Expectations of Fundamentals and Stock Market Puzzles

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Abstract

We revisit several leading aggregate stock market puzzles by incorporating into a standard dividend discount model survey expectations of earnings of S&P 500 firms. Using survey expectations, while keeping discount rates constant, explains a significant part of "excess" stock price volatility, fluctuations in price earnings and price dividend ratios, and return predictability. Expectations about long term earnings growth emerge as a key driver of these pricing anomalies. The evidence is consistent with a mechanism in which good news about fundamentals leads to excessively optimistic long term earnings forecasts, inflating stock prices and leading to subsequent low returns. Relaxing rational expectations of fundamentals in a standard asset pricing model, guided by empirical measures of expectations and in line with accumulating evidence on overreaction, yields a parsimonious account of stock market puzzles.

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I. Introduction

In the dividend discount model, the price of a stock at time t is given by:

$$P_t = \sum_{s=t+1}^{\infty} \frac{\mathbb{E}_t(D_s)}{R^{s-t}},$$

where R is the constant required return and $\mathbb{E}_t(D_s)$ is the rational expectation of the dividend per share at time s. Research over the last few decades has shown that this model is a poor description of stock market movements. There are three main problems. First, as shown by Shiller (1981) and Leroy and Porter (1981), stock prices are much more volatile than the present value of dividends or earnings. In the dividend discount model, all price volatility should be due to news about these fundamentals. Second, the price dividend ratio has a low correlation with future growth in dividends or earnings (Campbell and Shiller 1988). This is also inconsistent with the dividend discount model, in which the price dividend ratio reflects rational forecasts of future dividends. Third, stock returns are predictable: a high price dividend ratio today predicts low stock returns over a three to five year horizon (Campbell and Shiller 1988). This is inconsistent with another key assumption of the model: constant required returns.

The Campbell-Shiller decomposition shows that these puzzles are related, and that they can be reconciled under rational expectations if the required return is time-varying. Several models of time varying returns have been proposed, based on disaster risk, recursive utility, and habit formation (e.g., Rietz 1988, Campbell and Cochrane 1999, Bansal and Yaron 2004, Barro 2006, Gabaix 2012, Wachter 2013). This approach is not without problems. First, it relies on changes in risk attitudes, which are hard to measure directly. Second, it predicts that investors should expect low returns when stocks are expensive. In survey data, however, the opposite is true: in good times investors expect high, not low, returns (Greenwood and Shleifer 2014). Contrary to rational expectations, such optimism is systematically disappointed in the future.

In this paper we address stock market puzzles by taking an orthogonal route: we hold required returns constant and assess how far we can get by relaxing belief rationality. We discipline departures from rationality by using measured expectations of future fundamentals. Recent work shows the promise of using such data. Bordalo, Gennaioli, La Porta, and Shleifer (BGLS 2019) find that analyst forecasts of firms' long-term earnings growth overreact to news about firm-level performance, and that this overreaction helps explain the cross section of returns. De la O and Myers (DM 2020) show that analyst short-term earnings forecasts for S&P 500 firms have strong explanatory power for the price earnings and price dividend ratios.

We systematically investigate the explanatory power of measured expectations of fundamentals for stock prices. Relative to previous work, we incorporate expectations of long term earnings growth into our analysis, and jointly assess all three puzzles. We also investigate systematic departures of measured expectations from rationality, and explore the connection between systematic errors in expectations and predictability of stock returns. These steps prove critical to account for the puzzles, and highlight the mechanism linking beliefs to mispricing.

We report three main findings. First, expectations of fundamentals exhibit a remarkable ability to account for stock price volatility, time variation in the valuation ratios, and return predictability. The leading behavioural approach to these anomalies stresses extrapolative price expectations (Cutler, Poterba, and Summers 1990, DeLong et al. 1990b, Barberis et al. 2015, 2018, Jin and Sui 2019), but we show that beliefs about fundamentals play a key role.

Second, beliefs about *long term* earnings are the unifying element. Not only are these beliefs important in accounting for excess volatility and price dividend ratio variation, they prove essential to generate predictability of returns, which is the key marker of price anomalies. No predictability of returns is obtained from short term beliefs. This confirms the centrality of long run perceptions (e.g., Bansal and Yaron 2004), but in shaping beliefs, not risk attitudes.

Third, the mechanism driving price anomalies is overreaction to fundamental news. At times of high earnings growth, expectations about long term earnings are too optimistic, which suggests that at these times stock prices are inflated. Consistent with this mechanism, we show that high growth periods are followed by predictable disappointment in the form of forecast errors, and that these errors in turn correlate with low realized returns.

These results are in line with our earlier work on Diagnostic Expectations and analysts' firm level forecasts (BGLS 2019): a firm's high earnings growth is followed by upward revisions of its expected long term earnings growth, subsequent disappointment of these expectations, and low stock returns for the firm. Just like for the aggregate stock market, the overreaction of firm-level forecast revisions is associated with a large return differential: over 40 years of data, firms in the highest decile of long term growth forecasts earn on average 12pp lower one-year returns than firms in the lowest decile (La Porta 1996, BGLS 2019). Here, for the aggregate stock market, we find that the average one-year return differential between years with long term growth forecasts at the 95th percentile vs 5th percentile is 20pp. Overreaction of long-term growth forecasts is thus a quantitatively large driver of both aggregate and cross-sectional stock market puzzles. It offers a disciplined departure from rationality that may improve the explanatory power of asset pricing models.

Conceptually, the approach in this paper is closest to models featuring mis-specified beliefs about the earnings data generating process (DeLong et al. 1990a, DeBondt and Thaler 1990, Lakonishok et al. 1994, Barberis, Shleifer, and Vishny 1998, Daniel, Hirshleifer, and Subrahmanyam 1998, 2001, Odean 1998, Greenwood and Hanson 2015). Barsky and DeLong (1993) and Luo and Wachter (2020) apply related ideas to the aggregate stock market.

A growing literature uses measured beliefs to shed light on asset prices (e.g., La Porta 1996, Frankel and Lee 1998, Lee, Myers, and Swaminathan 1999, Lee and Swaminathan 2000, Bachetta et al 2009, Koijen and Nieuwerburgh 2011). Nagel and Xu (2019) show that a weighted average of past dividend growth negatively predicts future returns, is uncorrelated with measured expectations of returns, and positively correlates with expectations of fundamental growth, captured by a measure similar to the one we use. They offer a model of learning with recency effects, in which investors effectively hold adaptive expectations of future dividends.[2] Relative to this work, we provide the first systematic and unified assessment of the three stock market puzzles, identify overreaction to fundamentals as the driver of pricing anomalies, and connect systematic forecast errors to predictable returns.

The paper is organized as follows. Section 2 offers a formulation of beliefs about the growth of cashflows that nests several departures from rationality: noise, overreaction, and underreaction to fundamentals. We show that, under constant required returns, the puzzles can be explained if beliefs deviate from rationality, and identify the conditions under which this may be the case. These include, but are not limited to, overreaction to news.

Section 3 presents the data, with an emphasis on measured analyst expectations of short and long term earnings growth for S&P 500 firms. Using this data, in Section 4 we compute an expectations-based stock price index, assuming a constant required return. We show that, consistent with the model in Section 2, measured expectations have strong explanatory power for the puzzles. In particular, beliefs about long term growth connect stock price volatility and variation in the price dividend or earnings ratios to predictability of future returns.

In Section 5 we investigate the mechanism linking beliefs and price anomalies. Guided by the model of Section 2, we study systematic errors in the formation of long term beliefs and connect these predictable errors to predictable returns. Overreaction of investor expectations of long term payouts to fundamental news emerges as the key driver of anomalies.

2. Expectations and Stock Market Puzzles

Following Campbell and Shiller (1988), the log return r_{t+1} obtained by holding a stock between t and $t+1$ is given by the log linearized expression:

$$r_{t+1} = \alpha p_{t+1} + (1-\alpha) d_{t+1} - p_t + k, \tag{1}$$

[2] Other papers that consider the implications of learning frictions for stock market excess volatility and return predictability include Timmermann (1993), Alti and Tetlock (2014), Hirshleifer, Li and Yu (2015), Ravi and Liu (2019), and Guo and Wachter (2020).

where p_t and p_{t+1} are the log stock prices at t and $t+1$, d_{t+1} is the log dividend at $t+1$, and $\alpha = e^{pd}/(1+e^{pd}) < 1$ depends on the average log price dividend ratio pd and k is a constant that depends on α.

By iterating Equation (1) forward and imposing the transversality condition, one obtains the Campbell-Shiller decomposition:

$$p_t - d_t = \frac{k}{1-\alpha} + \sum_{s\geq 0} \alpha^s g_{t+s+1} - \sum_{s\geq 0} \alpha^s r_{t+s+1}, \tag{2}$$

where $g_{t+s+1} \equiv d_{t+s+1} - d_{t+s}$ is dividend growth between $t+s$ and $t+s+1$. Dividend growth follows a covariance stationary process with MA representation $g_t = \sum_{j\geq 0} \eta_j \epsilon_{t-j}$, where ϵ_t is an i.i.d. Gaussian shock with mean zero and variance σ^2. η_j is the impulse response for j periods ahead, satisfying square summability $\sum_{j\geq 0} |\eta_j|^2 < \infty$ and $\eta_0 = 1$.[3]

In Equation (2), variation in the log price to dividend ratio is due to expected variation in dividend growth, in required returns, or both. In the dividend-discount model, the required return is constant, so price movements only reflect expectations of dividend growth g_{t+s+1}. As a result, if expectations are rational, realized excess returns cannot be predicted. Here we keep required returns constant and derive the conditions on beliefs about future dividend growth that can help address the puzzles described in the Introduction. In the rest of the paper, we then show that 1) measured expectations do help account for the puzzles empirically, and 2) they satisfy the conditions for doing so established in this section.

Let $\mathbb{E}_t(.)$ denote rational expectations and $\mathbb{E}_t^M(.)$ denote the possibly non-rational market expectations. Taking the market expectation of Equation (2) yields the (log) stock price:

$$p_t^M = d_t + \frac{k-r}{1-\alpha} + \sum_{s\geq 0} \alpha^s \mathbb{E}_t^M(g_{t+s+1}). \tag{3}$$

where r is the constant required return. We allow for departures from rationality of the form:

[3] We use this very flexible specification because it allows for the observed short-term reversals in earnings growth. Much of our analysis focuses on earnings, for which we have more detailed data on beliefs.

$$\mathbb{E}_t^M(g_{t+s+1}) = \mathbb{E}_t(g_{t+s+1}) + \epsilon_{Mt} \tag{4}$$

Beliefs are forward looking but are distorted by a mistake ϵ_{Mt}. We assume ϵ_{Mt} follows an AR(1) process $\epsilon_{Mt} = \rho\epsilon_{Mt-1} + u_{Mt}$, where $\rho \in [0,1]$ and u_{Mt} is i.i.d. normal with mean zero and variance σ_M^2. When $\sigma_M^2 = 0$, beliefs are rational.

Our model nests several well-known departures from rationality, and the nature of biases depends on the correlation σ_{MF} between ϵ_{Mt} and fundamental news ϵ_t. If the expectations shock is independent of fundamentals, so $\sigma_M^2 > 0$ but $\sigma_{MF} = 0$, then forecasts are distorted by persistent noise, as in noise trading models (Black 1986, DeLong et al. 1990a). If instead $\sigma_{MF} < 0$, the belief distortion is negatively correlated with the current news ϵ_t, so it tempers any positive autocorrelation in fundamentals. This can yield a muted response to news, as with rational inattention (Sims 2003, Huang and Liu 2007, Bouchaud et al. 2019). Finally, if $\sigma_{MF} > 0$, the belief distortion entails excess optimism after high growth $\epsilon_t > 0$. This can produce overreaction to news as in models of diagnostic expectations (Bordalo, Gennaioli, and Shleifer 2018, BGLS 2019), but also in earlier models (e.g., DeLong et al. 1990b, Barberis et al. 1998).[4]

We next restate the aggregate stock market puzzles in our model, and ask under what assumptions on belief volatility σ_M^2 and on mis-reaction to fundamental news σ_{MF} the model can account for these puzzles.[5]

We begin with excess volatility, according to which annual price changes are too volatile relative to what is implied by rational expectations and constant returns (Shiller 1981, LeRoy and Porter 1981, Campbell and Shiller 1987). Define excess volatility by $\Delta V_{M,RE} = Var(p_t^M -$

[4] The expectations shock could capture "intangible news", as described by Daniel and Titman (2006), namely distortions that are entirely forward looking, and are not measurable from past growth. In contrast, our model does not nest the purely backward-looking adaptive expectations, due to the rational component in Equation (4).

[5] We prove our main results with a more general version of Equation (4) in which the distortion changes across horizons s (replacing ϵ_{Mt} by $\delta_{s+1}\epsilon_{Mt}$). The basic formulation of Diagnostic Expectations (DE, BGS 2018) falls in this case, as it assumes that δ_{s+1} equals the true impulse response function. DE further assume that: i) the belief shock is collinear with fundamentals, $u_{Mt} = \theta_M\epsilon_t$ with $\theta_M > 0$, and ii) the belief distortion is transient, $\rho = 0$. Recent work shows that an empirically more valid formulation of DE allows for persistent distortions $\rho > 0$, and for stronger overreaction in the long run (BGLS 2019, D'Arienzo 2020). This is confirmed by our analysis here.

$p^M_{t-1}) - Var(p^{RE}_t - p^{RE}_{t-1})$, where p^{RE}_t is the (log) price prevailing under rational expectations in Equation (3). Shiller's finding is the statement that $\Delta V_{M,RE} > 0$.

Consider next the price dividend ratio puzzle. In the dividend discount model, regressing future discounted growth $\sum_{s\geq 0} \alpha^s g_{t+s+1}$ on $p_t - d_t$ should yield a coefficient of 1. This is strongly rejected in the data, where the coefficient is well below 1 (Campbell and Shiller 1988, Cochrane 2011). Errors in beliefs can account for this finding if they entail prices p^M_t such that the regression coefficient $\hat{\beta}_{MP} = \frac{cov(\sum_{s\geq 0} \alpha^s g_{t+s+1}, p^M_t - d_t)}{var(p^M_t - d_t)}$ is below 1.

Finally, return predictability holds that when stocks are expensive ($p_t - d_t$ is high) future discounted returns $\sum_{s\geq 0} \alpha^s r_{t+s+1}$ are low (Campbell and Shiller 1987, 1988). This cannot happen in a rational model with constant required returns. In our model, errors in beliefs help explain this finding if the regression coefficient $\hat{\beta}_{MR} = \frac{cov(\sum_{s\geq 0} \alpha^s r_{t+s+1}, p^M_t - d_t)}{var(p^M_t - d_t)}$ is negative.

Proposition 1 describes the conditions under which beliefs about fundamentals can account for the three puzzles with constant expected returns (proofs appear in Appendix A).

Proposition 1 *Suppose that* $\frac{\partial p^{RE}_t}{\partial \epsilon_t}$ *is positive and sufficiently large. Then there exist positive constants μ and ω such that the dividend discount model in Equations (3) and (4) yields:*

a) Excess volatility $\Delta V_{M,RE} > 0$ *when* $\sigma^2_M + \mu(1-\alpha)\sigma_{MF} > 0$.

b) The price dividend ratio puzzle, $\hat{\beta}_{MP} \in (0,1)$*, and the return predictability puzzle,* $\hat{\beta}_{MR} < 0$*, when* $\sigma^2_M + \omega(1-\alpha)\sigma_{MF} > 0$.

Proposition 1 shows that the puzzles can be accounted for by two possible mechanisms. First, when belief distortions are pure noise, i.e. display excess volatility uncorrelated with fundamentals ($\sigma_M^2 > 0$ and $\sigma_{MF} = 0$). Second, the puzzles can be explained if beliefs overreact to fundamentals, in that they become excessively optimistic after good news, $\sigma_{MF} > 0$.[6]

To assess whether stock market fluctuations are driven mostly by random noise or by excessive optimism in good times, we exploit a key additional prediction of our model.

Proposition 2 *Positive revisions of growth forecasts* $[\mathbb{E}_t^M(g_{t+s+1}) - \mathbb{E}_{t-1}^M(g_{t+s+1})] > 0$ *after good fundamental news* ϵ_t *predict low returns, and negative revisions after bad news predict high returns, if and only if* $\sigma_{MF} > 0$.

Proposition 2 shows that predictability of returns is crucial for pinning down the mechanism giving rise to the pricing anomalies. Suppose that the econometrician finds that strong fundamentals are positively correlated with upward revisions in growth expectations. If these upward revisions predict lower future returns, it means that expectations become too optimistic after good news, $\sigma_{MF} > 0$. That is, stock prices are distorted by "overreaction to news": strong fundamentals lead to excess optimism, leading to an inflated price p_t^M. Optimism is then systematically reversed in the future, leading to negative forecast errors and low returns.

Section 3 presents our measures of expectations, which are equity analysts' earnings and dividend forecasts. In Section 4 we incorporate these measures into a dividend discount model and consider their explanatory power for price volatility, price dividend ratio variation, and future stock returns, in line with Proposition 1. In Section 5 we address the mechanism of overreaction by analyzing how expectations react to news and how forecast revisions predict returns, in line with Proposition 2.

3. Data

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[6] The price to dividend ratio puzzle and the return predictability puzzle rely on the same condition, which follows from the Campbell-Shiller decomposition. Excess volatility relies on a related but distinct condition. Intuitively, excess volatility relies on the volatility of expectations shocks, while the price dividend ratio puzzle and return predictability puzzles also depend on their persistence.

