Investment Hangover and the Great Recession*

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Abstract

We present a model of investment hangover motivated by the Great Recession. Overbuilding of durable capital such as housing requires a reallocation of productive resources to other sectors, which is facilitated by a reduction in the interest rate. If monetary policy is constrained, overbuilding induces a demand-driven recession with limited reallocation and low output. Investment in other capital initially declines due to low demand, but later booms and induces an asymmetric recovery in which the overbuilt sector is left behind. Welfare can be improved by ex-post policies that stimulate investment (including in overbuilt capital), and ex-ante policies that restrict investment.

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1 Introduction

After 2008, the US economy went through the worst macroeconomic slump since the Great Depression. Real GDP per capita declined from more than $49,000 in 2007 (in 2009 dollars) to less than $47,000 in 2009, and surpassed its pre-recession level only in 2013. The civilian employment ratio, which stood at about 63% in January 2008, fell below 59% by the end of 2009, and remained below 59.5% by the end of 2015.

Recent macroeconomic research emphasizes the boom bust cycle in house prices—and its effects on financial institutions, firms, and households—as the main culprit for these developments. The collapse of home prices arguably affected the economy through at least two principal channels. First, financial institutions that suffered losses related to the housing market cut back their lending to firms and households (Brunnermeier (2009), Ivashina and Scharfstein (2010)). Second, homeowners that had borrowed against rising collateral values during the boom faced tighter borrowing constraints and had to reduce their outstanding leverage (Eggertsson and Krugman (2012), Mian and Sufi (2014), Guerrieri and Lorenzoni (2016)). Both channels reduced aggregate demand, plunging the economy into a Keynesian recession. The recession was exacerbated by the zero lower bound on the nominal interest rate, also known as the liquidity trap, which restricted the ability of monetary policy to counter demand shocks (Hall (2011), Christiano, Eichenbaum, Trabandt (2014)).

A growing body of evidence shows that these views are at least partially correct: the financial and the household crises both appear to have played a part in the Great Recession.[1] But these views also face a challenge in explaining the nature of the recovery after the Great Recession. The recovery has been quite asymmetric across components of aggregate private spending. As the right panel of Figure 1 illustrates, nonresidential investment—measured as a fraction of output—almost reached its pre-recession level by 2015, while residential investment remained depressed. One explanation for this pattern is that households were unable to buy homes due to ongoing deleveraging. But the right panel of Figure 1 casts doubt on this explanation: sales of durables not directly related to housing such as cars—which should also be affected by household deleveraging—rebounded strongly while sales of new homes lagged behind. Another potential explanation is that the US residential investment generally lags behind in recoveries. This explanation is also incorrect: Leamer (2007)

[1] Several recent papers, such as Campello, Graham, and Harvey (2010) and Chodorow-Reich (2014), provide evidence that financial crisis affected firms' investment before 2010. Mian, Rao, Sufi (2013) and Mian and Sufi (2014, 2015) provide evidence that household deleveraging reduced household consumption and employment between 2007 and 2009.

