positively predict forecast errors as in the underreaction models. In contrast, diagnostic expectations overreact to news regardless of the persistence of the data-generating process, which is confirmed in the data (Table IA.IV in the Internet Appendix).

[30] Expectations may be formed, at least in part, by rational learning from prices. Suppose that investors' required return is unobservable and follows a mean-reverting process. Price increases then signal an improvement in fundamentals but also a decrease in investors' required return, so that expectations of earnings growth and expectations of returns should be negatively correlated. To test this prediction, we construct a measure of analysts' expectations of returns by gathering IBES data on the projected price level forecasted by analysts within a 12-month horizon. We define target returns as the ratio of the mean target price across all analysts following the stock to the current stock price. Using this measure, the correlation between analysts' expectations of LTG and their expected returns is 0.23 (significant at the 1% level), in contrast to the prediction above. Our model does not provide a meaningful counterpart to this finding, as (diagnostic) expectations of returns are constant and equal to $R$.

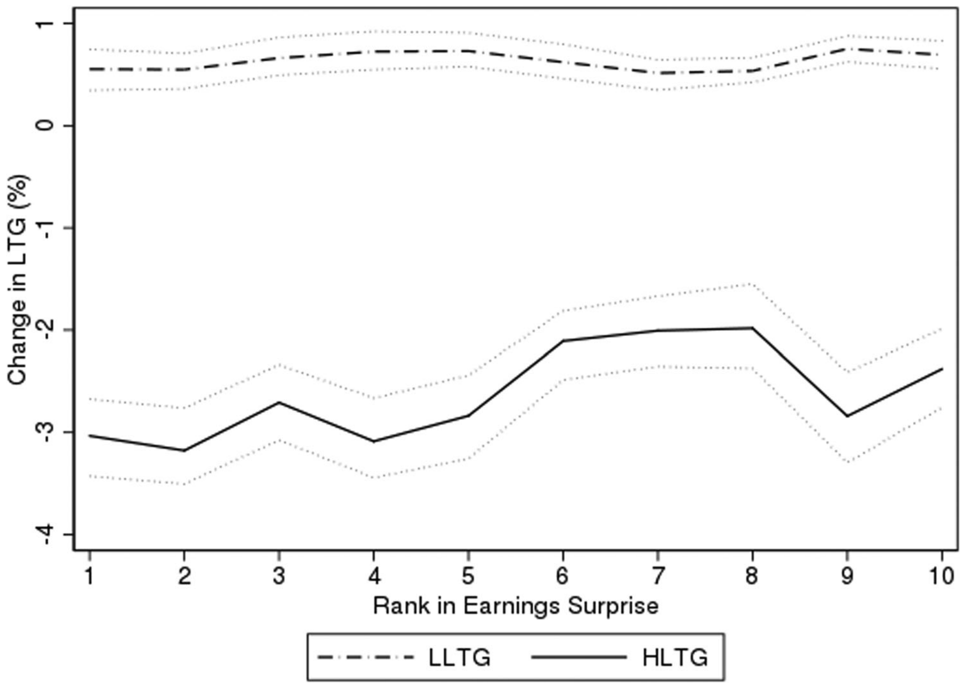


**Figure 9. Evolution of analysts' beliefs in response to earnings' announcements.** For each analyst $j$, firm $i$, and fiscal year $t$, we compute the difference between the LTG forecasts made 45 days following the announcement of earnings in (1) fiscal year $t$ ($wsj_t$+45) and (2) fiscal year $t-1$ ($wsj_{t-1}$+45). We measure forecast errors for earnings based on forecasts 45 days following the announcement for earnings in fiscal year $t-1$ ($wsj_{t-1}$+45). We rank observations into deciles based on the ratio of the forecasting error for EPS in fiscal year $t$ to the stock price when the forecast was made ($wsj_{t-1}$+45). The figure plots the bootstrapped mean LTG change between $wsj_{t-1}$+45 and $wsj_t$+45 for the HLTG and LLTG portfolios. The dotted lines indicate 5th and 95th confidence levels determined via bootstrapping.