Forecasts of Dividends and Earnings. We gather monthly data on stock market analyst forecasts for S&P500 firms from the IBES Unadjusted US Summary Statistics file, which surveys analysts during the third Wednesday of each month. We focus on (median) annual forecasts of dividends per share (DPS), earnings per share (EPS), and long-term earnings growth (LTG). IBES data on earnings is more extensive than on dividends: coverage starts on 3/1976 for EPS, 12/1981 for LTG, and on 10/2002 for DPS.[7] In principle, IBES tracks annual forecasts for fiscal years one (typically, 4 months into the future) through five (typically 52 months into the future). In practice, *EPS (DPS)* forecasts beyond the third (second) fiscal year are often missing. In our analysis, we fill in for missing *EPS* forecasts by assuming that analysts expect *EPS* to grow at the rate LTG starting with the last non-missing positive *EPS* forecast. This is a sensible assumption since IBES defines LTG as the "…expected annual increase in operating earnings over the company's next full business cycle. In general, these forecasts refer to a period of between three to five years." In the text, we only consider one-year ahead *DPS* forecasts.

We aggregate *EPS* and *DPS* forecasts across firms in the S&P500 index. We first linearly interpolate *EPS* (*DPS)* forecasts for each firm *i* and focus on forecasts at horizons ranging from one to five years (in one-year increments). Next, for each firm *i* and month *t*, we compute forecasts for the level of earnings (dividends) by multiplying the EPS_{it} (DPS_{it}) forecast by the number of shares of firm i outstanding at time t. We then sum these forecasts across all firms in the index to obtain aggregate earnings (dividends) for the index. Finally, to compute the index level EPS_t and DPS_t we divide these summed forecasts by the total number of shares in the S&P500 index, the index divisor S_t.[8] In line with the model in Section 2, we use these forecasts in Section 4 to generate forecasts or earnings (dividends) growth one or two-years ahead.[9]

[7] Forecasts for *DPS* start on 1/1991. We require observations to include forecasts for firms totaling at least 90% of the market of the S&P. Following this rule, the first observation with a non-missing forecast is in 10/2002.
[8] The divisor S_t is the ratio of the market capitalization of S&P500 and the S&P500 index. It equals 100 in the base year and is adjusted to reflect changes in shares outstanding, in the composition of the index, and corporate actions.
[9] We compute growth forecasts at the aggregate level, and not at the firm level, because many firm-level observations have zero or very low current earnings. We set an observation in a given month to missing if the market cap of the firms for which we have forecasts at a given horizon is less than 90% of the market cap of the index.

We aggregate LTG forecasts by value-weighting firm level forecasts:

$$LTG_t = \sum_{i=1}^{S} LTG_{i,t} \frac{P_{i,t} \cdot Q_{i,t}}{\sum_{i=1}^{S} P_{i,t} \cdot Q_{i,t}}$$

where S is the number of firms in the S&P500 index with IBES data on LTG_{it}, P_{it} is the stock price of firm *i* at time *t*, and $Q_{i,t}$ is the number of shares outstanding of firm *i* at time *t*.[10]

Analysts may distort their forecasts due to agency conflicts. As we showed in previous work (BGLS 2019), this is unlikely to affect the time series variation in forecasts, which is key here. Furthermore, all brokerage houses typically cover S&P500 firms, so investment banking relationships and analyst sentiment are less likely to play a role in the decision to cover firms in the S&P500.[11] To further alleviate the concern about agency conflicts, and in particular to reduce the impact of outliers, we focus on median forecasts across analysts.

Bordalo et al (2020) show that consensus beliefs such as the median forecast are not ideal to study departures from rationality because informational frictions bias the consensus forecast toward under-reaction even if individual analysts over-react to their individual information. We do not address this issue here, but stress that – if anything – it makes our results stronger.

Earnings surprise/returns data. From the CRSP/COMPUSTAT merged file data, we collect data on earnings (income before extraordinary items) and dates when the *Wall Street Journal* published quarterly earnings releases (*rdq*). We aggregate earnings for the S&P500 in the same way as EPS forecasts. From CRSP, we get shares outstanding, stock prices, data on S&P500 index membership and returns.

[10] Nagel and Xu (2019) weigh $LTG_{i,t}$ using firm level earnings, $LTG_t^{EW} = \sum_{i=1}^{s} LTG_{i,t} \frac{Q_{i,t} \cdot \mathbb{E}_t^O[EPS_{i,FYR0}]}{\sum_{i=1}^{S} Q_{i,t} \cdot \mathbb{E}_t^O[EPS_{i,FYR0}]}$, where $\mathbb{E}_t^O[EPS_{i,FYR0}]$ is the IBES median forecast for the earnings of firm *i* in the current fiscal year, and *s* is the set of firms with positive earnings forecasts that are members of the S&P500. The correlation between LTG_t and LTG_t^{EW} is 95.44%. Since stocks with high LTG often have low earnings, our preferred measure is LTG_t.

[11] For example, in December of 2018, nineteen analysts followed the median S&P500 firm, while four analysts followed the median firm not in index. Analysts are also less likely to rate as "buy" firms in the S&P500 index.

We obtain monthly data on price dividend and price earnings ratios and dividends for the S&P500 from Robert Shiller's website.[12] Data on expected returns for the S&P500 come from the quarterly survey of CFOs administered by John Graham and Campbell Harvey. Starting in October 2010, the survey tracks, among other things, the returns that CFOs expect for the S&P500 over the following 12 months and over the following ten years.

4. Growth Expectations, Excess Volatility and Price Dividend Variation

To assess the ability of measured expectations to account for market prices and valuation ratios, we construct a price index by plugging measured expectations of earnings or dividend growth into a dividend discount model with a constant discount rate r, following Equation (3).

In our main specification, we construct a price index based on expectations of earnings growth, for both short and long-term, because these are available for a longer sample period and also at longer horizons than for dividends. Formally:

$$p_t^O \approx e_t + \frac{\tilde{k} - r}{1 - \alpha} + \sum_{s \geq 0} \alpha^s \mathbb{E}_t^O \left(\Delta e_{t+j+1} \right) \tag{5}$$

where $\tilde{k} = k + (1 - \alpha)\ de$ and $de =$ is the average log payout ratio. This expression holds in the limit where α is close to 1 (in the data, $\alpha = \frac{1}{1+e^{-pd}} \sim 0.9774$). We use the superscript O to denote the observed expectations $\mathbb{E}_t^O \left(\Delta e_{t+j+1} \right)$, which may be positively correlated with, but are not necessarily identical to, market expectations $\mathbb{E}_t^M \left(\Delta e_{t+j+1} \right)$.

Empirically, we generate p_t^O as:

$$p_t^O = e_t + \frac{\tilde{k} - r}{1 - \alpha} + \ln \left(\frac{\mathbb{E}_t^O EPS_{t+1}}{EPS_t} \right) + \sum_{j=1}^{10} \alpha^{j-1}\ \mathbb{E}_t^O \left(\Delta \mathrm{e}_{t+j+1} \right) + \frac{\alpha^{10}}{1 - \alpha}\ g \tag{6}$$

where we set r to 8.48% (the sample mean) and $\tilde{k}$ to 0.0927. We measure expected growth between t and $t + 1$ and between $t + 1$ and $t + 2$ using forecasted earnings. For longer horizons

[12] http://www.econ.yale.edu/~shiller/data.htm

we use LTG. Recall that LTG captures earnings growth over the next business cycle, and in particular over the next 3 to 5 years, so we do not have data for very long run growth expectations. We thus make the reasonable assumption that forecasts for the longer term gradually revert from LTG toward an average long-run level g.

We implement this approach in two ways. In our main specification, we use LTG to proxy for the expected growth rate between $t+3$ and $t+10$. This corresponds to the average duration of a business cycle in our data. We then set the expected growth rate beyond $t+11$ such that the level our price index p_t^O is consistent with the actual observed stock price, which yields $g = 6.35\%$.[13] In the main analysis we use nominal values, but in Appendix B (Figure B.1) we show that our results are robust when we account for inflation. In a robustness exercise, we use LTG until $t+5$ and infer growth expectations for longer horizons by applying the estimated decay of observed cyclically adjusted earnings to LTG_t. This yields virtually identical results (see Appendix B, Table B.1).

We also construct a price index using expectations about dividends one year ahead for the period where such expectations are available (starting in 2002), using:

$$p_t^{OD} = d_t + \frac{k-r}{1-\alpha} + \ln\left(\frac{\mathbb{E}_t^O DPS_{t+1}}{DPS_t}\right) + \frac{1-\alpha^{10}}{1-\alpha} LTG_t + \frac{\alpha^{10}}{1-\alpha}\ g \qquad (7)$$

Here we assume that expectations of long run dividend growth are also described by LTG. Using the same method as in Equation (6), we set $g = 5.5\%$.[14,15]

We compare our expectations-based indices with a rational (log) price benchmark p_t^{RE} computed following Shiller's (2014) methodology. Starting from the terminal price $p_T^* =$

[13] Specifically, g is the average of the growth rate g_t obtained by solving, at each time t, the equation $p_t = e_t + \frac{\tilde{k}-r}{1-\alpha} + \ln\left(\frac{\mathbb{E}_t^O EPS_{t+1}}{EPS_t}\right) + \sum_{j=1}^{10} \alpha^{j-1}\, \mathbb{E}_t^O \Delta e_{t+j+1} + \frac{\alpha^{10}}{1-\alpha}\, g_t$.

[14] The value of g affects only the level of prices, and thus does not affect our accounting for price fluctuations. The difference in values of g in Equations (6) and (7) is due partly to the adjustment for the average payout ratio, which shapes the first term and constant $\tilde{k}$.

[15] One year ahead dividend forecasts are available from 2002, those for 2 years ahead are more commonly available from 2005. To maximize sample size, we build p_t^{OD} using one year ahead forecasts and inpute longer term forecasts from LTG. In the Appendix we show that two year ahead forecasts yield similar results in the restricted sample.

$\ln\left(\frac{D_T}{r-g}\right)$ at $T = 2019$, the index p_t^{RE} is computed backwards, using the actual dividends over time, and setting $g = 5.81\%$ and $r = 8.48\%$ to reflect sample averages. We obtain:

$$p_t^{RE} = d_t + \sum_{s=t}^{T} a^{s-t}(d_{s+1} - d_s) + \alpha^{T-t} * (p^*_{2019} - d_{2019}) + \sum_{s=t}^{T-1} a^{s-t}(k-r). \quad (8)$$

The analysis proceeds as follows. In Section 4.1 we address Shiller's puzzle by assessing whether our expectations based indices p_t^O and p_t^{OD} display time series volatility comparable to that of market prices. In Section 4.2 we assess the correlation between the beliefs-based indices and the actual market price p_t, in particular whether beliefs-based prices capture variation in the price to earnings and the price to dividend ratios. In Section 4.3 we check the ability of expectations-based indices. We also check the ability of measured expectations themselves to predict future returns. The latter step allows us to separately examine the roles of short and long term expectations and to relax the parametric assumptions needed to construct price indices.

4.1 Excess Volatility Puzzle

To assess whether measured beliefs can account for price volatility, we compare the standard deviation of annual price changes computed using p_t^O, p_t^{OD} and p_t^{RE} to the standard deviation of annual changes in the actual price p_t. Table 1 below reports the results.

Table 1.
Volatility of log price changes

Panel A reports the standard deviation of one-year change in: (1) the log of the price of the S&P500 index, Δp, (2) the rational benchmark index, Δp_t^{RE} (Equation 8), and (3) the price index based on earnings forecasts, Δp_t^O (Equation 6). The sample period ranges from 12/1982 to 12/2018 and has 419 monthly observations. Panel B reports the standard deviation of one-year change in: (1) the log of the price of the S&P500 index, Δp, (2) the rational benchmark index, Δp_t^{RE}, and (3) the price index based on dividend forecasts, Δp_t^O (Equation 7). The sample period ranges from 10/2002 to 12/2018 and has 152 monthly observations.

Panel A: Earnings-based index

	Δp	Δp^{RE}	Δp^{O}
σ	15.2%	0.3%	13.9%
95th Confidence Interval	14.2%-16.3%	0.3%-0.3%	13.0%-14.9%

Panel B: Dividend-based index

	Δp	Δp^{RE}	Δp^{OD}
σ	15.1%	0.2%	17.3%

95th Confidence Interval	13.6%-17.1%	0.2%-0.3%	15.5%-19.5%

The large gap between volatility of actual prices (Column 1) and the volatility of rational prices (Column 2) is Shiller's puzzle. Our expectations-based index p_t^O dramatically improves the prediction for price volatility (Column 3), explaining nearly all of the observed volatility. Panel B shows that results are similar if we use the dividends-based price index p_t^{OD}. The results suggest that measured beliefs are highly volatile, an issue we come back to in Section 5.[16]

In sum, measured beliefs about future fundamentals are sufficiently volatile to account for Shiller's excess volatility puzzle. A distinct question is whether measured beliefs can also help account for the actual trajectory of stock prices. For this to occur under our assumption of constant required returns, it must be that measured beliefs are not only volatile but that they actually track market beliefs. This is what we study next.

4.2 Time Variation in Price Ratios

Figure 1 plots the actual market price p_t (green line) and the expectations-based price index p_t^O (red line) against the rational price p_t^{RE} (blue line).

[16] Campbell and Shiller (1987) offer an alternative approach to excess volatility, which also addresses the non-stationarity of prices (Marsh and Merton 1986). Under plausible assumptions, and appropriate choice of r, dividends and prices are co-integrated, so that $P_t - \frac{D_t}{r}$ is stationary, where $P_t - \frac{D_t}{r} = \frac{1}{r}\sum_{s\geq 0}\left(\frac{1}{1+r}\right)^s \mathbb{E}_t(g_{t+s+1})$. We can then compare the volatility $P_t - \frac{D_t}{r}$ to that of $P_t^O - \frac{D_t}{r}$ where $P_t^O = e^{p_t^O}$. Table B.5, panel A shows that, using $r = 5.5\%$ as in Equation (7), the rational benchmark failst to capture all observed variation in $P_t - \frac{D_t}{r}$ (59%). Incorporating expectations of long-term dividends significantly increases variation (138%). The results of this exercise are shown in Table B.5 panel B. A well-known problem with this method is that the results are extremely sensitive to the r (Campbell and Shiller 1987). For this reason, we prefer the approach of Table 1 (Shiller 2014).

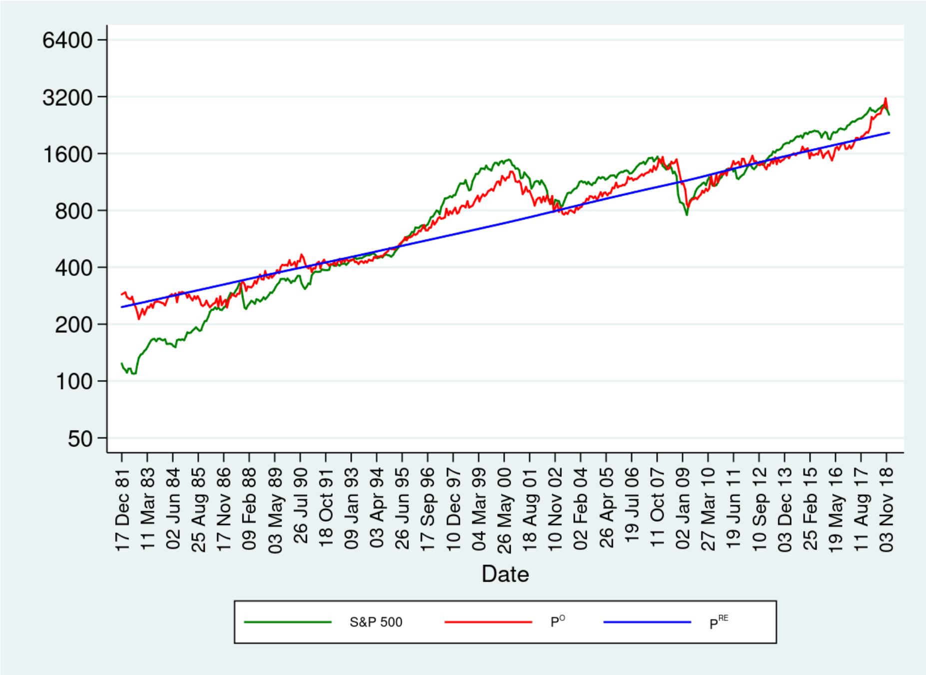

Figure 1. We plot in logscale the levels of the the S&P500 index (green line), the rational benchmark index (p_t^{RE}, blue line, Equation 8), and the price index based on earnings forecasts (p_t^O , red line, Equation 6).

Expectation-based prices are remarkably well aligned with the actual price, especially at low frequencies. Crucially, p_t^O and p_t move in tandem relative to Shiller's rational price p_t^{RE}: when the actual price is above the rational benchmark, so is the expectations-based price; and vice versa when the actual price is below the rational benchmark.[17] This is a strong indication that measured beliefs are a good proxy for market beliefs. Figure B.2 in Appendix B plots the corresponding – and very similar -- results for the dividend-based index p_t^{OD} after 2002.