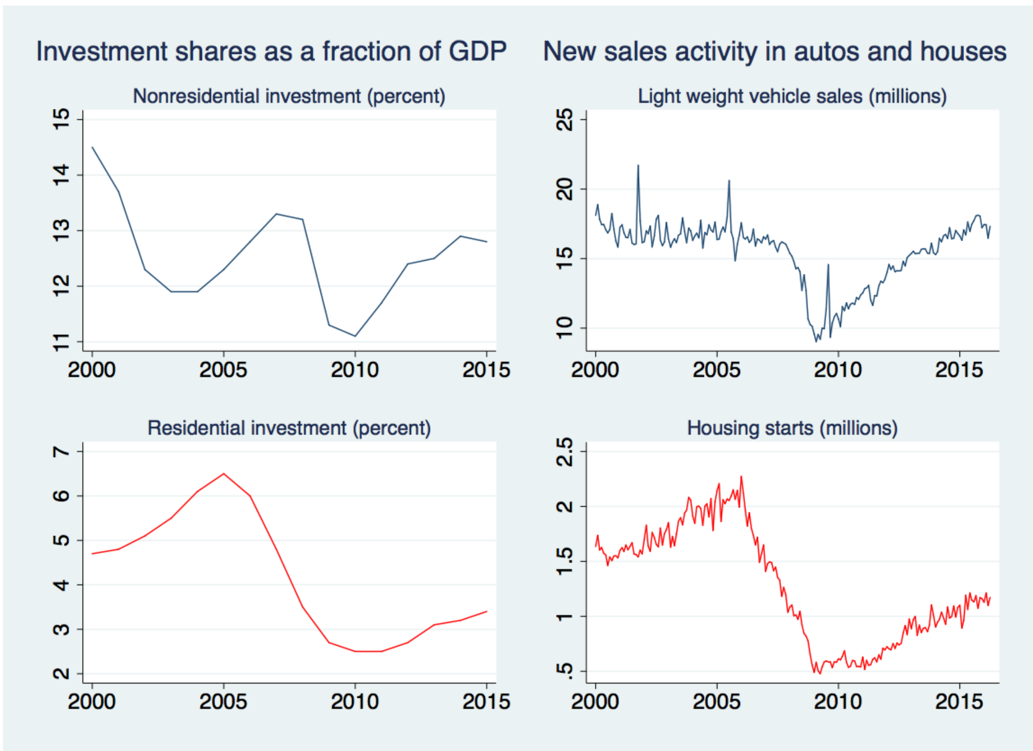

Figure 1: The left panels plot the two components of investment in the US as a share of GDP. The right panels plot new sales of autos and light trucks (top) and housing starts (bottom). Source: St. Louis Fed.

analyzes the post-war recessions in the US and shows that residential investment typically recovers before nonresidential investment and other consumer durables.

In this paper, we supplement the two accounts of the Great Recession with a third channel, which we refer to as the *investment hangover*. Our key hypothesis is that there was also an *investment boom* in the housing market in addition to the price boom, which lead to an overbuilding of housing capital by 2005. This hypothesis is supported by economic theory as well as empirical evidence. Standard investment theories (e.g., the Q theory) would suggest that an asset price boom driven by optimism about asset valuations should also be associated with an investment boom. Moreover, once the valuations are revised downwards, past investment would appear to be excessive in retrospect—which is what we refer to as overbuilding. Consistent with theory, Figure 1 illustrates a sharp increase in residential investment and housing starts before 2005. Since housing capital is highly durable, the housing capital was arguably overbuilt by the Great Recession.[2]

Motivated by our hypothesis, we use a stylized macroeconomic model to analyze how the economy behaves after overbuilding a durable type of capital—such as housing, structures, or infrastructure (e.g., railroads). Our model's first prediction is that investment in overbuilt capital declines. Intuitively, an excess of initial stock substitutes for new investment. Figure 2 provides evidence from the Great Recession consistent with this prediction. The sales of newly constructed homes, which have historically changed in proportion to the sales of existing homes, fell disproportionately starting around 2005.

This logic is reminiscent of the Austrian theory of the business cycle, in which recessions are times at which the economy liquidates the excess capital built during boom years (Hayek (1931)). The Hayekian view, however, faces a challenge in explaining how liquidating capital in one sector reduces aggregate output and employment. As noted by Krugman (1998), the economy has a natural adjustment mechanism that facilitates the reallocation of labor (and other productive resources) from the liquidating sector to other sectors. As economic activity in the liquidating sector declines, the interest rate falls and stimulates spending in other sectors, which keeps employment from falling. This reallocation process can be associated with some increase in frictional unemployment. But it is unclear in the Austrian theory how employment can fall in *both* the liquidating and the nonliquidating sectors, which

[2] See Haughwout et al. (2013) for an in-depth empirical analysis of the housing supply during this period. Using multiple methodologies and data sets, they estimate that around 3 million housing units were overbuilt by 2009.