Barberis, Shleifer, and Vishny (1998) model of investor sentiment. That model is motivated by representativeness but does not model it explicitly. Rather, it assumes that investors hold dogmatic, yet incorrect, priors in a way that is designed to produce excess optimism after a string of good news. Although the true process driving a firm's earnings is a random walk, analysts perform Bayesian updating across two incorrect models, one in which earnings are believed to trend and one in which they are believed to mean revert. Overreaction occurs because fast earnings growth leads the analyst to attach too high a probability that the firm is of the "trending type," even though no firm is actually trending.

Diagnostic expectations capture this key intuition from Barberis, Shleifer, and Vishny (1998): after good performance analysts place disproportionate weight on strong fundamentals, and the reverse after bad performance. In our view, the new model has two main advantages relative to its antecedent. First, it seeks to provide a more explicit model of representativeness. The kernel-of-truth property, which holds that beliefs overreact to objective information, is key for the model's generality and tractability, and provides novel predictions for the cross-section of stock returns that are supported in the data. Second, and relatedly, diagnostic expectations are portable across different domains: unlike Barberis, Shleifer, and Vishny (1998) they are not designed for a specific finance setting such as learning about a firm's earnings growth, and so they can be easily applied to macroeconomic expectations, laboratory experiments, or social stereotyping. This allows us to assess and quantify kernel-of-truth distortions in a variety of contexts, and to identify robust departures from rational expectations.

## VII. Conclusion

This paper revisits what has perhaps been the most basic challenge to rational asset pricing since Shiller (1981): overreaction to news and the resulting excess volatility and mean reversion. We investigate this phenomenon in the context of individual stocks, for which we have extensive evidence on security prices, fundamentals, and—crucially—expectations of future fundamentals. La Porta (1996) shows empirically that securities whose long-term earnings growth analysts are most optimistic about earn low returns going forward. Here, we propose a theory of belief formation that delivers this finding, and also provide a characterization of joint evolution of fundamentals, expectations, and returns that can be taken to the data.

A central feature of our theory is that investors are forward looking, in the sense that they react to news. However, their reaction is distorted by representativeness, the fundamental psychological principle that people put too much probability weight on states of the world that the news they receive aligns with. In psychology, this is known as the kernel-of-truth hypothesis: people react to information in the right direction, but too strongly. We refer to such belief formation as diagnostic expectations, and show that a theory of security prices based on this model of beliefs can explain not just previously documented return anomalies, but also the joint evolution of fundamentals, expectations, and returns.

The theory is portable in the sense that the same model of belief distortions has been shown to work in several other contexts. At the same time, the model can be analyzed using a variation of Kalman filter techniques used in models of rational learning. The theory yields a number of strong empirical predictions that have not been considered before and that we bring to the data. These predictions distinguish the theory from adaptive expectations, and show that investors and analysts are forward looking in forming their beliefs. The model can also be estimated using the method of moments, and does a reasonable job fitting several moments of the data. Significantly for our purposes, the critical representativeness parameter that we estimate—our measure of overreaction—is comparable to estimates that we obtain with very different data sets.

Of course, this is just a start. Our approach to expectation formation can be taken to other contexts, most notably aggregate stock prices but also macroeconomic time series. Here, we focus on distortions of beliefs about the means of future fundamentals, but the kernel-of-truth idea could be applied to thinking about other moments as well, such as variance or skewness. We hope to pursue these ideas in future work, but stress what we see as the central point: the theory of asset pricing can incorporate fundamental psychological insights while retaining the rigor and the predictive discipline of rational expectations models. And it can explain the data not just on the joint evolution of fundamentals and security prices, but also on expectations, in a unified dynamic framework. Relaxing the rational expectations assumption does not entail a loss of rigor; to the contrary it allows for a disciplined account of additional features of the data. An estimation exercise suggests, moreover, that the model can replicate several quantitative, and not just qualitative, features of the data.

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## Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

**Appendix S1**: Internet Appendix.
**Replication Code**