The fact that the expectations-based price p_t^O closely tracks the market price (Figure 1) and matches its volatility (Table 1) suggests that measured beliefs may also help account for the variation in the price dividend or price earnings ratios, as in Proposition 1b. To check this, we use the price indices of the previous section to build valuation ratios $p_t^O - e_t$, $p_t^{OD} - d_t$ and $p_t^{RE} - e_t$. Following Cochrane (2011), we then regress the synthetic ratios $p_t^O - e_t$ and $p_t^{RE} - e_t$ on the contemporaneous price earnings ratio $p_t - e_t$. Likewise, we regress the dividend ratio indices

[17] The fact that the departures of p_t^O and p_t from p_t^{RE} are persistent suggests that expectations errors are themselves persistent, which is captured in Equation (3) when $\rho > 0$.

$p_t^{OD} - d_t$ and $p_t^{RE} - d_t$ on the contemporaneous price dividend ratio $p_t - d_t$. The OLS coefficients in these regressions quantify the share of variation in $p_t - e_t$ and $p_t - d_t$ accounted for by the relevant index.

We also examine the explanatory power of expectations of future stock returns for the contemporaneous price earnings and price dividend ratio. To do so, we regress measures of future cumulative returns on the current price earnings ratio $p_t - e_t$ and price dividend ratio $p_t - d_t$. This exercise allows us to assess whether discount rate variation or price extrapolation help explain time variation in the ratios. With time varying expected returns, high valuations should be associated with low expected returns. With extrapolative price expectations, high valuations should be associated with high expected returns. Table 2 reports the results.

Table 2.
Price ratio decompositions

In Panel A, the dependent variable are: (1) the difference between the log rational benchmark index p_t^{RE} (Equation 8) and log earnings e_t, (2) the difference between the log of the price index based on earnings forecasts p_t^O(Equation 6) and log earnings, (3) the log of the gross one-year expected return ($\mathbb{E}_t^O[r_{t+1}]$), and (4) the discounted value of future expected discounts based on an AR(1) model for $\mathbb{E}_t^O[r_{t+1}]$ and $\mathbb{E}_t^O[r_{t+10}]$. The dependent variables in the first two columns of Panel B are: (1) the difference between p_t^{RE} and log dividends d_t, and (2) the difference between the log of the price index based on dividend forecasts p_t^{OD} (Equation 7) and d_t. The dependent variables in the last two columns of Panel B are the same as in the corresponding columns of Panel A. The independent variables are the log price-to-earnings ratio in Panel A and the log price-to-dividend ratio in Panel B. Forecasts for earnings and dividends are available monthly while data on expected returns is quarterly. Each regression uses as many observations as possible. In each panel, the last row reports the sample period for each regression. Standard errors are not adjusted for serial correlation. Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.

Panel A: Price earnings ratio

	(1)	(2)	(3)	(4)
	$p_t^{RE} - e_t$	$p_t^{O} - e_t$	$\mathbb{E}_t^O r_{t+1}$	$\sum \mathbb{E}_t^O \alpha^j r_{t+j}$
pe_t	0.5323[a]	0.6169[a]	0.0025	0.0069
	(0.0808)	(0.0528)	(0.0054)	(0.0146)
Observations	437	437	73	73
Adjusted R^2	37%	64%	-1%	-1%
Sample period	1981:12-2018:11	1981:12-2018:11	2010:10-2018:12	2010:10-2018:12

Panel B: Price dividend ratio

	(1)	(2)	(3)	(4)
	$p_t^{RE} - d_t$	$p_t^{OD} - d_t$	$\mathbb{E}_t^O r_{t+1}$	$\sum \mathbb{E}_t^O \alpha^j r_{t+j}$
pd_t	0.0755[c]	0.6454[a]	0.0516[a]	0.1411[a]

	(0.0427)	(0.0703)	(0.0087)	(0.0237)
Observations	173	173	73	73
Adjusted R^2	0%	38%	46%	46%
Sample period	2002:10-2018:12	2002:10-2018:11	2010:10-2018:12	2010:10-2018:12

Column (2) in Panels A and B show that measured expectations of earnings and dividend growth account for a large part of the variation in the price earnings and price dividends ratios. In panel A, $p_t^O - e_t$ captures 62% of the actual price earnings ratio variation. As shown in column (1), the rational index $p_t^{RE} - e_t$ explains a sizable 53% of the variation in $p_t - e_t$, but measured expectations significantly improve upon this. Figure 2 illustrates the remarkable ability of measured beliefs to account for variation in $p_t - e_t$. Panel B shows that measured expectations bring about an even larger improvement with respect to price dividend ratio variation. Here the index $p_t^{OD} - d_t$ explains roughly 65% of price dividend ratio variation, compared to the 8% accounted for by the rational model.

The strong explanatory power of beliefs data raises a concern: analysts may, at least in part, use current stock prices to infer market expectations of earnings growth. If required returns are truly time varying, and analysts ignore that, this inference may contaminate LTG forecasts with variation in required returns. This, in turn, could generate a mechanical correlation between beliefs and prices. In Section 5 we study the determinants of LTG revisions and show that the evidence is not consistent with the mechanical extraction of earnings growth from stock prices.

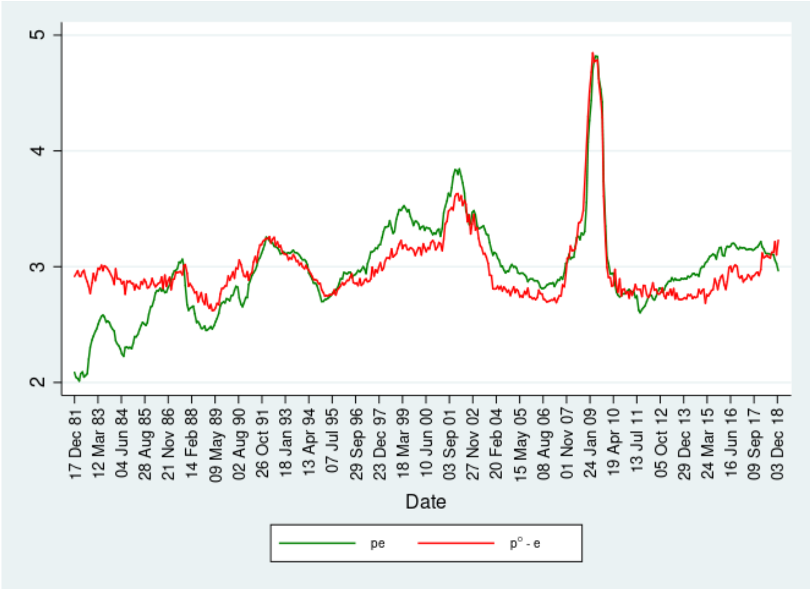

Figure 2. We plot the log price-to-earnings ratio (green line) and $p_t^O - e_t$ (red line), the difference between the index based on earnings forecasts p_t^O (Equation 6) and e_t .

Consider next columns (3) and (4). In Panel A, expectations of returns do not co-vary with the price earnings ratio. In Panel B, they display positive co-movement with the price dividend ratio. This is the opposite of what rational models of time varying required returns predict, and more consistent with price extrapolation. [18] Even in this case, the explanatory power of expectations of returns is much lower than that of expected fundamentals (14% vs 64%).

The ability of our expectations-based indices to track price ratios is consistent with the results of De la O and Myers (2020), who show that short term expectations of earnings and dividend growth help explain a sizable share of variation in price to earnings and price to dividend ratios. A key innovation in our analysis is to use expectations of long term earnings growth, *LTG*. To see why this matters, we compute short-term price indices, denoted p_t^{OS} and p_t^{OSD}, which incorporate one year ahead earnings growth and dividend growth expectations in Equation (6) and assume constant growth after that (see Appendix B). As reported in Table B.3, the index $p_t^{OS} - e_t$ explains roughly 52% of variation in the price earnings ratio, while $p_t^{OSD} - d_t$ explains roughly

[18] Overall, measured expectations of fundamentals and of returns fall short of accounting for 100% of price variation. This could be in part due to measurement error in expectations, and in part to genuine variation in market attitudes toward risk not captured by measures of expectations of returns.

34% of variation in the price dividend ratio, in both cases significantly less than the indices that include expectations about the long term.

It is not just that LTG significantly improves on the explanatory power of short term expectations (by 20% for the price earnings ratio and 90% for the price dividend ratio). More fundamentally, the explanatory power of short-term expectations is fairly close to that of the rational model, which suggests that LTG may be the main driver of pricing anomalies. In the following, we flesh out this link: in Section 4.3 we show that LTG predicts returns, and in Section 5 we show how errors in long-term expectations are predictable and can account for the pricing puzzles, in line the overreaction mechanism of Proposition 2.

4.3. Predictability of Returns

A well-known implication of the Campbell-Shiller decomposition is that any stock price variation unaccounted for by future fundamentals should predict future returns. The fact that measured expectations account for price variation is however only a necessary, but not sufficient, condition for them to predict returns. In fact, measured beliefs may explain little price variation beyond future fundamentals if they are close to rational. And even if measured beliefs depart from rationality, return predictability could mostly come from time varying discount rates, leaving little scope for beliefs about future fundamentals to predict future returns.

To evaluate whether measured expectations account for price anomalies, it is therefore crucial to assess their ability to predict returns. We do this in two ways. First, we regress cumulative stock returns over 1 to 5 years on the expectations-based price indices $p_t^O - e_t$ and $p_t^{OD} - d_t$. Second, we directly regress cumulative stock returns on expectations of short and long term earnings growth. This exercise allows us to see which measures of expectations have more explanatory power and is also robust to the parametric assumptions used to compute price indices. Table 3 reports the estimation results (here we use raw returns but the results are similar if we use excess returns, see Table D.1 Appendix D).

Table 3.
Return Predictability

The dependent variable is the log return between year t and $t+1$ in column 1 and the discounted value of the cumulative return between year t and $t+3$ and $t+5$ in columns 2 and 3. The independent variables are: (1) the difference between the price index based on earnings forecasts p_t^O (Equation 6) and log earnings e_t, (2) the difference between the price index based on dividend forecasts p_t^{OD} (Equation 7) and dividends d_t, (3) the forecast for earnings growth in the long run LTG_t, (4) the time t forecast for one-year growth in earnings in year $t+1$, $\mathbb{E}_t^O[e_{t+1}-e_t]$, (5) the time-*t* forecast for one-year for growth in earnings in year $t+2$, $\mathbb{E}_t^O[e_{t+2}-e_{t+1}]$. The sample period is 1981:1-2013:12 except in Panel B where it is 2002:10-2013:12. We adjust standard errors for serial correlation using the Newey-West correction (number of lags ranges from 12 in the first column to 60 in the last one). Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.

	(1)	(2)	(3)
	r_{t+1}	$\sum_{j=1}^{3}\alpha^{j-1}r_{t+j}$	$\sum_{j=1}^{5}\alpha^{j-1}r_{t+j}$
Panel A: Returns and $p^O - e$			
$p_t^O - e_t$	-0.0638	-0.0495	0.0766
	(0.0848)	(0.1491)	(0.1649)
Adjusted R^2	1%	0%	0%
Observations	378	378	378
Panel B: Returns and $p^{OD} - d$			
$p_t^{OD} - d_t$	-0.4344[c]	-0.8931[b]	-1.7730[a]
	(0.2195)	(0.3503)	(0.2544)
Adjusted R^2	9%	16%	52%
Observations	114	114	114
Panel C: Returns and LTG			
LTG_t	-3.3411[a]	-8.7214[a]	-10.8570[a]
	(1.1390)	(1.9357)	(2.1247)
Adjusted R^2	11%	25%	26%
Observations	385	385	385
Panel D: Returns and Short Term Earnings Growth I			
$\mathbb{E}_t^O[e_{t+1}-e_t]$	-0.0273	0.0545	0.2080[b]
	(0.0731)	(0.0878)	(0.0842)
Adjusted R^2	0%	0%	3%
Observations	378	378	378
Panel E: Returns and Short Term Earnings Growth II			
$\mathbb{E}_t^O[e_{t+2}-e_{t+1}]$	-0.4077	0.5269	2.8528
	(0.6096)	(1.7635)	(2.1259)
Adjusted R^2	0%	0%	6%
Observations	378	378	378

Panel A shows that the expectations-based price earnings ratio $p_t^O - e_t$ does not predict returns in our sample. In a sense, this result is expected: it is well known that in the sample period considered, the actual price earnings ratio $p_t - e_t$ does not predict future returns (Table D.1 Appendix D). Moreover, as we saw in Table 2, the explanatory power of the expectations-based $p_t^O - e_t$ is not much higher than that of the rational $p_t^{RE} - e_t$, which also reduces the scope for return predictability.

Panel B shows that the expectations-based price dividend ratio $p_t^{OD} - d_t$ negatively predicts realized returns, especially at long horizons. A high $p_t^{OD} - d_t$ today predicts disappointing returns in the future, suggesting that overly optimistic expectations about future growth may indeed cause overpricing of stocks that subsequently reverses. The expectations based price dividend ratio account for a sizable 52% of return variation over the next five years.

The more interesting results are in Panels C, D and E, where we predict returns using our measures of expectations. Panel C shows that high current expectations of long term earnings growth strongly predict low future returns. LTG can account for 26% of variation in realized returns over the next five years, or roughly two thirds of the return variation accounted for by $p_t^O - d_t$ at the same horizon.[19] Panels D and E confirm an important finding from the analysis of price ratios variation in Section 4.2: expectations of short term earnings do not improve much over the rational model, and hence are unable to significantly predict returns.[20]

In sum, this Section yields two messages. First, measured expectations of fundamentals offer a parsimonious account of the three leading stock market puzzles. Second, expectations of long-term growth play a key role in accounting for the pricing anomalies, not only because they

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[19] The predictive power of current long-term growth forecasts for subsequent returns comes from the persistent movements of the LTG time series. A decomposition of LTG into components with different persistence levels, following Ortu, Tamoni and Tebaldi (2013), reveals that shocks to LTG that have a half-life of 5 quarters or more predict returns at all horizons, while higher frequency oscillations do not predict returns.

[20] It is well known that the OLS estimator in predictive regressions using lagged stochastic regressors, such as *LTG*, may be biased (Stambaugh 1999). The bias arises since the disturbances in the regression for returns may be correlated with future values of *LTG*. To address this issue, we follow the methodology of Kothari and Shanken (1997). Specifically, we use simulations to compute the coefficient that we would estimate under the null hypothesis of no predictability and bootstrap a p-value for the OLS value in Table 3. We find that, under the null hypothesis that the true LTG coefficient is zero, the predicted values of the LTG coefficients are: -0.33 in column 1, -0.90 in column 2, and -0.97 in column 3. The p-values for the LTG coefficients in Table 3 under the null hypothesis of no predictability are: 3.20% in column 1, 3.16% in column 2, and 1.80% in column 3.

significantly improve the explanatory power for the variation in valuations, but also because they account for the predictability of future returns. We next ask why this is the case.

5. Expectation Formation and Predictability of Returns

To analyze the mechanism behind stock market anomalies, we ask two sets of questions.[21] In Sections 5.1 and 5.2 we ask what drives changes in expectations and whether they are rational. In Section 5.3 we investigate the mechanism of price anomalies by testing the predictions of Proposition 2, which connect the overreaction of forecasts to future returns. Table 3 shows that only LTG can account for return predictability, but for completeness we also analyze expectations of short term earnings growth.

5.1 Expectation Formation

We begin with the determinants of changes in expectations. We consider three leading candidates: i) fundamental news, ii) past stock returns, and iii) expected future stock returns. Assessing the role of fundamental news is informative about the rationality of the reaction of beliefs to news, captured by ϵ_t in our model, in line with Equation (4). Assessing the role of past stock returns and of expectations of returns addresses the concern that analysts' stated expectations of fundamentals may in part be contaminated by time varying expected returns. If this were the case, the predictive power of LTG would reflect time varying expected returns, and not the impact of beliefs about future payouts on prices, which is our main focus here.

Specifically, controlling for past returns helps evaluate the possibility that analysts mechanically infer their expectations of long term fundamentals, LTG, by looking at the actual stock price. When analysts see the stock price go up, they might decide to upgrade their forecasts of long term earnings growth in a way that justifies the price increase. As we argued in Section

[21] We focus on earnings expectations since revisions of dividend expectations are only available for short term forecasts and only since 2005.

4.2, this implies that LTG might be contaminated by time varying discount rates.[22] Considering expectations of future stock returns is another way to control for discount rate variation, potentially also due to price extrapolation. In Appendix D, we also consider the role of several other proxies for time varying discount rates.

We measure fundamental news using earnings surprises relative to cyclically adjusted earnings, $e_t - cae_{t-5}$. We measure past returns using the cumulative stock return in the past year, and we measure expectations of returns using the contemporaneous one-year ahead expected return from the Graham and Harvey survey (see Section 3). Table 4 reports the regressions of forecast revisions on fundamental news. We also regress forecast revisions on lagged returns and expectations of returns.

Table 4.
Predicting Changes in Growth Expectations

The dependent variables in the first two columns are revisions in the following growth forecasts: (1) $\Delta\mathbb{E}_t^O[e_{t+1} - e_t] = \left(\mathbb{E}_t^O - \mathbb{E}_{t-1}^O\right)[e_{t+1} - e_t]$, the revision between year $t-1$ and year t in the forecast for the one-year earnings growth rate in year $t+1$, and (2) $\Delta\mathbb{E}_t^O[e_{t+2} - e_{t+1}] = \left(\mathbb{E}_t^O - \mathbb{E}_{t-1}^O\right)[e_{t+2} - e_{t+1}]$, the revision between year $t-1$ and year t in the forecast for the one-year earnings growth rate in year $t+2$. Finally, $\Delta LTG_t = LTG_t - LTG_{t-1}$ is the change between year $t-1$ and t in the forecast for earnings growth in the long run. The independent variables are: (1) the log of earnings in year *t* relative to the cyclically-adjusted log earnings in year $t-5$, $e_t - cae_{t-5}$, (2) the S&P500 return between years $t-1$ and t, $r_{t-1,t}$, and (3) the change between $t-1$ and t in the forecast for the one-year return on the S&P500, $\Delta\mathbb{E}_t^O r_{t+1}$. Columns 1, 3 and 5 use monthly data for 1982:12-2018:11 while Columns 2, 4 and 6 use quarterly data for 2001:12-2018:12. Newey-West standard errors are shown in parentheses (with 12 lags). Note: [a] significant at the 1% level and [b] significant at the 5% level.