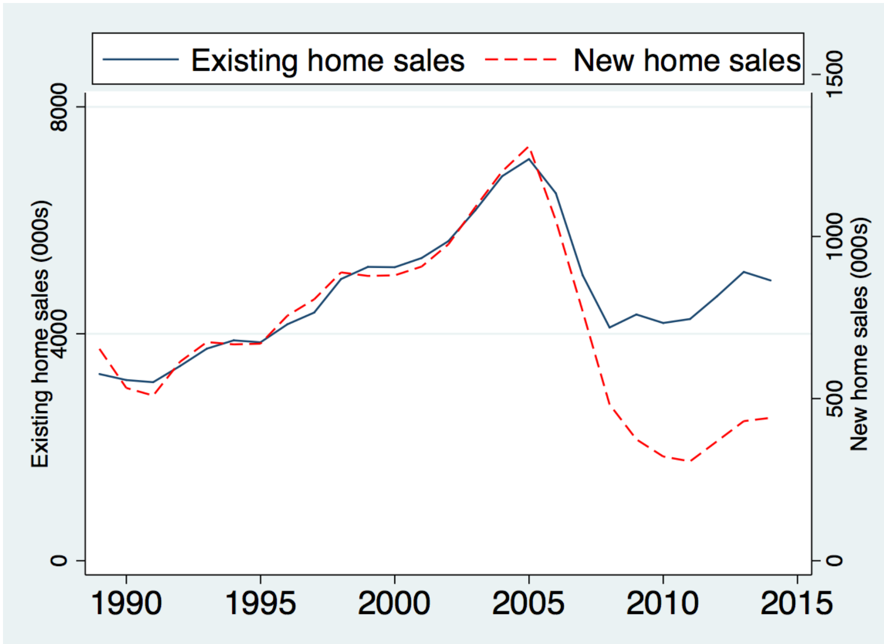

Figure 2: The top panel plots the homeownership rate in the US (source: US Bureau of the Census). The bottom panel plots the total sales of existing and new homes (source: National Association of Realtors).

seems to be the case for major recessions such as the Great Recession. To fit that evidence, an additional—Keynesian—aggregate demand mechanism is needed.

Accordingly, we depart from the Hayekian view by emphasizing that monetary policy plays a central role in the aggregate reallocation mechanism. If inflation cannot increase in the short run—an assumption that we maintain—then the real interest rate can fall and counter the demand shock only if the monetary policy lowers the nominal interest rate. In practice, many constraints on monetary policy might prevent this from happening. In the aftermath of the Great Recession, the monetary policy in developed economies was constrained by the zero lower bound on the nominal interest rate. In economies with fixed exchange rates (e.g., the Euro Zone), or exogenously determined money supply (e.g., the gold standard), the monetary policy is often constrained even if the nominal interest rate is above zero. We show that these types of constraints undermine the aggregate reallocation mechanism. If the initial overbuilding is sufficiently large, then the interest rate does not decline sufficiently and overbuilding induces a demand-driven recession.

Our model also shows how a slowdown in the overbuilt sector can naturally spill over to other sectors. The recession reduces the return to other types of capital—

such as equipment and machines—which were not necessarily overbuilt but which are used in the production of overbuilt capital. Thus, other types of investment can also decline, in line with the acceleration principle of investment (see, for instance, Samuelson (1939)), despite the low cost of capital implied by the low interest rate. As the economy decumulates the overbuilt capital, other investment gradually recovers in anticipation of a recovery in output. Through the lens of our model, then, the recession can be roughly divided into two phases. In the first phase, all types of investment decline, generating a severe and widespread slump. In the second phase, investment in overbuilt capital remains low but other investment increases, generating a partial recovery. In the context of the Great Recession, this implies that housing investment is left behind in the recovery, as in Figure 1.

We also investigate the implications of our analysis for policies directed towards regulating investment. A naive intuition would suggest that the planner should not stimulate investment in overbuilt capital, such as housing during the Great Recession, since the problems originate in this sector. We find that this intuition is incorrect: if the recession is sufficiently severe, then the planner optimally stimulates investment and slows down the decumulation of overbuilt capital. This result is driven by two observations. First, the planner recognizes that raising investment in a demand-driven recession stimulates aggregate demand and employment. In view of these *aggregate demand externalities*, the planner perceives a lower cost of building compared to the private sector. The lower cost, by itself, is not sufficient reason for intervention—the planner also considers the benefits. The second observation is that the return from investment in overbuilt capital need not be low—especially for long-lived capital such as housing or infrastructure. New investment will generate low flow utility in the short run but it will be useful in the future. Stimulating investment in overbuilt capital is beneficial because it helps to economize on future investment.

We also find that, before the economy enters the liquidation episode, it is optimal for the planner to reduce the accumulation of capital, so as to stimulate investment and aggregate demand during the recession. This result is also driven by aggregate demand externalities, and it holds as long as the agents in our model assign a positive probability to the recession (that is, the planner does not need to fully anticipate the recession to intervene). Our model also suggests that the intervention is more desirable for investment in more durable types of capital, because durability is the link by which past investment affects future economic activity. Taken together, our welfare analysis supports policies that intertemporally substitute investment towards periods that feature deficient demand, especially for long-lived capital.