	(1)	(2)	(3)	(4)	(5)	(6)
	$\Delta\mathbb{E}_t^O[e_{t+1} - e_t]$	$\Delta\mathbb{E}_t^O[e_{t+1} - e_t]$	$\Delta\mathbb{E}_t^O[e_{t+2} - e_{t+1}]$	$\Delta\mathbb{E}_t^O[e_{t+2} - e_{t+1}]$	ΔLTG_t	ΔLTG_t
$e_t - cae_{t-5}$	-0.6025[a]	-0.6861[a]	-0.6119[a]	-0.5255[a]	0.5412[a]	0.4179[b]
	(0.1014)	(0.1282)	(0.0738)	(0.1487)	(0.1010)	(0.1619)
$r_{t-1,t}$		-0.0294		-0.2186		0.1296
		(0.2488)		(0.2332)		(0.1691)
$\Delta\mathbb{E}_t^O r_{t+1}$		0.1480		0.0179		-0.0534
		(0.0923)		(0.0814)		(0.1746)
Obs	419	63	419	63	433	67

[22] Inferring market expectations from prices is not a problem if analysts know the time varying discount rates. In this case, even if they infer market beliefs about future earnings from prices, they would do so correctly. Problems arise if analysts erroneously assume that the rate of return is constant. In this case, the inferred expected growth is directly pinned down by the dividend price ratio $p_t - d_t$, which is in turn contaminated by discount rate variation.

Adjusted R^2	36%	48%	37%	64%	29%	28%

Short and long run earnings growth forecasts behave in sharply different ways: after good fundamental shocks, beliefs about one or two years ahead earnings growth are revised downward (columns 1 through 4). In contrast, beliefs about long term earnings growth are revised upward (columns 5 and 6). This difference may seem puzzling, but is in fact consistent with the data generating process for earnings growth. This process displays short term reversals followed by a long run recovery. To see this, Table 5 reports aggregate earnings growth in the years after periods of earnings booms (top 30% of surprise $e_t - cae_{t-5}$) and after periods of earnings busts (bottom 30% of surprises).

Table 5.
Short Term Reversals in Earnings Growth

In December of each year *t* between 1981 and 2014, we construct deciles based on the log of earnings in year *t* relative to the cyclically-adjusted log earnings in year $t-5$, $e_t - cae_{t-5}$, and report the average one-year growth rate of earnings in $t+1$, $t+2$, …, $t+5$ for observations in the top 30% and bottom 30% of $e_t - cae_{t-5}$.

		$e_{t+1} - e_t$	$e_{t+2} - e_{t+1}$	$e_{t+3} - e_{t+2}$	$e_{t+4} - e_{t+3}$	$e_{t+5} - e_{t+4}$
$e_t - cae_{t-5}$	Low	0.30	0.22	0.13	0.06	0.01
	High	-0.23	-0.05	0.03	0.05	0.29
	High-Low	-0.53	-0.27	-0.10	-0.01	0.28

As Table 5 shows, large positive earnings surprises are associated with sharp growth reductions in the short run and progressive recovery in the long run, and vice-versa for large negative earnings surprises. In terms of our model, the earnings data generating process displays short term reversals, $\eta_1, \eta_2 < 0$, and a gradual convergence to the long run mean or even overshooting, $\eta_3, \eta_4 \ldots \geq 0$.[23] This explains why, after rapid earnings growth, analysts revise their expectations of short term growth down (columns 1 and 3 of Table 4), and those of long term growth up (column 5 of Table 4). Growth expectations are thus revised in a forward looking way, in line with Equation (4).

[23] We confirm this finding by estimating the MA model of Section 2 in Table C.1, Appendix C.

Column (6) in Table 4 indicates that LTG is not contaminated by discount rate variation. Revisions in LTG are explained by fundamental news, but are not predicted either by the market return over the past year, nor by the expectations of future returns. The coefficient of past fundamentals is stable across specifications (5) and (6), reinforcing the idea that the co-movement between LTG revisions and past fundamentals has little to do with past and expected returns. The same applies if we consider cumulative returns over a longer period, say $t-5$ to t (Table C.2 in Appendix C), which allows for inference from prices over longer time scales. Overall, this evidence is inconsistent with the possibility that analysts mechanically infer LTG from realized stock prices or that LTG is contaminated by expectations of future stock returns. The same applies for expectations of short term growth (columns 2 and 4 in Table 4).

One may argue that other kinds of discount rate variation, captured neither by past stock returns nor by expectations of returns, contaminate LTG revisions. While direct measures of market risk aversion over time are not available, research has proposed a variety of theory-based measures of time varying risk premia. As a robustness exercise, we consider whether such measures can account for changes in LTG (Table C.3 in Appendix C). We try the surplus consumption ratio (Campbell and Cochrane 1999), the consumption-wealth ratio (cay, Lettau and Ludvigson 2001), the term spread defined as the log difference between the gross yield of 10-year and 1-year US government bonds from the St. Louis Fed, the credit spread defined as the log difference between the gross yield of BAA and AAA bonds from the St. Louis Fed, the high yield share measure of credit market sentiment (Greenwood and Hanson 2013), the price of volatile stocks defined as the book-to-market ratio of low-volatility stocks minus that of high-volatility stocks (Plueger, Siriwardane and Sunderam 2020), the Kelly and Pruitt (2013) optimal forecast of aggregate equity market returns, the economic policy uncertainty index in Baker et al. (2016), and the Miranda-Aggripino et al. (2020) global factor. The evidence is strong: no proxy for risk premia significantly changes the association between LTG revisions and fundamentals news, and very few of these proxies contribute to revisions of LTG.

Finally, to the extent that revisions in LTG are associated to changes in required returns, they should predict changes in realized returns. We assess return predictability in Section 5.3, and Table 7 shows that only the component of LTG revisions that is predicted from earnings surprises (as in Table 4 column 5) helps predict subsequent returns.

Altogether, the results suggest that growth expectations are revised in a forward looking way, with considerable weight put on recent earnings growth. After accounting for fundamentals, these revisions are not correlated with available measures of time varying discount rates. Revisions in LTG thus appear to be a reliable measure of changes in expectations of future fundamentals.[24] We next assess whether these revisions depart from rationality.

5.2 Are Expectations Rational?

Following Coibion and Gorodnichenko (CG 2015), we assess whether analyst forecast revisions predict subsequent forecast errors, defined as realized minus expected growth. A positive regression coefficient indicates underreaction: an insufficient positive revision after good news entails a forecast systematically below the realization (a positive forecast error). A negative coefficient implies overreaction: an excessive positive revision after good news predicts a forecast systematically above the realization (the forecast error is negative).[25]

Table 6 presents regressions of forecast errors on forecast revisions for earnings growth at short (one and two years ahead) and long horizons. We compute realized long term growth as the average yearly earnings growth in the next 3, 4, and 5 years following the definition of LTG.

Table 6.
Forecast Errors and CG Revisions

The dependent variables are the errors in forecasting the annual earnings growth between: year t and $t+1$ in column 1, year $t+1$ and $t+2$ in column 2, and year t and $t+h$ in columns $h=3,4,5$. Forecast errors beyond year $t+2$

[24] This conclusion is also consistent with existing work showing that analyst beliefs about future earnings growth are a much better predictor of aggregate investment than measures of firm-level q based on market prices (Cummins, Hasset, Oliner 2006).

[25] Here we consider consensus (i.e. median) beliefs. A positive consensus coefficient is compatible with rationality of individual forecasts when forecasters' information is noisy (CG 2015). Instead, a negative consensus coefficient is unambiguously indicative of overreaction (BGLS 2019).

are defined relative to the forecast for long run earnings growth, LTG. $\Delta\mathbb{E}_t^O[e_{t+1} - e_t]$ is the revision between year t-1 and year t in the forecast for the one-year earnings growth in year $t+1$. $\Delta\mathbb{E}_t^O[e_{t+2} - e_{t+1}]$ is the revision between year $t-1$ and year t in the forecast for the one-year earnings growth in year $t+2$. Finally, ΔLTG_t is the change in LTG between year $t-1$ and t. We use monthly expectations data starting on December of 1982 (the first period with ΔLTG_t) and data on realized earnings through December of 2018. Newey-West standard errors are reported in parentheses (the number of lags ranges from 12 in the first column to 60 in the last column). Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, and [c] significant at the 10% level.

	$(e_{t+1} - e_t) - \mathbb{E}_t^O[e_{t+1} - e_t]$	$(e_{t+2} - e_{t+1}) - \mathbb{E}_t^O[e_{t+2} - e_{t+1}]$	$\frac{(e_{t+3} - e_t)}{3} - LTG_t$	$\frac{(e_{t+4} - e_t)}{4} - LTG_t$	$\frac{(e_{t+5} - e_t)}{5} - LTG_t$
$\Delta\mathbb{E}_t^O[e_{t+1} - e_t]$	0.0169				
	(0.1453)				
$\Delta\mathbb{E}_t^O[e_{t+2} - e_{t+1}]$		3.2971[c]			
		(1.9961)			
ΔLTG_t			-10.0734[a]	-6.8184[a]	-5.1003[a]
			(2.6286)	(1.8928)	(1.8928)
Observations	408	396	397	385	373
Adjusted R^2	0%	3%	25%	21%	19%

In the case of one or two year ahead expectations, the regression coefficients are positive, though not statistically significant. If anything, revisions at short term horizons are insufficient rather than excessive, consistent with underreaction. At longer horizons, in contrast, there is strong evidence of overreaction: upward revisions of LTG predict future disappointment, while downward revisions predict positive surprises. Figure 3 illustrates these results. Table 6 confirms, at the level of the S&P index, the overreaction of firm-level LTG forecasts originally documented by BGLS (2019).[26] Overreaction of long term growth forecasts is a robust funding, which arises also at the level of individual firms, shaping the cross section of their stock returns.

[26] The fact that some coefficients in Table 6 have magnitudes above one reflects the fact that movements in LTG are on average followed by movements in growth rates in the opposite direction. This is a non-linear phenomenon concentrated in cases of strong recoveries after poor performance and drops in LTG.

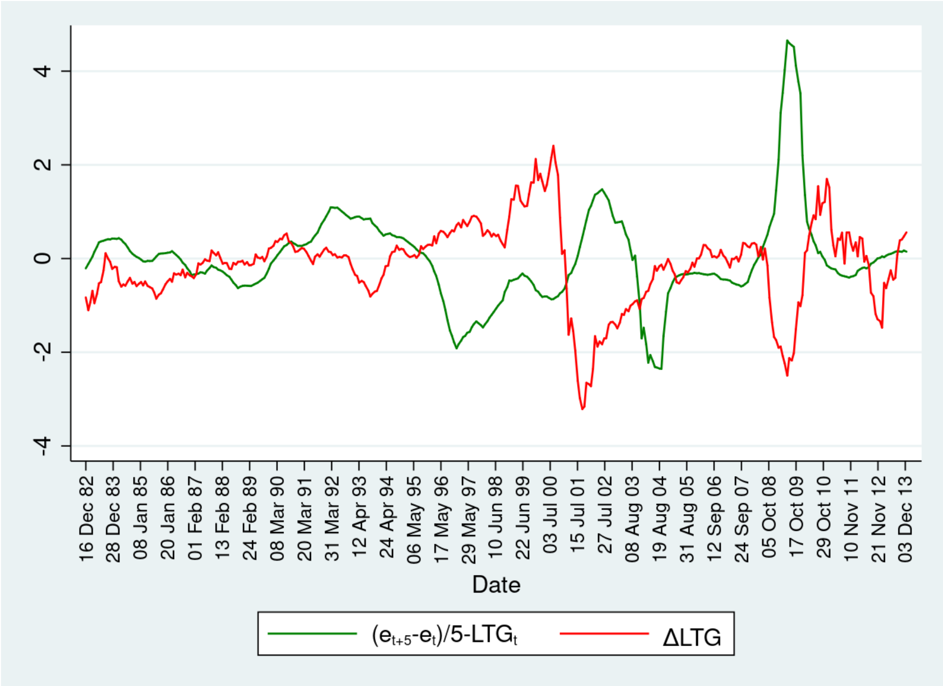

Figure 3. We plot the 5-year forecast error (green line) and the one-year change in expectations of long term earnings growth (red line). Both variables are normalized to have mean zero and standard deviation equal to 1.

The differential predictability of short and long term forecast errors is consistent with our model. If beliefs are described by Equation (4), the coefficient of error predictability estimated for earnings growth at horizon $s+1$, denoted $\beta_{CG,s+1}$, is proportional to (see Appendix A):

$$\beta_{CG,s+1} \propto -[(1+\rho)\eta_{s+1}\sigma_{OF} + \sigma_O^2]. \tag{9}$$

If beliefs overreact to fundamentals, namely $\sigma_{OF} > 0$, equation (9) can account for the evidence in Table 6. Short term reversals in earnings growth can generate a positive or near zero coefficient for one and two-year ahead forecasts provided there is sufficient short term reversal, namely $\eta_1, \eta_2 < -\frac{\sigma_O^2}{(1+\rho)\sigma_{OF}}$. Long term growth recovery ($\eta_3, \eta_4 \ldots \geq 0$) generates a negative predictability coefficient for LTG.

Intuitively, after good news analysts become excessively optimistic at all horizons, due to $\sigma_{OF} > 0$. This implies that they do not fully account for short term reversals, so short term beliefs underreact, while they exaggerate long term recovery, so long term beliefs overreact.

The analysis of measured expectations leads to two key findings. First, measured expectations of future fundamentals react in a forward looking way to fundamental news, in line with Equation (4), and do not depend on measures of time varying risk premia, suggesting that they reliably measure beliefs about future payouts. Second, short term expectations are near rational while long term expectations overreact to news, i.e. $\sigma_{OF} > 0$. That is, long term expectations become too optimistic after strong growth. We now show empirically that overreacting expectations are crucial for thinking about price anomalies.

5.3. LTG Revisions and Predictability of Returns

To analyze the link between overreacting expectations and return predictability along the lines of Proposition 2, we implement a two stage estimation strategy. In the first stage we regress forecast revisions on fundamental news. This stage, shown in Table 4, allows us to isolate upward forecast revisions due to good fundamental news and downward revisions due to bad news. In the second stage, we regress future stock returns on the forecast revisions predicted from the first stage. In line with Proposition 2, if fundamental-driven upward forecast revisions predict low future returns, it must be due to belief overreaction, $\sigma_{OF} > 0$: good fundamental news lead to excessively optimistic beliefs about long term growth, which lead to inflated stock prices but also systematic disappointment and low returns. The reverse occurs after bad fundamental news, when LTG is revised downward excessively. Table 7 reports the results of the second stage regression.

Table 7.
Predicted Forecast Revisions and Returns

We report second-stage results for IV regressions using two- and five-year stock returns as the dependent variable. The independent variables are the instrumented values of: (1) revision between year $t-1$ and year t in the forecast for the one-year earnings growth rate in year $t+1$, $\Delta\mathbb{E}_t^O[e_{t+1} - e_t]$, (2) the revision between year $t-1$ and year t in the forecast for the one-year earnings growth rate in year $t+2$, $\Delta\mathbb{E}_t^O[e_{t+2} - e_{t+1}]$, (3) the change in the long-term growth forecast between year $t-1$ and t, ΔLTG_t. The instrument is the log of earnings in year t relative to the cyclically-adjusted earnings in year $t-5$, $e_t - cae_{t-5}$. See Table 6 for first-stage estimates. Independent variable (4) is the residual change in the long-term growth forecast. All variables are normalized to have mean zero and standard deviation equal to 1. We adjust standard errors for serial correlation using the Newey-West correction (with 24 lags in the first two columns and 60 in the last one). Superscripts: [a] significant at the 1% level.

	Dep Variable: Log return between years t and:			
	$t+2$	$t+2$	$t+5$	$t+5$
$\mathbb{E}\left[\Delta\mathbb{E}_t^O[e_{t+1} - e_t] \middle\vert e_t - cae_{t-5}\right]$	0.4201 (0.2735)			

$\mathbb{E}\left[\Delta\mathbb{E}_t^O[e_{t+2} - e_{t+1}]\middle\vert e_t - cae_{t-5}\right]$		0.4321		
		(0.2753)		
$\mathbb{E}[\Delta LTG_t \vert e_t - cae_{t-5}]$			-0.9073[a]	
			(0.2670)	
$\Delta LTG_t - \mathbb{E}[\Delta LTG_t \vert e_t - cae_{t-5}]$				0.1095
				(0.2878)
Obs	400	400	373	373
Modified F-stat	53.95	135.58	42.01	n.a.
AR Confidence Interval	[-.05, 1.08]	[-.06, 1.05]	[-1.60,-.39]	n.a.
Reduced form Adj R^2	6%	6%	23%	23%

Revisions of short run expectations predicted from fundamental news do not predict future returns (Columns 1 and 2), consistent with the finding of Table 6 that such revisions do not help predict forecast errors to begin with.

In contrast, Column 3 shows that predicted revisions of long run earnings growth account for a significant share of return predictability. An increase in the predicted value of ΔLTG_t by one standard deviation entails a reduction in 5-year log returns of 0.32 (=0.9037 x 0.35; the standard deviation of 5-year log returns is 0.35). Since the average yearly log return is 8.1%, this corresponds to losing roughly 47 months' worth of returns over the five years. Finally, Column 4 shows that the component of LTG revisions not predicted by fundamentals does not predict returns. Overreacting expectations of long-term growth thus take the center stage in explaining stock market puzzles.[27]

As a final exercise, we trace the expectations-based mechanism in greater detail by linking overreaction to fundamentals to predictable forecast errors, and then predictable errors to future returns. To this end, we perform a three-stage decomposition, where the predicted forecast revisions in Table 4 (first stage) are used to predict forecast errors (second stage). We then use

[27] In part, non-fundamental LTG revisions may reflect the sluggishness inherent in the *consensus* LTG when analysts receive noisy information (Bordalo, Gennaioli, Ma, and Shleifer 2020), as well as information already reflected in prices. More broadly, this result suggests that strong fundamentals may play a significant role in propelling excess optimism. Of course, this result is fully consistent with softer information also helping create over-reaction as in Daniel Titman (2006).

predicted forecast errors to predict future realized returns (third stage). Table 8 reports the estimation results for stages two and three.

Table 8.
Predicted Forecast Errors and Returns

The dependent variables in Panel A are the errors in predicting the values of the: (1) growth rate in earnings in year $t+1$, $(e_{t+1}-e_t)-\mathbb{E}_t^O[e_{t+1}-e_t]$, (2) growth rate in earnings between year $t+1$ and $t+2$, $(e_{t+2}-e_{t+1})-\mathbb{E}_t^O[e_{t+2}-e_{t+1}]$ in column 2, and (3) errors in forecasting the annual earnings growth between year t and $t+h$ in columns $h=3,4{,}5$. Forecast errors beyond year $t+2$ are defined relative to LTG_t. All the independent variables are based on the revisions in earnings forecasts from the corresponding regression in Table 4. The dependent variables in panel B are two- and five-year stock market returns. The independent variables are based on the predicted forecasts errors from the corresponding regression in Panel A. We adjust standard errors for serial correlation using the Newey-West correction (with 24 lags in the first two columns and 60 in the remaining ones). Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, and [c] significant at the 10% level.

Panel A: IV Regressions for forecast errors

	(1)	(2)	(3)	(4)	(5)
	$(e_{t+1}-e_t)-\mathbb{E}_t^O[e_{t+1}-e_t]$	$(e_{t+2}-e_{t+1})-\mathbb{E}_t^O[e_{t+2}-e_{t+1}]$	$\frac{(e_{t+3}-e_t)}{3}-LTG_t$	$\frac{(e_{t+4}-e_t)}{4}-LTG_t$	$\frac{(e_{t+5}-e_t)}{5}-LTG_t$
$\mathbb{E}[\Delta\mathbb{E}_t^O[e_{t+1}-e_t] \mid e_t-cae_{t-5}]$	0.2060				
	(0.1547)				
$\mathbb{E}[\Delta\mathbb{E}_t^O[e_{t+2}-e_{t+1}] \mid e_t-cae_{t-5}]$		0.5095			
		(0.3114)			
$\mathbb{E}[\Delta LTG_t \mid e_t-cae_{t-5}]$			-1.5311[a]	-1.4731[a]	-1.4798[a]
			(0.3251)	(0.3079)	(0.3355)
Obs	408	396	397	385	373
Modified F-stat	71.546	201.514	64.168	54.579	37.326
AR Confidence Interval	[-.07, .54]	[0.00,1.25]	[-2.68,-1.11]	[-2.61,-1.07]	[...,-1.04]
Reduced form Adj R^2	2%	11%	53%	55%	59%

Panel B: IV Regressions for returns

	Dependent Variable: Log return between years t and:				
	$t+2$	$t+2$	$t+5$	$t+5$	$t+5$
$\mathbb{E}\left[(e_{t+1}-e_t)-\mathbb{E}_t^O[e_{t+1}-e_t]\right]$	1.8966[c]				
	(1.1300)				
$\mathbb{E}\left[(e_{t+2}-e_{t+1})-\mathbb{E}_t^O[e_{t+2}-e_{t+1}]\right]$		0.8525[b]			
		(0.3960)			
$\mathbb{E}\left[\frac{(e_{t+3}-e_t)}{3}-LTG_t\right]$			0.6131[a]		
			(0.2044)		
$\mathbb{E}\left[\frac{(e_{t+4}-e_t)}{4}-LTG_t\right]$				0.6074[a]	
				(0.1752)	
$\mathbb{E}\left[\frac{(e_{t+5}-e_t)}{5}-LTG_t\right]$					0.6131[a]

					(0.2044)
Obs	396	396	373	373	373
Modified F-stat	7.42	12.86	64.29	61.14	65.28
AR Confidence Interval	[entire grid]	[-.04,...]	[0.22, 1.01]	[0.22, 0.92]	[0.22, 0.92]
Reduced form Adj R^2	6%	6%	23%	23%	23%

Because short term beliefs display at most weak underreaction to fundamentals, the entailed forecast errors do not account for future returns (Columns 1 and 2). In contrast, there is a positive and significant association between predicted long-term forecast errors and subsequent returns (columns 3, 4 and 5), further validating the overreaction mechanism. When excessive upward LTG revisions are followed by negative forecast errors, returns are low; when excessive downward revisions are followed by positive forecast errors, returns are high. According to Table 8, this mechanism explains nearly all the predictability of 5 year ahead returns from LTG (23% vs 26% in Panel C of Table 3). The same set of overreacting long-term growth forecasts that accounts for cross sectional return anomalies (BGLS 2019) also accounts for the time series predictability of aggregate stock returns.

The results in this section close the loop of the argument laid out in the Introduction: expectations of long-term growth overreact, particularly to fundamental shocks, and the corresponding forecast errors predict returns. Overreacting beliefs about long term growth can explain the puzzling patterns of stock market volatility, valuations, and returns.

6. Conclusion

We showed that measured expectations of fundamentals help explain in a parsimonious way the leading stock market puzzles even with constant discount rates, and without price extrapolation. The analysis has two key takeaways. First, the non-rationality of expectations of long-term fundamentals is central for thinking about pricing anomalies. Not only do these expectations help account for excess price volatility and time variation in valuations, but they exhibit systematic errors that help predict future returns. Second, the mechanism for pricing anomalies that emerges from the analysis is one in which good fundamental news cause investors to become too optimistic about long term fundamentals. This inflates stock prices, and leads to systematically low returns as high expectations are disappointed. The exact same mechanism has been documented at the firm level, accounting for large and systematic differentials in returns between firms with high vs low growth expectations (BGLS 2019).

A skeptic may question that measured long term expectations surreptitiously embody variation in discount rates. We consider this possibility, but do not find support for it. In particular, beliefs about long term growth are mostly driven by earnings news; they do not mechanically follow the dynamics of stock prices and are not explained by conventional measures of time varying risk premia such as the surplus consumption ratio, the consumption-wealth ratio and several others. In addition, it is the component of forecast revisions correlated with fundamental news that negatively predicts returns. These results further strengthen our belief-based interpretation of the evidence and confirm the usefulness of beliefs data for advancing our understanding of asset prices. In future work, it will be important to try to measure directly time variation in risk aversion so as to compare the explanatory power of different mechanisms.

Our analysis also raises some foundational questions. First, what is the psychology of long term beliefs, accounting for the overreaction to fundamental news ($\sigma_{OF} > 0$)? This feature is consistent with the diagnostic expectations model proposed by BGLS (2019) in their study of the cross section of returns. In that model, a firm's strong earnings growth causes analysts to drastically revise up the probability that it is a "Google", which entails excess optimism about earnings growth at all horizons. In future work it would be interesting to connect cross-sectional and aggregate price anomalies starting from this basic formulation of overreacting beliefs.

A second challenge raised by our analysis is that beliefs display persistence, in that they take some time to overreact. Diagnostic expectations exhibit this feature in the presence of sluggish updating, due for instance to limited attention or real rigidities, or when different forecasters have dispersed information as in Bordalo, Gennaioli, Ma and Shleifer (2020a) and

Bordalo, Gennaioli, Kwon, and Shleifer (2020c). Another way to think about rigidity is to view it as a consequence of imperfect memory (Bordalo et al 2020b, Azeredo da Silveira, Woodford and Sung, 2020, Afrouzi et al 2020). These approaches offer promising avenues to develop realistic yet manageable models of beliefs that can help asset pricing research make progress.

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Appendix

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Appendix A. Proofs.

Proof of Proposition 1. The MA representation of the data generating process implies that:

$$g_{t+s+1} = \sum_{j\geq 0} \eta_j \epsilon_{t+s+1-j}.$$

which in turn implies:

$$\mathbb{E}_t(g_{t+s+1}) = \sum_{j\geq s+1} \eta_j \epsilon_{t+s+1-j} = \sum_{j\geq 0} \eta_{j+s+1} \epsilon_{t-j}.$$

Likewise, the expectations shock admits a moving average representation:

$$\epsilon_{M,t} = \sum_{j\geq 0} \rho^j u_{M,t-j},$$

Using the more general formulation of expectations errors:

$$\mathbb{E}_t^M(g_{t+s+1}) = \mathbb{E}_t(g_{t+s+1}) + \delta_{s+1}\epsilon_{M,t}$$

allowing differential impact $\delta_{s+1}\epsilon_{M,t}$ at different horizons s, we obtain:

$$\mathbb{E}_t^M(g_{t+s+1}) = \sum_{j\geq 0} \eta_{j+s+1}\epsilon_{t-j} + \rho^j \delta_{s+1} u_{M,t-j}.$$

Expectations have two components: a rational part which responds to current and past shocks ϵ_{t-j}, with a propagation coefficient η_{j+s+1}, and a distortion part which responds to current and past expectational shocks $u_{M,t-j}$ with propagation coefficient $\rho^j \delta_{s+1}$. This means that an expectational shock has an initial term structure given by δ_{s+1} which is persistent over time t but decays at a rate ρ.

By Equation (5), then, the log stock price at time t is equal to:

$$p_t^M = d_t + \frac{k-r}{1-a} + \sum_{s\geq 0}\sum_{j\geq s+1} a^s \eta_j \epsilon_{t+s+1-j} + \sum_{s\geq 0}\sum_{j\geq 0} a^s \delta_{s+1}\rho^j u_{M,t-j},$$

which can be written as:

$$p_t^M = d_t + \frac{k-r}{1-a} + \sum_{j\geq 0} \epsilon_{t-j} \sum_{s\geq 0} a^s \eta_{j+s+1} + \sum_{j\geq 0} \rho^j u_{M,t-j} \sum_{s\geq 0} a^s \delta_{s+1}$$

$$= d_t + \frac{k-r}{1-a} + \sum_{j\geq 0} H_j \epsilon_{t-j} + \Delta \sum_{j\geq 0} \rho^j u_{M,t-j}$$

where we have defined $H_j \equiv \sum_{s\geq 0} a^s \eta_{j+s+1}$ and $\Delta \equiv \sum_{s\geq 0} a^s \delta_{s+1}$ as the "average" fundamental impulse response for time j, and the "average" impulse response for expectations distortions. In particular, rationality corresponds to the case $\Delta = 0$.

Consider now the Propositions part a). The log price change is then equal to:

$$p_t^M - p_{t-1}^M = d_t - d_{t-1} + \sum_{j\geq 0} \epsilon_{t-j} H_j - \sum_{j\geq 0} \epsilon_{t-1-j} H_j + \Delta\left(\sum_{j\geq 0} \rho^j u_{M,t-j} - \sum_{j\geq 0} \rho^j u_{M,t-1-j}\right)$$

$$= \sum_{j\geq 0} \eta_j \epsilon_{t-j} + \epsilon_t H_0 + \sum_{j\geq 1} \left((1-a)H_j - \eta_j\right) \epsilon_{t-j} + \Delta u_{M,t}$$

$$+ (\rho - 1)\Delta\left(\sum_{j\geq 1} \rho^{j-1} u_{M,t-j}\right)$$

$$= \epsilon_t (1 + H_0) + (1-a)\sum_{j\geq 1} H_j \epsilon_{t-j} + \Delta u_{M,t} + (\rho - 1)\Delta\left(\sum_{j\geq 1} \rho^{j-1} u_{M,t-j}\right)$$

where we used $d_t - d_{t-1} = \sum_{j\geq 0} \eta_j \epsilon_{t-j}$ as well as $\eta_0 = 1$ and

$$H_j - H_{j-1} = \sum_{s\geq 0} a^s \eta_{j+s+1} - \sum_{s\geq 0} a^s \eta_{j+s} = (1-a)H_j - \eta_j$$

We then have:

$$var(p_t^M - p_{t-1}^M)$$

$$= \left[(1+H_0)^2 \sigma^2 + (1-a)^2 \sum_{j\geq 1} H_j{}^2\right] \sigma^2 + \Delta^2 \left[1 + (1-\rho)^2 \sum_{j\geq 0} \rho^{2j}\right] \sigma_M^2$$

$$+ 2\Delta\left[(1+H_0) + (\rho - 1)(1-a)\sum_{j\geq 1} \rho^{j-1} H_j\right] \sigma_{MF}$$

$$= var(p_t^{RE} - p_{t-1}^{RE})\sigma^2 + \frac{2}{1+\rho}\Delta^2 \sigma_M^2$$

$$+ 2\Delta\left[(1+H_0) - (1-\rho)(1-a)\sum_{j\geq 1} \rho^{j-1} H_j\right] \sigma_{MF}$$

So there is excess volatility if:

$$\sigma_M^2 + \frac{1+\rho}{\Delta}\left[\sum_{j\geq 0} \rho^j H_j + \left(1 - \left(1 - a(1-\rho)\right)\sum_{j\geq 1} \rho^{j-1} H_j\right)\right] \sigma_{MF} > 0$$

In the benchmark case where $\delta_{s+1} = 1$ for all s, we have $\Delta = \frac{1}{1-a}$ and the condition above can be rewritten:

$$\sigma_M^2 + (1-a)\mu\sigma_{MF} > 0$$

where $\mu = (1+\rho)\left[\sum_{j\geq 0}\rho^j H_j + \left(1-\left(1-a(1-\rho)\right)\sum_{j\geq 1}\rho^{j-1}H_j\right)\right]$. The condition $\mu > 0$ is then equivalent to:

$$1 + H_0 > \sum_{j\geq 1}\rho^j H_j\left(\frac{1}{\rho} - 1\right)(1-a),$$

which is fulfilled provided the rational price response to a fundamental shock $\frac{\partial p_t^{RE}}{\partial \epsilon_t} = 1 + H_0$ is large enough. Because $a \approx 0$ and because the long run impulse response converges to zero, so that $\sum_{j\geq 1}\rho^j H_j$ is low, the condition $\mu > 0$ is satisfied provided $\frac{\partial p_t^{RE}}{\partial \epsilon_t}$ is not much above zero.

Consider now the Proposition's part b). The log price dividend ratio is equal to:

$$p_t^M - d_t = \frac{k-r}{1-a} + \sum_{j\geq 0} H_j\epsilon_{t-j} + \Delta\sum_{j\geq 0}\rho^j u_{M,t-j},$$

which in conventional tests is used as an explanatory variable for future realized dividend growth rates:

$$\sum_{s\geq 0} a^s g_{t+s+1} = \sum_{j\geq 0}\sum\nolimits_{s\geq 0} a^s\eta_j\epsilon_{t+s+1-j} = \sum_{j\geq 0} H_j\epsilon_{t-j} + v',$$

where v' is a combination of future shocks. Then:

$$cov\left[p_t^M - d_t, \sum_{s\geq 0} a^s g_{t+s+1}\right] = cov\left[\sum_{j\geq 0} H_j\epsilon_{t-j} + \Delta\sum_{j\geq 0}\rho^j u_{M,t-j}, \sum_{j\geq 0} H_j\epsilon_{t-j}\right]$$

$$= \sum_{j\geq 0} H_j^2\,\sigma^2 + \Delta\sum_{j\geq 0}\rho^j H_j\,\sigma_{MF}$$

while

$$var(p_t^M - d_t) = \sum_{j\geq 0} H_j^2\,\sigma^2 + \left(\frac{\Delta^2}{1-\rho^2}\right)\sigma_M^2 + 2\Delta\sum_{j\geq 0}\rho^j H_j\,\sigma_{MF}$$

So the coefficient from regressing the future discounted dividend growth on the log price dividend is:

$$\beta = \frac{\sum_{j\geq 0} H_j^2\,\sigma^2 + \Delta\sum_{j\geq 0}\rho^j H_j\,\sigma_{MF}}{\sum_{j\geq 0} H_j^2\,\sigma^2 + \left(\frac{\Delta^2}{1-\rho^2}\right)\sigma_M^2 + 2\Delta\sum_{j\geq 0}\rho^j H_j\,\sigma_{MF}}$$

Note that under rational expectations, $\Delta = 0$, we have $\beta = 1$. Instead, the coefficient is smaller than 1 if

$$\sigma_M^2 + (1-\rho^2)\frac{\sum_{j\geq 0}\rho^j H_j}{\Delta}\sigma_{MF} > 0$$

Again, in the benchmark case where $\delta_{s+1} = 1$ for all s, we have:

$$\sigma_M^2 + (1-a)\omega\sigma_{MF} > 0$$

with $\omega = (1-\rho^2)\sum_{j\geq 0}\rho^j H_j$.