Although our model is motivated by the Great Recession, it is more widely applicable and sheds some light on historical business cycles associated with overbuilding. A prime example is the American business cycle of 1879-1885, which is often linked with an investment cycle in railroads—a highly durable type of capital that constituted a sizeable fraction of aggregate investment at the time. Fels (1952) provides a narrative of the episode and describes many of the effects predicted by our model, including an asymmetric recovery in which the railroad sector is left behind. He writes:

> Construction of railroads was the principal factor in the upswing. The number of railroad miles built rose spectacularly from 2665 in 1878 to 11569 in 1882...The downswing gathered momentum slowly in 1883. The decline in railroad construction not only eliminated the jobs of many workers directly employed in railroad building but also spread depression to other industries. As one would expect from the theory of the acceleration principle, the iron and steel industry was particularly affected... Nevertheless, contraction gradually came to an end and gave way to weak revival in the course of 1885... But railroad-building by itself, which was to experience a great boom in 1886 and especially 1887, was at its lowest ebb at the time of the cyclical upturn. Thus, the upturn occurred not because of the behavior of railroad construction but in spite of it.

Fels (1952) also concludes that the episode can be best understood by combining Austrian and Keynesian business cycle theories of the time (he discusses Schumpeter's and Hicks' views as opposed to Hayek's and Keynes'). This suggests that the mechanisms we formalize in this paper have also been relevant for a different type of capital and under a different constraint on monetary policy (namely, the gold standard).

The rest of the paper is organized as follows. The next subsection discusses the related literature. Section 2 describes the equilibrium in the baseline model in which the economy decumulates the overbuilt capital in a single period. Section 3 characterizes this equilibrium, and presents our main result that excessive overbuilding induces a recession. Section 4 considers a variant of the baseline model (with adjustment costs) in which the decumulation is spread over multiple periods. This section investigates investment in other capital during the decumulation episode, and discusses the relationship of our model with the acceleration principle of investment. Section 5 analyzes the ex-post and ex-ante policy implications of our analysis using variants of the baseline model. Section 6 concludes. The online appendices A and B contain the omitted extensions of the baseline model as well as the omitted proofs.

1.1 Related literature

Our paper makes contributions to several strands of the literature. First, we identify the ex-ante overbuilding of housing as an important source of deficient aggregate demand during the Great Recession. A large literature emphasizes other types of demand shocks such as those driven by financial frictions or household deleveraging.[3] Other papers emphasize long-run factors that might have lowered demand more persistently (Summers (2013), Eggertsson and Mehrotra (2014), Caballero and Farhi (2014)). Our paper complements this literature and provides an explanation for why residential investment has lagged behind in the recovery.

Another strand of the literature investigates the role of housing during the Great Recession, but often focusing on channels other than overbuilding. Many papers, e.g., Iacoviello and Pavan (2013), emphasize the collateral channel by which house price shocks might have tightened household borrowing constraints. Boldrin et al. (2013) also emphasize overbuilding, but they do not analyze the resulting deficient demand problem. Instead, they focus on the supply-side input-output linkages by which the slowdown in construction spills over to other sectors.[4]

Second, and more broadly, we illustrate how overbuilding a durable type of capital can trigger a recession. As DeLong (1990) discusses, Hayekian (or liquidationist) views along these lines were quite popular before and during the Great Depression, but were relegated to the sidelines with the Keynesian revolution in macroeconomics. Our paper illustrates how Hayekian and Keynesian mechanisms can come together to generate a recession. The Hayekian mechanism finds another modern formulation in the recent literature on news-driven business cycles. A strand of this literature argues that positive news about future productivity can generate investment booms, occasionally followed by liquidations if the news is not realized (see Beaudry and Portier (2013) for a review). This literature typically generates business cycles without nominal rigidities (see, for instance, Beaudry and Portier (2004), Jaimovich and Rebelo (2009)), whereas we emphasize nominal rigidities and a demand side channel.

In recent and complementary work, Beaudry, Galizia, Portier (BGP, 2014) also investigate whether overbuilding can induce a recession driven by deficient demand.

[3]See also Gertler and Karadi (2011), Midrigan and Philippon (2011), Jermann and Quadrini (2012), He and Krishnamurthy (2014) for quantitative dynamic macroeconomic models that emphasize either banks', firms', or households' financial frictions during the Great Recession.

[4]There is also a large literature that develops quantitative business cycle models with housing, but without focusing on the Great Recession or overbuilding, e.g., Greenwood and Hercowitz (1991), Gervais (2002), Iacoviello (2005), Campbell and Hercowitz (2005), Davis and Heathcote (2005), Fisher (2007), Piazzesi, Schneider, and Tuzel (2007), Kiyotaki, Michaelides, and Nikolov (2011).

In BGP, aggregate demand affects employment due to a matching friction in the labor market, whereas we obtain demand effects through nominal rigidities. In addition, BGP show how overbuilding increases the (uninsurable) unemployment risk, which exacerbates the recession due to households' precautionary savings motive. We describe how overbuilding exacerbates the recession due to the endogenous response of other types of investment.[5]

Third, our analysis illustrates how a constrained monetary policy, e.g., due to the liquidity trap or exogenous money supply, restricts the efficient reallocation of resources between sectors. A large macroeconomics literature investigates the role of reallocation shocks relative to aggregate activity shocks in generating unemployment (see, for instance, Lilien (1982), Abraham and Katz (1986), Blanchard and Diamond (1989), Davis and Haltiwanger (1990)). Our paper shows that the constrained monetary policy blurs the line between reallocation and aggregate activity shocks. In our setting, reallocation away from investment in overbuilt capital triggers a Keynesian recession. Moreover, other types of investment also decline earlier in the recession, generating sectoral comovement that resembles an aggregate activity shock. Caballero and Hammour (1996) describe a supply-side channel by which reallocation is restricted because the expanding sectors are constrained due to a hold-up problem.

Fourth, we obtain several positive and normative results for investment when the economy experiences a demand-driven recession. These results apply regardless of whether the episode is driven by overbuilding or some other (temporary) demand shock.[6] On the positive side, we show that investment can decline earlier in the recession, even if the real interest rate remains low and there are no financial frictions, because low aggregate demand also lowers the return to investment. This mechanism is reminiscent of the acceleration principle of investment that was emphasized in an

[5] The literature on lumpy investment has also considered the possibility of a hangover (or conversely, pent-up demand), driven by past aggregate shocks that can shift the latent distribution of firms' investment imbalances (see Caballero, Engel, Haltiwanger (1995)). Thomas (2002) argued that the lumpiness, and the associated latent investment distribution, does not affect aggregate investment much once the cost of capital is endogenized. House (2014) clarified that this result is driven by the feature of standard neoclassical models—with or without lumpy investment—that the timing of investment is highly elastic with respect to the changes in cost of capital. However, most empirical evidence suggests that investment timing is not so elastic, especially over short and medium horizons (see Caballero (1999)). As House (2014) also notes, "the key property of the model which generates the irrelevance results—the infinite elasticity of investment demand—is a feature of the models and may not be a feature of reality."

[6] A growing theoretical literature investigates various aspects of the liquidity trap, but often abstracts away from investment for simplicity (see, for instance, Krugman (1998), Eggertsson and Woodford (2003), Auerbach and Obstfeld (2005), Adam and Billi (2006), Jeanne and Svennson (2007), Werning (2012)).

older literature (see Clark (1917) or Samuelson (1939)), but there are also important differences that we clarify in Section 4.1.[7] On the normative side, we show that the private investment decisions during or before the recession are typically inefficient, and characterize the constrained optimal interventions. These results complement a recent literature that analyzes the inefficiencies driven by aggregate demand externalities. Korinek and Simsek (2015) and Farhi and Werning (2013) focus on ex-ante financial market allocations, such as leverage and insurance, whereas we establish inefficiencies associated with physical investment.[8]

2 Baseline model

The economy is set in infinite discrete time $t \in \{0, 1, ...\}$ with a single consumption good, and three factors of production: a special type of durable capital, h_t, other capital, k_t, and labor, l_t. Our prime example for the special capital is housing, and thus, we refer to it also as the housing capital. Other examples are structures or infrastructure (e.g., railroads). For brevity, we also refer to nonhousing capital as "capital." Each unit of housing capital produces one unit of housing services. Capital and labor are combined to produce the consumption good according to a neoclassical technology that we describe below.

Absent shocks, the economy converges to a level of housing capital denoted by h^*, which we refer to as the target level (see Eq. (3) below). We analyze situations in which the economy starts with an initial housing capital that exceeds the target, $h_0 > h^*$, which we refer to as *overbuilding.*

The overbuilding assumption, $h_0 > h^*$, can be interpreted in several ways. Our favorite interpretation is that it captures an unmodeled overbuilding episode that took place before the start of our model. In particular, suppose the (expected) housing demand increased in the recent past relative to its historical level. The economy has built housing capital to accommodate this high level of demand, captured by h_0. At date 0, the economy receives news that that the high demand conditions are not sustainable. The stock of housing capital needs to adjust to its historical average, captured by h^*. Section 5.2 introduces an ex-ante period and formalizes this

[7] The mechanism is also present in many other New Keynesian models with capital and constrained monetary policy, but it is not always emphasized. Schmitt-Grohe and Uribe (2012) also show that the liquidity trap can generate an investment slump driven by low return.