Finally, consider the predictability of returns. By Equation (1), the one period stock return is equal to:

$$r_{t+1} = a(p_{t+1} - d_{t+1}) + g_{t+1} - (p_t - d_t),$$

where we have set $k = 0$ for convenience. By iterating the equation forward until $t + T$ we obtain:

$$\sum_{s=0}^{T-1} a^s r_{t+s+1} = a^T(p_{t+T} - d_{t+T}) + \sum_{s=0}^{T-1} a^s g_{t+s+1} - (p_t - d_t).$$

By using the price rule (where for convenience we have also set $r = 0$), it is immediate to obtain:

$$\sum_{s=0}^{T-1} a^s r_{t+s+1} = \sum_{s\geq 0} a^{s+T}\mathbb{E}_{t+T}^M(g_{t+T+s+1}) + \sum_{s=0}^{T-1} a^s g_{t+s+1} - \sum_{s\geq 0} a^s \mathbb{E}_t^M(g_{t+s+1}),$$

which can be written as:

$$\sum_{s=0}^{T-1} a^s r_{t+s+1} = \sum_{s\geq T} a^s[\mathbb{E}_{t+T}^M(g_{t+s+1}) - \mathbb{E}_t^M(g_{t+s+1})] + \sum_{s=0}^{T-1} a^s[g_{t+s+1} - \mathbb{E}_t^M(g_{t+s+1})],$$

so that T-period ahead returns combine the forecast revisions up until T as well as the term structure of forecast errors made at time t. Note that:

$$\begin{aligned}\mathbb{E}_{t+T}^M(g_{t+s+1}) - \mathbb{E}_t^M(g_{t+s+1}) &= \sum\nolimits_{j\geq T}\rho^j\delta_{s+1-T}u_{M,t+T-j} - \sum\nolimits_{j\geq 0}\rho^j\delta_{s+1}u_{M,t-j} + \nu \\ &= \sum\nolimits_{j\geq 0}\left(\rho^{j+T}\delta_{s+1-T} - \rho^j\delta_{s+1}\right)u_{M,t-j} + \nu\end{aligned}$$

where ν captures shocks that occur after t. From the perspective of t, realized future returns are:

$$\begin{aligned}&\mathbb{E}_t\left[\sum_{s=0}^{T-1} a^s r_{t+s+1}\right] \\ &\quad = \sum_{s\geq T} a^s \sum\nolimits_{j\geq 0}\left(\rho^{j+T}\delta_{s+1-T} - \rho^j\delta_{s+1}\right)u_{M,t-j} \\ &\quad - \left(\sum_{s=0}^{T-1} a^s\delta_{s+1}\right)\left(\sum\nolimits_{j\geq 0}\rho^j u_{M,t-j}\right) = \left[\rho^T\sum_{s\geq T} a^s\delta_{s+1-T} - \Delta\right]\left(\sum\nolimits_{j\geq 0}\rho^j u_{M,t-j}\right) \\ &\quad = (\rho^T a^T - 1)\Delta\left(\sum\nolimits_{j\geq 0}\rho^j u_{M,t-j}\right)\end{aligned}$$

This implies that regressing the T-period return on the current price dividend ratio $p_t^M - d_t$ yields a coefficient $cov(\sum_{s=0}^{T-1} a^s r_{t+s+1}, p_t^M - d_t)/var(p_t^M - d_t)$, where

$$cov\left(\sum_{s=0}^{T-1} a^s r_{t+s+1}, p_t^M - d_t\right)$$
$$= (\rho^T a^T - 1)\Delta cov\left(\sum\nolimits_{j\geq 0} \rho^j u_{M,t-j}, \sum_{j\geq 0} H_j \epsilon_{t-j} + \Delta \sum_{j\geq 0} \rho^j u_{M,t-j}\right)$$
$$= -\frac{1-\rho^T a^T}{1-\rho^2}\Delta^2\left((1-\rho^2)\frac{\sum_{j\geq 0}\rho^j H_j}{\Delta}\sigma_{M,F} + \sigma_M^2\right)$$

The coefficient is negative provided

$$(1-\rho^2)\frac{\sum_{j\geq 0}\rho^j H_j}{\Delta}\sigma_{M,F} + \sigma_M^2 > 0$$

Again, in the benchmark case where $\delta_{s+1} = 1$ for all s, we have:

$$\sigma_M^2 + (1-a)\omega\sigma_{MF} > 0$$

with $\omega = (1-\rho^2)\sum_{j\geq 0}\rho^j H_j$.

■

Proposition 2. The one period forecast revision about g_{t+s+1} can be written as:

$$\mathbb{E}_t^M(g_{t+s+1}) - \mathbb{E}_{t-1}^M(g_{t+s+1}) = \eta_{s+1}\epsilon_t + \delta_{s+1}u_{M,t} + \left(\delta_{s+1} - \frac{\delta_{s+2}}{\rho}\right)\sum\nolimits_{j\geq 1}\rho^j u_{M,t-j}.$$

The forecast revision that is explained by the current fundamental shock ϵ_t is:

$$\frac{cov[\mathbb{E}_t^M(g_{t+s+1}) - \mathbb{E}_{t-1}^M(g_{t+s+1}), \epsilon_t]}{var(\epsilon_t)} = \left[\eta_{s+1} + \delta_{s+1}\frac{\sigma_{MF}}{\sigma^2}\right] \equiv \vartheta.$$

Positive shocks cause positive revisions on average if and only if $\vartheta > 0$. The forecast revision at t about growth $s+1$ periods ahead caused by current fundamentals ϵ_t is then

$$FR_{t,s+1} = \vartheta\epsilon_t.$$

The covariance of future returns with this "fundamentals-based revision" is equal to:

$$cov\left[\mathbb{E}_t\left[\sum_{s=0}^{T-1} a^s r_{t+s+1}\right], \vartheta\epsilon_t\right] = (\rho^T a^T - 1)\Delta\vartheta\sigma_{MF}.$$

Given that $\rho^T a^T - 1 < 0$, $\Delta > 0$, and we consider by assumption the case $\vartheta > 0$, the upward forecast revision predicts lower future returns provided $\sigma_{MF} > 0$.

■

Proof of Equation (9). We begin by deriving the Coibion Gorodnichenko coefficient that links forecast errors to forecast revisions. From Equation (4), the expected forecast error at time t is:

$$\mathbb{E}_t[g_{t+s+1} - \mathbb{E}_t^O(g_{t+s+1})] = -\delta_{s+1}\left(\sum\nolimits_{j\geq 0}\rho^j u_{O,t-j}\right) = -\delta_{s+1}u_{O,t} - \delta_{s+1}\left(\sum\nolimits_{j\geq 1}\rho^j u_{O,t-j}\right)$$

while the revision at t is has two components, one driven by the shocks at t (both fundamental and to expectations) and another driven by the change in the impact of past expectation shock on the forecast:

$$\mathbb{E}_t^O(g_{t+s+1}) - \mathbb{E}_{t-1}^O(g_{t+s+1}) = \eta_{s+1}\epsilon_t + \delta_{s+1}u_{O,t} + \frac{\delta_{s+1}\rho - \delta_{s+2}}{\rho}\left(\sum\nolimits_{j\geq 1}\rho^j u_{O,t-j}\right).$$

so that:

$$cov[g_{t+s+1} - \mathbb{E}_t^O(g_{t+s+1}), \mathbb{E}_t^O(g_{t+s+1}) - \mathbb{E}_{t-1}^O(g_{t+s+1})]$$

$$= -\delta_{s+1}(\eta_{s+1}\sigma_{OF} + \delta_{s+1}\sigma_O^2) - \delta_{s+1}(\delta_{s+1}\rho - \delta_{s+2})\frac{\rho}{1-\rho^2}\sigma_O^2$$

$$var[\mathbb{E}_t^O(g_{t+s+1}) - \mathbb{E}_{t-1}^O(g_{t+s+1})]$$

$$= \eta_{s+1}^2\sigma^2 + \delta_{s+1}^2\sigma_O^2 + \eta_{s+1}\delta_{s+1}\sigma_{OF} + (\delta_{s+1}\rho - \delta_{s+2})\frac{\rho}{1-\rho^2}\sigma_O^2.$$

The CG coefficient is negative provided:

$$\delta_{s+1}[(\eta_{s+1}\sigma_{OF} + \delta_{s+1}\sigma_O^2)(1-\rho^2) + (\delta_{s+1}\rho - \delta_{s+2})\rho\sigma_O^2] > 0$$

which is equivalent to:

$$\delta_{s+1}[\eta_{s+1}\sigma_{OF}(1-\rho^2) + (\delta_{s+1} - \delta_{s+2}\rho)\sigma_O^2] > 0.$$

In the benchmark case where $\delta_{s+1} = 1$ for all s, this becomes:

$$(1+\rho)\eta_{s+1}\sigma_{OF} + \sigma_O^2 > 0. \qquad (A.1)$$

which is reminiscent of the conditions in Propositions 1. Belief updating at horizon $s+1$ is excessive when the distortion co-moves with rational updating ($\sigma_{OF} > 0$ and $\eta_{s+1} > 0$) or when beliefs are noisy (large σ_O^2). In the first case analysts over-react to fundamental news, in the second they over-react to noise.

We can now interpret the results of Tables 4 and 7. The results in Table 4 arise if a positive shock to earnings growth displays short term reversal, namely $\eta_1, \eta_2 < 0$, and long-term higher growth, namely $\eta_{s+1} > 0$ for $s > 2$. In turn, predictability of forecast errors depends on the combination of analyst optimism in reaction to a good shock, σ_{OF}, with the impact of that shock on subsequent growth, η_{s+1}. According to Equation (A.1), the patterns for short and long term forecasts are reconciled if analysts become sufficiently optimistic after a good growth shock:

$$\sigma_{OF} > -\frac{1}{\eta_{s+1}} \frac{\sigma_O^2}{(1+\rho)}, s = 1,2$$

If this condition is met, after a positive shock analysts revise their forecasts downward, anticipating mean reversion, but because $\sigma_{OF} > 0$ they do not revise enough. Insufficient reversion of beliefs creates short term under-reaction. On the other hand, after the same positive growth shock, analysts revise up their long run beliefs due to both the rational and irrational components in Equation (5), causing over-reaction of long term forecasts.

The same logic explains the results of Table 8, where σ_{OF} is assessed following a two-stage approach. In a first stage we regress the revision of growth forecasts on our proxy for fundamental news (as in Table 4). In a second stage, we regress the forecast error on the revision predicted from the first stage.

We now show that a positive first stage coefficient $\varphi_s > 0$ at horizon $s + 1$ means that $\eta_{s+1}\sigma^2 + \sigma_{O,F} > 0$: expectations move in the direction of the shock provided the shock is persistent ($\eta_{s+1} > 0$) and expectational distortions correlate with the shock itself ($\sigma_{O,F} > 0$). Recall that the forecast revision is:

$$\mathbb{E}_t^O(g_{t+s+1}) - \mathbb{E}_{t-1}^O(g_{t+s+1}) = \eta_{s+1}\epsilon_t + \delta_{s+1}u_{O,t} + \frac{\delta_{s+1}\rho - \delta_{s+2}}{\rho}\left(\sum\nolimits_{j\geq 1} \rho^j u_{O,t-j}\right).$$

It follows that:

$$\varphi_{fs} = \frac{cov[\mathbb{E}_t^O(g_{t+s+1}) - \mathbb{E}_{t-1}^O(g_{t+s+1}), \epsilon_t]}{var[\epsilon_t]} = \frac{\eta_{s+1}\sigma^2 + \delta_{s+1}\sigma_{O,F}}{\sigma^2}.$$

The predicted forecast revision is then equal to:

$$\widehat{\mathbb{E}}_t^O(g_{t+s+1}) - \widehat{\mathbb{E}}_{t-1}^O(g_{t+s+1}) = \varphi_{fs}\epsilon_t$$

If news proxies indeed predict forecast revisions, the second stage coefficient is given by:

$$\frac{cov\left[g_{t+s+1} - \mathbb{E}_t^O(g_{t+s+1}), \widehat{\mathbb{E}}_t^O(g_{t+s+1}) - \widehat{\mathbb{E}}_{t-1}^O(g_{t+s+1})\right]}{var\left[\widehat{\mathbb{E}}_t^O(g_{t+s+1}) - \widehat{\mathbb{E}}_{t-1}^O(g_{t+s+1})\right]} = -\frac{\delta_{s+1}\sigma_{OF}}{\varphi_{fs}\sigma^2}.$$

To derive this equation, we use the forecast error:

$$\mathbb{E}_t[g_{t+s+1} - \mathbb{E}_t^O(g_{t+s+1})] = -\delta_{s+1}\left(\sum\nolimits_{j\geq 0} \rho^j u_{O,t-j}\right) = -\delta_{s+1}u_{O,t} - \delta_{s+1}\left(\sum\nolimits_{j\geq 1} \rho^j u_{O,t-j}\right)$$

The analysis yields $\sigma_{OF} > 0$ if, for horizons s such that news positively predict revisions, $\varphi_{fs} > 0$, the second stage is negative. Intuitively, in this case analysts become too optimistic after good shocks, as suggested by Table 4 for beliefs about long run growth. But $\sigma_{OF} > 0$ also holds if the first stage coefficient is negative, $\varphi_{fs} < 0$ and the second stage coefficient is positive. In this case, after a positive shock, beliefs get revised downwards, as in the case of mean reversion, but insufficiently so. Insufficient mean reversion after good news is also a sign of excess optimism, which entails $\sigma_{OF} > 0$.

■

Appendix B. Further Results on Price Indices

In this Appendix, we collect several results that complement the analysis of the expectations-based price indices in Section 4.

a) Alternative specifications of long-term forecasts in the dividend discount model. Here we consider an alternative definition of expectation-based prices where expectations at time t of growth beyond year $t+5$ is inferred by applying the observed decay of observed cyclically adjusted earnings to LTG_t. Regressing $caeps_t - caeps_{t-t}$ on $caeps_{t-5} - caeps_{t-10}$ yields a slope coefficient of roughly 0.4. Thus, for a ten-year forecasting horizon we set:

$$p_t^{O10} = e_t + \frac{\tilde{k} - r}{1-\alpha} + \ln\left(\frac{E_t^O \, EPS_{t+1}}{EPS_t}\right) + \sum_{j=1}^{5} \alpha^{j-1} \, E_t^O \Delta \mathrm{e}_{t+j+1}$$

$$+ \sum_{j=6}^{10} \alpha^{j-1} \ln\left(1 + 0.4 * E_t^O \Delta \mathrm{e}_{t+5}\right) + \frac{\alpha^{10}}{1-\alpha} \, g_{10}.$$

and similarly for a 15 and 20-year forecasting horizon, as well as for an alternative dividend based index p_t^{ODT} (where long term growth is assumed to be described by LTG). Table B.1 shows the results.

Table B.1

Panel A reports the standard deviation of one-year change in: (1) the log of the price of the S&P500 index Δp, (2) the index based on dividends forecasts Δp_t^{OD} (Equation 7), and (3) the alternative index based on dividend forecasts p_t^{ODT}, for T=10, 15 and 20 (please see equation in text of Appendix B). The sample period ranges from 10/2003 to 11/2018 and has 152 monthly observations. Panel B reports the standard deviation of one-year change in: (1) the log of the price of the S&P500 index Δp, (2) the index based on earnings forecasts Δp_t^O (Equation 7), and (3) the alternative index based on earnings forecasts Δp_t^{OT} for $T = 10, 15,$ and 20 (see text of Appendix B). The sample period ranges from 12/1982 to 12/2018 and has 419 monthly observations.

Panel A: Dividend based synthetic price indices

	Δp	Δp^{OD}	Δp^{OD10}	Δp^{OD15}	Δp^{OD20}
σ	15.1%	17.3%	15.1%	15.6%	15.8%
95th Confidence Interval	13.6%-17.1%	15.5%-19.5%	13.5%-17.0%	14.0%-17.6%	14.2%-17.8%

Panel B: Earnings based synthetic price indices

	Δp	Δp^{O}	Δp^{O10}	Δp^{O15}	Δp^{O20}

σ	15.2%	13.9%	12.1%	12.6%	12.7%
95th Confidence Interval	14.2%-16.3%	13.0%-14.9%	11.3%-13.0%	11.8%-13.5%	11.9%-13.7%

b) Indices based on expectations of short-term growth. We define a price index based on expectations for short term earnings growth

$$p_t^{OS} = e_t + \frac{\tilde{k} - r}{1-\alpha} + \ln\left(\frac{E_t^O \; EPS_{t+1}}{EPS_t}\right) + \frac{1}{1-\alpha}\, g_1$$

and on expectations for short term dividend growth

$$p_t^{OSD} = d_t + \frac{k - r}{1-\alpha} + \ln\left(\frac{E_t^O \; DPS_{t+1}}{DPS_t}\right) + \frac{1}{1-\alpha}\, g_{1,D}$$

Table B.2 adds to Table 1 the volatility of annual price changes as computed with p_t^{OS} and p_t^{ODS}.