[8] A separate literature emphasizes the inefficiencies in physical investment driven by pecuniary externalities (see, for instance, Lorenzoni (2008), Hart and Zingales (2011), Stein (2011), He and Kondor (2014), Davila (2015)).

interpretation.

An alternative and mathematically equivalent interpretation is that h_0 corresponds to the historical housing demand, whereas h^* represents "the new normal" with permanently low housing demand (e.g., due to a permanent change in household preferences, beliefs, or constraints in the aftermath of a housing crisis). One could also imagine intermediate interpretations in which the past overbuilding and the current low demand both play some role in driving the adjustment. We would like to understand how the economy adjusts to an excess stock of housing capital.

In our baseline model (until Section 4), we assume that one unit of the consumption good can be converted into one unit of housing or nonhousing capital, or vice versa, without adjustment costs. Thus, the two types of capital evolve according to,

$$h_{t+1} = h_t \left(1 - \delta^h\right) + i_t^h \text{ and } k_{t+1} = k_t \left(1 - \delta^k\right) + i_t^k. \tag{1}$$

Here, i_t^h (resp. i_t^k) denote housing (resp. nonhousing) investment, and δ^h (resp. δ^k) denotes the depreciation rate for housing (resp. nonhousing) capital. These assumptions also imply that the relative price of housing capital is constant and normalized to one. Hence, overbuilding in this setting will affect housing investment without changing house prices. This is only for simplicity. With adjustment costs, overbuilding would also affect (typically reduce) house prices but the effect on housing investment would be qualitatively unchanged.

Households The economy features a representative household whose problem can be written as,

$$\max_{\left\{l_t, \hat{c}_t, a_{t+1}, i_t^h\right\}_t} \sum_{t=0}^{\infty} \beta^t \left(u\left(\hat{c}_t - v\left(l_t\right)\right) + u^h \mathbf{1}\left[h_t \geq h^*\right]\right), \tag{2}$$

$$\text{s.t.} \quad \hat{c}_t + a_{t+1} + i_t^h = w_t l_t + \Pi_t + a_t\left(1 + r_t\right)$$

$$h_{t+1} = h_t\left(1 - \delta^h\right) + i_t^h$$

The household earns wage income per labor (w_t), and receives profits from firms that will be described below (Π_t). She chooses labor (l_t), consumption ($\hat{c}_t$), financial asset holdings (a_{t+1}) and housing investment to maximize discounted utility. The functions in her per-period utility, $u(\cdot), v(\cdot)$, satisfy the standard regularity conditions. She also receives utility from housing services as captured by the separable term, $u^h \mathbf{1}\left[h_t \geq h^*\right]$. This is equal to u^h if $h_t \geq h^*$ and zero otherwise, where we take u^h to be a large constant.

Our specification of household preferences in (2) relies on two simplifying assumptions. Note that households receive a large utility from investing up to h^* but zero marginal utility from additional units. This implies that when u^h is sufficiently large (and the interest rate is not too negative, $r_{t+1} > -\delta^h$) the household's optimal housing investment is,

$$h_{t+1} = h^*, \text{ which implies } i_t^h = h^* - h_t \left(1 - \delta^h\right). \tag{3}$$

Intuitively, households invest or disinvest so as to reach the target level of housing capital in a single period.[9] In particular, starting with some $h_0 > h^*$, the economy decumulates the excess residential capital in one period. Hence, a period in this setting should be thought of as long as necessary (several years) to adjust the housing capital to its steady-state level. In Section 4.1, we introduce downward adjustment costs on residential investment—more specifically, a lower bound on investment—so as to spread the decumulation over multiple periods.

The second simplification in household preferences is the functional form $u(\hat{c}_t - v(l_t))$, which implies that the household's labor supply decision does not depend on its consumption (see Greenwood, Hercowitz and Huffman (GHH, 1988)). Specifically, the optimal labor solves the static problem,

$$e_t = \max_{l_t} w_t l_t - v(l_t). \tag{4}$$

Here, e_t denotes households' labor income net of the disutility of labor. We also define $c_t = \hat{c}_t - v(l_t)$ as *net consumption*. In terms of the net variables, the household solves a standard consumption and savings problem,

$$\max_{\{c_t, a_{t+1}\}_t} \sum_{t=0}^{\infty} \beta^t u(c_t) \text{ s.t. } c_t + a_{t+1} + i_t^h = e_t + \Pi_t + a_t(1 + r_t). \tag{5}$$

The optimal household behavior is summarized by Eq. (3) and problems (4) and (5).