TABLE B.2

Panel A reports the standard deviation of one-year change in: (1) the log of the price of the S&P500 index, Δp, (2) the rational benchmark index, Δp_t^{RE} (Equation 8), (3) the index based on earnings forecasts Δp_t^{O} (Equation 6), and (4) the price index based on expectations for short-term earnings growth $\Delta \mathrm{p}^{\mathrm{OS}}$ (see text of Appendix B). The sample period ranges from 12/1982 to 11/2018 and has 419 monthly observations. Panel B reports the standard deviation of one-year change in: (1) the log of the price of the S&P500 index Δp, (2) the rational benchmark index Δp_t^{RE} (Equation 8), (3) the index based on dividend forecasts Δp_t^{OD} (Equation 7), and (4) the price index based on expectations for short-term dividend growth $\Delta \mathrm{p}^{\mathrm{OSD}}$ (see text of Appendix B). The sample period ranges from 10/2002 to 1/2018 and has 152 monthly observations.

Panel A: Earnings-based price indices

	Δp	Δp^{RE}	Δp^{O}	Δp^{OS}
σ	15.2%	0.3%	13.9%	12.0%
95th Confidence Interval	14.2%-16.3%	0.3%-0.3%	13.0%-14.9%	11.2%-12.9%

Panel B: Dividend-based price indices

	Δp	Δp^{RE}	Δp^{OD}	Δp^{OSD}
σ	15.1%	0.3%	17.3%	11.7%
95th Confidence Interval	13.6%-17.1%	0.2%-0.3%	15.5%-19.5%	10.5%-13.2%

Table 3 in Section 4 examined the explanatory power of our indices $p_t^O - e_t$ and $p_t^{RE} - e_t$ for the price earnings ratio, and correspondingly $p_t^{OD} - d_t$ and $p_t^{RE} - d_t$ for the price dividend ratio. Table B.3 extends Table 2 by adding the short term indices above.

TABLE B.3

In Panel A, the dependent variables are: (1) the difference between the log rational benchmark index p_t^{RE}(Equation 8) and log earnings (e_t), (2) the difference between the price index based on expectations for short-term earnings growth p_t^{OS} (see text of Appendix B) and e_t, and (3) the difference between the price index based on earnings forecasts p_t^O (Equation 6) and e_t. In Panel B, the dependent variables are: (1) the difference between p_t^{RE} and dividends (d_t), (2) the difference between the price index based on expectations for short-term earnings growth p_t^{OSD}(see text of Appendix B) and d_t, and (3) the difference between the log of the index based on dividend forecasts p_t^{OD} (Equation 7) and d_t. The independent variables are the log price-to-earnings ratio in Panel A and the log price-to-dividend ratio in Panel B. In each panel, the last row reports the sample period for each regression. Standard errors are not adjusted for serial correlation. Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.

Panel A: Earnings-based index

	(1)	(2)	(3)
	$p_t^{RE} - e_t$	$p_t^{OS} - e_t$	$p_t^O - e_t$
pe_t	0.5453[a]	0.5241[a]	0.6169[a]
	(0.0796)	(0.0610)	(0.0528)
Observations	437	437	437
Adjusted R^2	39%	55%	64%
Sample period	1981:12-2018:12	1981:12-2018:12	1981:12-2018:12

Panel B: Dividend-based index

	(1)	(2)	(3)
	$p_t^{RE} - d_t$	$p_t^{ODS} - d_t$	$p_t^{OD} - d_t$
pd_t	0.1534[a]	0.3454[a]	0.7178[a]
	(0.0106)	(0.0485)	(0.0872)
Observations	445	178	134
Adjusted R^2	23%	42%	42%
Sample period	1981:12-2018:12	2002:10-2018:12	2005:11-2018:11

Table B.4 presents the correlations across the various indices, computed over the appropriate sample periods.

Table B.4

Pairwise correlations between the following variables: (1) log of the price dividend ratio pd_t, (2) the difference between the rational benchmark index, p_t^{RE} (Equation 8) and log dividends (d_t), (3) the difference between the index based on dividend forecasts p_t^{OD} (Equation 7) and d_t, (4) the difference between the index based on short-term dividend growth p_t^{OSD} (see text of Appendix B) and d_t, (5) log of the price earnings ratio pe_t, (6) the difference between the index based on earnings forecasts p_t^O (Equation 6) and e_t, and (7) the difference between the index based on short-term earnings growth p_t^{OS} (see text of Appendix B) and e_t.

	pd_t	$p_t^{RE} - d_t$	$p_t^{OD} - d_t$	$p_t^{OSD} - d_t$	pe_t	$p_t^O - e_t$
$p_t^{RE} - d_t$	0.4850					
$p_t^{OD} - d_t$	0.6219	-0.3055				
$p_t^{OSD} - d_t$	0.6562	-0.0420	0.6935			
pe_t	0.5765	0.2308	-0.4589	-0.6582		
$p_t^O - e_t$	0.0620	0.0568	-0.4426	-0.6534	0.7974	

$p_t^{OS} - e_t$	-0.0438	0.1294	-0.5330	-0.6498	0.7418	0.9453

c) Price paths. Here we present two price paths: i) p_t^O adjusted for inflation using the CPI index (Figure B.1) and ii) the price index based on dividend forecasts p_t^{OD} (Figure B.2).

Figure B.1
Prices adjusted for inflation.

We plot in logscale the levels of the S&P500 index (green line), the rational benchmark index (p^{RE}, blue line) and our benchmark expectations-based price index (p^O, red line). All values are adjusted for inflation using the CPI index.

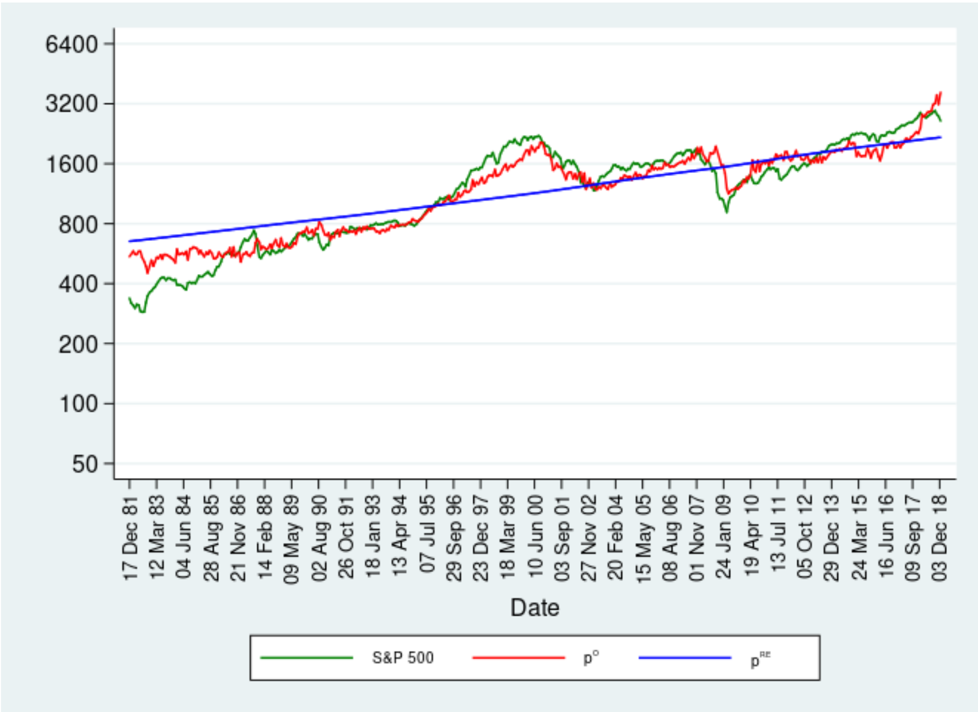
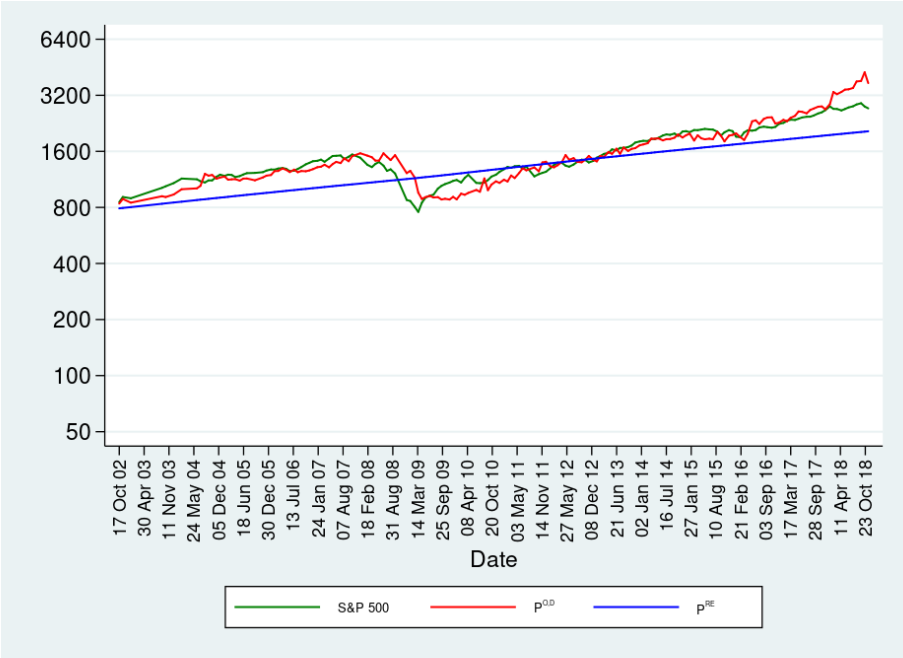


Figure B.2
Dividend expectations-based prices.

We plot in logscale the levels of the S&P500 index (green line), the rational benchmark index (p^{RE}, blue line) and our dividend expectations-based price index (p^{OD}, red line) over the sample where such forecasts are available, 2002 - 2018.



d) Co-integrated series. Following Campbell and Shiller (1987) we assess the volatility of the cointegrated series $P_t - \frac{D_t}{r}$ for different measures of prices. The sample period is 2002:10-2018-11 in the first panel and 1981:12-2018:11 in the other two panels. Table B.5 presents the results.

Table B.5

Panel A reports standard deviation and number of observations for different cointegrated series using 8.48% as the discount rate r. The variables in the first row of results are: (1) $P_t - \frac{D_t}{r}$, the difference between the S&P500 index P_t and the ratio of the dividends of the S&P500 index D_t to r, (2) $P_t^{RE} - \frac{D_t}{r}$, the difference between the level of the rational benchmark index (i.e. $\exp(p_t^{RE})$, see equation 8) and $\frac{D_t}{r}$, (3) $P_t^{OD} - \frac{D_t}{r}$, the difference between the level of the index based on dividend forecasts P_t^{OD} (i.e. $\exp(p_t^{OD})$, see Equation 7) and $\frac{D_t}{r}$, and (4) $P_t^{OSD} - \frac{D_t}{r}$ the difference between the level of the index based on short-term dividend growth P_t^{OSD} (i.e. $\exp(p_t^{OSD})$, see text of Appendix B) and $\frac{D_t}{r}$. The sample period is 10/2002-11/2018. The variables in the second row of results in Panel A are: (1) $P_t - \frac{D_t}{r}$, (2) $P_t^{RE} - \frac{D_t}{r}$, (3) $P_t^{O} - \frac{D_t}{r}$, the difference between the level of the index based on earnings forecasts P_t^{O} (i.e. $\exp(p_t^{O})$ and $\frac{D_t}{r}$, and (4) $P_t^{OD}$the level of the index based on short-term dividend growth P_t^{OS} (i.e. $\exp(p_t^{OS})$, see Appendix B) and $\frac{D_t}{r}$. The sample period is 12/1981-11/2018. The variables in the second and third rows are the same except that we replace D_t by E_t, the earnings of the S&P500 index. The sample period is 12/1981-11/2018. Panel B presents standard deviations for $P - \frac{D}{r}$, $P^{O} - \frac{D}{r}$, and $P^{OD} - \frac{D}{r}$ for values of r ranging between 3.5% and 8.5%.

Panel A: Standard deviation of the co-integrated series

$P_t - \frac{D_t}{r}$	$P_t^{RE} - \frac{D_t}{r}$	$P_t^{OD} - \frac{D_t}{r}$	$P_t^{OSD} - \frac{D_t}{r}$

Σ	416	244	576	395
N	173	173	173	173

	$P_t - \frac{D_t}{r}$	$P_t^{RE} - \frac{D_t}{r}$	$P_t^{O} - \frac{D_t}{r}$	$P_t^{OS} - \frac{D_t}{r}$
σ	548	374	425	433
N	437	437	437	437

	$P_t - \frac{E_t}{r}$	$P_t^{RE} - \frac{E_t}{r}$	$P_t^{O} - \frac{E_t}{r}$	$P_t^{OS} - \frac{E_t}{r}$
σ	362	220	247	231
N	437	437	437	437

Panel B: sensitivity to the co-integration parameter

	1981:12-2018:12		2002:12-2018:12	
	$P_t - \frac{D_t}{r}$	$P_t^{O} - \frac{D_t}{r}$	$P_t - \frac{D_t}{r}$	$P_t^{OD} - \frac{D_t}{r}$
$r = 3.5\%$	375.04	258.95	266.31	430.23
$r = 4.5\%$	437.73	317.46	320.44	483.35
$r = 5.5\%$	479.52	357.91	356.84	518.60
$r = 6.5\%$	509.09	386.88	382.65	543.53
$r = 7.5\%$	531.05	408.52	401.83	562.04
$r = 8.5\%$	547.84	425.25	416.63	576.31

Appendix C. Further results on expectations

In this Appendix, we collect several sets of results pertaining to the definition of : a) we estimate the earnings process according to the MA specification in Section 2, b) we consider a broader set of proxies for fundamental news, i.e. drivers of LTG revisions, c) we present further evidence suggesting that price levels do not drive LTG, including at the firm level.

a) Estimation of the earnings process. Here we complement the characterization of the impulse response in Table 5 by estimating an MA representation.

Table C.1

Panel A reports the results of estimating moving average processes of order one through five for the real one-year growth rate in earnings between 1981 and 2018. Panel B reports the impulse-response function for an MA(5) process and the associated asymptotic standard errors. We use annual (end-of-December) data for the sample period 1981-2018. Panel B reports the impulse-response function.

Panel A. Moving average representation of earnings growth

	(1)	(2)	(3)	(4)	(5)
ϵ_{t-1}	-1.0000[a]	-0.5794[a]	-0.6069[a]	-0.8893[a]	-0.8421[a]
	(0.0819)	(0.1583)	(0.2189)	(0.2182)	(0.2290)
ϵ_{t-2}		-0.4206[a]	-0.4321[a]	-0.6381[a]	-0.6519[a]
		(0.1436)	(0.1558)	(0.1833)	(0.1894)
ϵ_{t-3}			0.0390	-0.0287	-0.0469
			(0.2161)	(0.2478)	(0.2425)
ϵ_{t-4}				0.5687[a]	0.4664[c]
				(0.1834)	(0.2686)
ϵ_{t-5}					0.0904
					(0.1674)
Observations	38	38	38	38	38

Panel B. Impulse response of earnings growth

Step	IRF	SE_{IRF}
0	1	0
1	-0.36921	0.117794
2	-0.44205	0.124752
3	-0.15424	0.144386
4	0.17477	0.164236
5	0.07109	0.110311

Figure C.1 illustrates the impulse response documented above. The black curve represents the impulse response to a positive fundamental shock: a short term growth reversal, followed by a recovery. The red curve plots the corresponding response of beliefs. When $\sigma_{OF} > 0$, beliefs are too optimistic after good news at all horizons (the red curve is above the black one). Even though analysts revise short term prospects downwards, their revision is insufficient, generating under-reaction. At long horizons, analysts revise up too much and over-react.

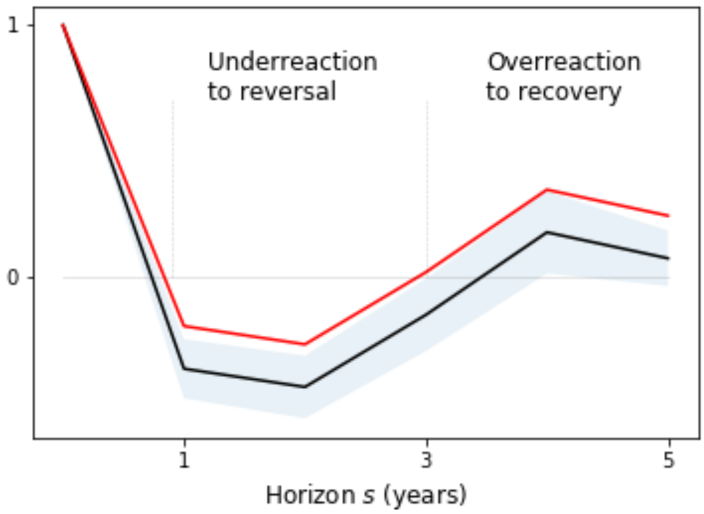


Figure C.1. Earnings growth impulse response (mean: black line, standard deviation: blue shade) and schematic growth forecasts given by Equation (4) (red line). Data on the earnings impulse response is in Table C.1 above.

b) Further evidence on determinants of LTG. We expand on the link between LTG and expectations of returns of Table 4, introducing further measures of expected returns, and showing that controlling for fundamentals, LTG is uncorrelated with expectations of returns.

Table C.2

The dependent variable is the one-year change in LTG. The independent variables are the (log) of earnings per share in year t dividend by cyclically-adjusted earnings in year $t-5$, $e_t - cae_{t-5}$, the cumulative returns for the S&P500 between years $t-5$ and t, and expected returns for the S&P500 at both the one- and ten-year horizon from the survey of CFOs administered by John Graham and Campbell Harvey ($\Delta\mathbb{E}_t^O r_{t+1}$ and $\Delta\mathbb{E}_t^O r_{t+10}$).