Remark 1 (Role of the Simplifying Preference Assumptions)**.** The GHH preferences do not play an important role beyond providing tractability. In Section 5, we show that our main results continue to apply in a version of the model with separable preferences at date 0, $u(c_0) - v_0(l_0)$. The kinked demand for housing services plays

[9] Housing capital not only provides housing services but it also represents an investment technology. Hence, Eq. (3) also requires the gross interest rate, $1 + r_{t+1}$, to be greater than the gross return (on unutilized houses), $1 - \delta^h$. This is ensured by the required condition, $r_{t+1} > -\delta^h$, which will be the case in equilibrium.

a more important role. First, this assumption considerably simplifies the housing investment part of our model, as illustrated by Eq. (3), and allows us to focus on the effect of overbuilding on the remaining equilibrium allocations. Second, the assumption also implies that housing investment in the short run does not react to the changes in the interest rate. With more elastic housing demand, housing investment would qualitatively follow a similar dynamic path as in our model (described in Section 3), but its initial decline would be dampened as monetary policy responds by lowering the interest rate. These additional effects are not a major concern for our analysis, because we will focus on scenarios in which the monetary policy is constrained.

Investment firms, production firms, and the constrained interest rate The capital stock of the economy is managed by a competitive investment sector. This sector invests up to the point at which the cost of capital is equated to the net return to physical capital,

$$r_{t+1} = R_{t+1} - \delta^k. \tag{6}$$

Here, R_{t+1} denotes the rental rate. The cost of capital is the same as the safe interest rate since there is no uncertainty (see Remark 3 below for a discussion of how introducing a risk premium would affect the analysis).The capital market clearing condition is given by $a_t = k_t$.

Our key ingredient is that the monetary policy is constrained so that the nominal interest rate does not react to demand shocks as much as in a real business cycle model. In practice, there might be several reasons why the monetary authority might be unable or unwilling to lower the interest rate sufficiently to counter demand shocks. In our baseline analysis, we consider the zero lower bound constraint on the nominal interest rate, $r^n_{t+1} \geq 0$, which appeared to be relevant during the Great Recession (see Appendix A.2 for an alternative setting with exogenous money supply). The zero lower bound constraint emerges because cash in circulation yields zero interest in addition to providing households with transaction services.[10] If the nominal interest rate fell below zero, then individuals would switch to hoarding cash instead of holding financial assets. Therefore, monetary policy cannot lower the nominal interest rate (much) below zero. The situation in which the nominal interest rate is at its lower bound is known as the liquidity trap.

Constraints on the nominal interest rate might not affect the real allocations by

[10] To simplify the notation and the exposition, however, we do not explicitly model money or its transaction services in the main text. Appendix A.2 analyzes a version of the model with these features.

itself. However, we also assume that nominal prices are sticky so that a constraint on the nominal rate translates into a constraint on the real rate. For analytical tractability, we assume prices are completely sticky (see Remark 2 below for an interpretation and a discussion of how the results would generalize). This ensures that the real interest rate is also bounded from below,

$$r_{t+1}^{n} = r_{t+1} \geq 0 \text{ for each } t. \tag{7}$$

As we will see, the lower bound on the interest rate will play a central role by creating an upper bound on investment as well as consumption.

We formally introduce nominal price rigidities with the standard New Keynesian model. Specifically, there are also two types of production firms. A competitive final good sector uses intermediate varieties $\nu \in [0,1]$ to produce the final output according to the Dixit-Stiglitz technology, $\hat{y}_t = \left(\int_0^1 \hat{y}_t(\nu)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)}$ where $\varepsilon > 1$. In turn, a unit mass of monopolistic firms labeled by $\nu \in [0,1]$ each produces the variety according to, $\hat{y}_t(\nu) = F(k_t(\nu), l_t(\nu))$, where $F(\cdot)$ is a neoclassical production function with standard regularity conditions.