	Dependent Variable: ΔLTG_t	
	(1)	(2)
$e_t - cae_{t-5}$	0.4204[a]	0.4335[a]
	(0.1027)	(0.0987)
$r_{t-5,t}$	0.2773	0.3041
	(0.2202)	(0.2217)
$\Delta\mathbb{E}_t^O r_{t+1}$	-0.0153	
	(0.1228)	
$\Delta\mathbb{E}_t^O r_{t+10}$		-0.1311
		(0.0966)

Observations	64	64
Adjusted R^2	32%	33%
Sample	2001:12: 2018:12	2001:12: 2018:12

We explore this point further by assessing to what extent changes in LTG are predicted by empirical measures of required returns or of risk, after controlling for shocks to fundamentals. Table C.4 shows that measures of risk do not predict changes in LTG, contrary to the hypothesis that LTG reflects changes in discount rates.

Table C.3

The table shows univariate regressions of one-year changes in the forecast for earnings growth in the long run, ΔLTG_t, on the following variables: (1) log of earnings in year t relative to cyclically-adjusted earnings in year $t-5$, $e_t - cae_{t-5}$, (2) the Plueger et al. (2020) price of volatile stocks defined as the book-to-market ratio of low-volatility stocks minus the book-to-market ratio of high-volatility stocks, (3) the Lettau and Ludvigson (2001) consumption-wealth ratio (cay), (4) the Kelly and Pruitt (2013) optimal forecast of aggregate equity market returns, (5) the Baker et al. (2016) economic policy uncertainty index, (6) the Gilchrist and Zakrajšek (2012) credit spread, (6) the Greenwood and Hanson (2013) measure of credit market sentiment (High Yield Share), (7) the term spread defined as the log difference between the gross yield of 10-year and 1-year US government bonds from the St. Louis Fed, (8) the credit spread defined as the log difference between the gross yield of BAA and AAA bonds from the St. Louis Fed, (9) the Campbell and Cochrane (1999) surplus consumption ratio (spc), and (10) the Miranda-Aggripino et al. (2020) global factor. All variables are normalized to have zero mean and standard deviation equal to 1. Newey-West standard errors are shown in parentheses (with lags for up to a year). Note: [a] significant at the 1% level, [b] significant at the 5% level, and [c] significant at the 10% level.[28]

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$e_t - cae_{t-5}$	0.4982[a]	0.4645[a]	0.3998[a]	0.4543[a]	0.4328[a]	0.4678[a]	0.4250[a]	0.4908[a]	0.5287[a]	0.5985[a]
	(0.0963)	(0.0999)	(0.1079)	(0.0892)	(0.1129)	(0.1036)	(0.0863)	(0.1218)	(0.1053)	(0.1563)
Price of Volatile Stocks$_t$	-0.0162									
	(0.1479)									
cay$_t$		-0.0577								
		(0.0977)								
Kelly Pruitt Forecast Market Return$_t$			-0.1912							
			(0.1175)							
Bloom Economic Political Uncertainty Index$_t$				-0.0523						
				(0.1133)						
Gilchrist Zakrajsek Bond spread$_t$					-0.0903					
					(0.1409)					
High Yield Share$_t$						-0.0318				
						(0.0711)				
Term Spread$_t$							-0.2204[b]			
							(0.1086)			
Credit Spread$_t$								-0.1189		

[28]The data for spc and cay are available at the two following sites, respectively:
https://faculty.chicagobooth.edu/john.cochrane/research/Data_and_Programs/index.htm
sites.google.com/view/martinlettau/data

								(0.1691)		
spc_t									0.0073	
									(0.0908)	
Global Factor										-0.1376
										(0.1213)
Observations	143	140	130	130	112	133	433	433	433	433
Adjusted R^2	32%	33%	40%	32%	41%	34%	36%	32%	32%	33%

Appendix D. Further results on predictability of returns

We next reproduce Table 3, which examines the predictability of returns on the basis of price indices and of expectations, using excess (as opposed to raw) returns.

Table D.1

The dependent variable is the log excess return between year *t* and *t+1* in column [1] and the discounted value of the cumulative excess return between year *t* and *t+3* and *t+5* in columns [2] and [3]. Excess returns are defined relative to the yield of one-year, three-year, and five-year government bonds. The independent variables are: (1) the log of the ratio of the dividend expectations-based index to dividends (p^{OD}-*d*), (2) the log of the ratio the earnings expectations-based index to earnings (p^{O}-*e*), (3) the forecast for earnings growth in the long run (LTG_t), (4) the forecast for real earnings growth in the long run ($LTG_t - \mathbb{E}_t^O \pi_{t+1}$), defined as LTG_t minus the forecast of one-year inflation from the Survey of Professional Forecasters ($\mathbb{E}_t^O \pi_{t+1}$), (5) the time-*t* forecast for one-year for growth in earnings in year *t+2* ($\mathbb{E}_t^O[e_{t+2}-e_{t+1}]$), (6) the time-*t* forecast for one-year real growth in earnings in year *t+2* ($\mathbb{E}_t^O[e_{t+2} - e_{t+1} - \pi_{t+1}]$), defined as ($E_t[e_{t+2}-e_{t+1}]$) minus the forecast of one-year inflation from the Survey of Professional Forecasters ($\mathbb{E}_t^O \pi_{t+1}$), (7) log price-to-dividend ratio for the S&P500, (8) log price-to-earnings ratio for the S&P500, (9) the term spread defined as the log difference between the gross yield of 10-year and 1-year US government bonds from the St. Louis Fed, (10) the credit spread defined as the log difference between the gross yield of BAA and AAA bonds from the St. Louis Fed, (11) the Campbell and Cochrane (1999) surplus consumption ratio (spc), and (12) the Lettau and Ludvigson (2001) consumption-wealth ratio (cay). We adjust standard errors for serial correlation using the Newey-West correction (number of lags ranges from 12 in the first column to 60 in the last one). Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.

	$xret_{t+1}$	$\sum_{j=1}^{3} \alpha^{j-1} xtret_{t+j}$	$\sum_{j=1}^{5} \alpha^{j-1} xret_{t+j}$
	(1)	(2)	(3)
	Panel A: Returns and p^O,D^-d **(2002:10-2013:12)**		
$p_t^{OD} - d_t$	-0.5043[b]	-1.0637[a]	-1.9959[a]
	(0.2258)	(0.3854)	(0.3277)
Observations	114	114	114
Adjusted R^2	12%	17%	47%
	Panel B: Returns and p^O,E^-d **(1981:12-2013:12)**		
$p_t^{O} - e_t$	-0.0404	0.0094	0.1588
	(0.0859)	(0.1509)	(0.1626)
Observations	378	378	378
Adjusted R^2	0%	0%	2%
	Panel C: Returns and LTG **(1981:12-2013:12)**		
LTG_t	-3.7288[a]	-9.6689[a]	-12.0820[a]
	(1.0798)	(1.6629)	(2.5057)
Observations	385	385	385
Adjusted R^2	14%	31%	34%

	***Panel D: Excess Returns and real LTG* (1981:12-2013:12)**		
$LTG_t - \mathbb{E}_t^O \pi_{t+1}$	-2.2707[b]	-6.0472[b]	-7.6012[a]
	(1.1005)	(2.3794)	(2.7100)
Observations	385	385	385
Adjusted R^2	8%	19%	21%
	***Panel E: Returns and Short-term growth* (1982:12-2013:12)**		
$\mathbb{E}_t^O[e_{t+2} - e_{t+1}]$	-0.3674	0.4747	2.5905
	(0.6509)	(1.8234)	(2.1737)
Observations	378	378	378
Adjusted R^2	0%	0%	6%
	Panel F: Returns and real Short-term growth (1982:12-2013:12)		
$\mathbb{E}_t^O[e_{t+2} - e_{t+1} - \pi_t]$	-0.2994	0.3891	2.1887
	(0.6275)	(1.5865)	(2.0750)
Observations	378	378	378
Adjusted R^2	0%	0%	4%
	***Panel G: Returns and price-to-dividend ratio* (1981:12-2013:12)**		
pd_t	-0.1055[b]	-0.2486[c]	-0.3629[b]
	(0.0536)	(0.1403)	(0.1798)
Observations	385	385	385
Adjusted R^2	7%	12%	19%
	***Panel H: Returns and price-to-earnings ratio* (1981:12-2013:12)**		
pe_t	-0.0633	-0.0872	-0.0451
	(0.0642)	(0.1227)	(0.1673)
Observations	385	385	385
Adjusted R^2	3%	2%	0%
	***Panel I: Returns, LTG, and term spread* (1981:12-2013:12)**		
LTG_t	-3.9778[a]	-7.5640[a]	-9.1335[a]
	(1.2399)	(2.2699)	(2.0495)
Term $Spread_t$	-0.9820	8.3026	11.6303[a]
	(1.9497)	(5.0916)	(3.7623)
Observations	385	385	385
Adjusted R^2	14%	38%	44%
	Panel J: Returns, LTG, and credit spread		

	(1)	(2)	(3)
		(1981:12-2013:12)	
LTG_t	-3.6620[a]	-9.6817[a]	-11.5629[a]
	(1.1111)	(1.8146)	(2.9327)
Credit spread$_t$	1.3253	-0.2542	10.2923
	(4.5237)	(9.7998)	(14.0005)
Observations	385	385	385
Adjusted R^2	13%	31%	35%
	Panel K: Returns, LTG, and Surplus consumption ratio (1981:12-2013:12)		
LTG_t	-4.0353[a]	-9.3265[a]	-9.6666[a]
	(1.2504)	(1.7539)	(2.2944)
spc$_t$	1.1638	-1.3000	-9.1715[a]
	(2.1880)	(2.1323)	(2.5970)
Observations	385	385	385
Adjusted R^2	14%	31%	41%
	Panel L: Returns, LTG, and consumption wealth ratio (1981:Q4-2013:Q4)		
LTG_t	-3.7928[a]	-9.5959[a]	-12.0517[a]
	(1.2841)	(1.9533)	(1.8449)
cay$_t$	2.0933	6.7643	4.1630
	(1.5127)	(4.1769)	(4.9533)
Observations	129	129	129
Adjusted R^2	16%	43%	36%

Table D.2

The dependent variable is the log return between year *t* and *t+1* in column [1] and the discounted value of the cumulative excess return between year *t* and *t+3* and *t+5* in columns [2] and [3]. All regressions include the forecast for earnings growth in the long run (LTG_t) as an independent variable. The independent variables also include: (1) the Plueger et al. (2020) price of volatile stocks defined as the book-to-market ratio of low-volatility stocks minus the book-to-market ratio of high-volatility stocks, (2) the Lettau and Ludvigson (2001) consumption-wealth ratio (cay), (3) the Kelly and Pruitt (2013) optimal forecast of aggregate equity market returns, (4) the Baker et al. (2016) economic policy uncertainty index, (5) the Gilchrist and Zakrajšek (2012) credit spread, (6) the Greenwood and Hanson (2013) measure of credit market sentiment (High Yield Share), (7) the term spread defined as the log difference between the gross yield of 10-year and 1-year US government bonds from the St. Louis Fed, (8) the credit spread defined as the log difference between the gross yield of BAA and AAA bonds from the St. Louis Fed, (9) the Campbell and Cochrane (1999) surplus consumption ratio (spc), and (10) the Miranda-Aggripino et al. (2020) global factor. Newey-West standard errors are shown in parentheses (with lags for up to a year). Note: [a] significant at the 1% level, [b] significant at the 5% level, and [c] significant at the 10% level.

	(1)	(2)	(3)
	r_{t+1}	$\sum_{j=1}^{3} \alpha^{j-1} r_{t+j}$	$\sum_{j=1}^{5} \alpha^{j-1} r_{t+j}$

	Panel A: Returns, LTG and Price Volatile Stocks (1981:12-2013:12)		
LTG_t	-3.7453[a]	-9.3993[a]	-10.9329[a]
	(1.2788)	(1.8841)	(2.0600)
Price of Volatile Stocks$_t$	0.0440	0.0767	-0.0151
	(0.0453)	(0.1342)	(0.1816)
Observations	129	129	129
Adjusted R-squared	11%	25%	25%
	Panel B: Returns, LTG and CAY (1981:Q4-2013:Q4)		
LTG_t	-3.3520[b]	-8.4885[a]	-10.5803[a]
	(1.3268)	(1.9063)	(1.6739)
cay$_t$	2.8712[c]	9.1095[b]	7.9245[c]
	(1.4737)	(3.6975)	(4.5558)
Observations	129	129	129
Adjusted R-squared	16%	47%	37%
	Panel C: Ret, LTG and Kelly-Pruitt Forecast Market Return (1981:Q4-2013:Q4)		
LTG_t	-3.3478	-11.6025[a]	-16.2611[a]
	(2.6157)	(2.1404)	(5.5204)
Kelly-Pruitt Forecast Market Return$_t$	0.0047	-0.0763	-0.1530
	(0.0589)	(0.0625)	(0.1383)
Observations	129	129	129
Adjusted R-squared	10%	25%	28%
	Panel D: Returns, LTG and Economic Political Uncertainty Index (1985:Q1-2013:Q4)		
LTG_t	-3.8274[a]	-8.9140[a]	-10.2834[a]
	(1.1604)	(2.5251)	(2.6182)
Economic Political Uncertainty Index$_t$	-0.0001	0.0007	0.0022
	(0.0006)	(0.0021)	(0.0016)
Observations	116	116	116
Adjusted R-squared	12%	29%	36%
	Panel L: Gilchrist-Zakrajsek bond spread (1981:Q4-2010:Q3)		
LTG_t	-3.3297[a]	-8.9338[a]	-11.0214[a]
	(1.1183)	(1.5538)	(1.7368)
Gilchrist-Zakrajsek bond spread$_t$	-0.0343[c]	-0.0574	-0.0367

	(0.0199)	(0.0370)	(0.0509)
Observations	116	116	116
Adjusted R-squared	13%	29%	26%

	Panel M: High Yield Share (1983:Q2-2013:Q4)		
LTG_t	-3.6536[a]	-9.3165[a]	-12.6656[a]
	(1.2185)	(1.3407)	(1.0942)
High Yield Share$_t$	0.0754	0.0314	-0.5340[b]
	(0.1241)	(0.1261)	(0.2329)
Observations	123	123	123
Adjusted R-squared	13%	27%	36%

	Panel A: Returns, LTG, and term spread (1981:12-2013:12)		
LTG_t	-4.0542[a]	-7.7182[a]	-9.3554[a]
	(1.2542)	(2.6979)	(2.2802)
Term Spread$_t$	-2.8130	3.9567	5.9213
	(2.0088)	(5.7495)	(4.6057)
Observations	385	385	385
Adjusted R-squared	13%	26%	29%

	Panel B: Returns, LTG, and credit spread (1981:12-2013:12)		
LTG_t	-3.2232[a]	-8.5759[a]	-10.0704[a]
	(1.2049)	(2.1470)	(2.8178)
Credit Spread$_t$	2.3353	2.8815	15.5884
	(5.2732)	(12.0531)	(17.0563)
Observations	385	385	385
Adjusted R-squared	11%	25%	30%

	Panel C: Returns, LTG, and Surplus consumption ratio (1981:12-2013:12)		
LTG_t	-3.9501[a]	-9.2888[a]	-9.8667[a]
	(1.3380)	(2.0251)	(2.3950)
spc$_t$	2.3128	2.1550	-3.7587
	(2.3222)	(2.3694)	(3.3475)
Observations	385	385	385
Adjusted R-squared	12%	25%	27%

	Panel O: Global Factor in risky asset prices (1981:12-2013:12)		
LTG_t	-2.8927[b]	-7.8212[a]	-9.4927[a]
	(1.2301)	(1.9764)	(1.7619)
Global Factor$_t$	-0.0613[b]	-0.1230[a]	-0.1864[a]

	(0.0249)	(0.0419)	(0.0305)
Observations	385	385	385
Adjusted R-squared	25%	45%	58%

Table D.3

Return predictability from price ratios and earnings growth

The dependent variable is the log return between year t and t+1 in column [1] and the discounted value of the cumulative return between year t and $t+3$ and $t+5$ in columns 2 and 3, respectively. The independent variables are: (1) log of earnings in year *t* relative to the cyclically-adjusted log earnings in year $t-5$ ($e_t - cae_{t-5}$), (2) the log price-dividend ratio (pd_t), and (3) the log price-earnings ratio (pe_t). We report results using monthly expectations data for the period 1981:12-2018:12. The last period with stock return data ranges from December of 2017 in column [1] to December 2013 in column [3]. We adjust standard errors for serial correlation using the Newey-West correction (number of lags ranges from 12 in the first column to 60 in the last one). Superscripts: [a] significant at the 1% level, [b] significant at the 5% level, [c] significant at the 10% level.

	(1)	(3)	(5)
	r_{t+1}	$\sum_{j=1}^{3} \alpha^{j-1} r_{t+j}$	$\sum_{j=1}^{5} \alpha^{j-1} r_{t+j}$
$e_t - cae_{t-5}$	-0.0997	-0.2757	-0.4226[a]
	(0.1071)	(0.1740)	(0.1518)
Adjusted R^2	1%	8%	20%
Observations	385	385	385
pd_t	-0.1508[a]	-0.3886[a]	-0.5934[a]
	(0.0504)	(0.1043)	(0.1183)
Adjusted R^2	13%	31%	49%
Observations	385	385	385
pe_t	-0.1030	-0.2002	-0.2197
	(0.0673)	(0.1279)	(0.1884)
Adjusted R^2	7%	9%	8%
Observations	385	385	385