Each monopolist has a preset and constant nominal price, $P_t(\nu) = P$ for each ν. This assumption implies that the aggregate price level is also constant, thereby formalizing the bound in (7).[11] The assumption also implies that monopolists are symmetric: they face the same real price (equal to one) and they choose the same level of inputs and outputs subject to an aggregate demand constraint. In particular, the representative monopolist's problem can be written as:

$$\Pi_t = \max_{k_t, l_t} F(k_t, l_t) - w_t l_t - R_t k_t \text{ s.t. } F(k_t, l_t) \leq \hat{y}_t. \tag{8}$$

Remark 2 (Role of the Price Stickiness Assumption)**.** The extreme price stickiness captures in reduced form a situation in which inflation is sticky in the *upward* direction during the decumulation episode. In practice, this type of stickiness could be driven by nominal rigidities at the micro level (in the goods market or the labor market), or due to constraints on monetary policy against creating inflation. It is also worth emphasizing that making the prices more flexible at the micro level does not necessarily circumvent the bound in (7). In fact, if the monetary policy can-

[11] Alternatively, we could also assume that firms set their prices at every period mechanically according to a predetermined inflation target, that is, $P_t(v) = P\Pi^t$ for some $\Pi \geq 1$. This formulation yields a very similar bound as in (7) and results in the same economic trade-offs. We normalize the inflation target to zero (or $\Pi = 1$) so as to economize on notation.

not credibly commit to creating inflation (e.g., due to an inflation targeting policy regime), then limited price flexibility exacerbates the bound in (7). This is because the negative output gap during the zero lower bound episode exerts a downward pressure on inflation. As the inflation (and expected inflation falls), the real interest rate increases and the demand shortage becomes more severe (see Remarks 1-3 in Korinek and Simsek (2015) for further discussion).

Remark 3 (Absence of a Risk Premium)**.** In practice, the cost of capital for investment projects would also feature a risk premium, which would appear as a second term on the left side of Eq. (6). For analytical tractability, we abstract away from risk and work with a deterministic model. Introducing risk premium into the model would leave our analysis qualitatively unchanged. The lower bound in (7) would apply to the safe interest rate, but it would constrain the cost of capital for all projects—including the risky projects. This is because the cost of capital is the sum of the safe interest rate and the risk premium, and the risk premium is largely independent of the level of the safe interest rate (as it is determined by factors such as uncertainty or risk aversion).

Efficient benchmark and the monetary policy In the equilibria we will analyze, the monopolists that solve problem (8) will find it optimal to meet all of its demand. Thus, the output satisfies $\hat{y}_t = F(k_t, l_t)$. In view of GHH preferences, we find it more convenient to work with *the net output*, that is, output net of labor costs, $y_t = \hat{y}_t - v(l_t)$. The net output is determined by the net aggregate demand, $y_t = c_t + i_t^k + i_t^h$. The net aggregate demand is in turn determined by monetary policy as well as other factors.

To describe monetary policy, we first define the efficient benchmark at some date t as the continuation allocation that maximizes households' welfare subject to the feasibility constraints (and given the state variables, $k_t, h_t \geq h^*$). Appendix B.1 shows that the efficient benchmark solves a standard neoclassical planning problem. In view of the GHH preferences, the efficient employment maximizes the net output in every period. We define

$$L(k_t) \in \arg\max_{\tilde{l}} F\left(k_t, \tilde{l}\right) - v\left(\tilde{l}\right) \text{ and } S(k_t) = F(k_t, L(k_t)) - v(L(k_t)) \qquad (9)$$

as respectively the efficient (or the supply-determined) level of the labor supply and net output.

Since the price level is fixed, we assume that the monetary policy focuses on stabilizing employment and output (analogous to a Taylor rule). In our setting, this corresponds to replicating the (statically) efficient allocations in (9) subject to the constraint in (7).[12] Specifically, the monetary policy follows the interest rate rule,

$$r_{t+1}^n = r_{t+1} = \max\left(0, r_{t+1}^*\right) \text{ for each } t \geq 0. \tag{10}$$

Here, r_{t+1}^* is recursively defined as the natural interest rate that obtains when the employment and the net output at date t are given by (9) and the monetary policy follows the rule in (10) at all future dates. This policy is constrained efficient in our environment as long as the monetary policy cannot use forward guidance: that is, it cannot commit to setting future interest rates beyond the current period.

Definition 1. *The equilibrium is a path of allocations,* $\left\{h_t, k_t, l_t, \hat{c}_t, c_t, i_t^h, i_t^k, \hat{y}_t, y_t\right\}_t$*, and real prices and profits,* $\{w_t, R_t, r_{t+1}, \Pi_t\}_t$*, such that the households and firms choose allocations optimally as described above, the interest rate is set according to* (10)*, and all markets clear.*

3 Investment hangover

We next characterize the equilibrium and establish our main result that excessive overbuilding triggers a demand-driven recession. We then analyze the comparative statics of the recession and discuss several extensions.

We start by establishing the properties of equilibrium within a period.

Lemma 1. *(i) If* $r_{t+1} > 0$*, then* $y_t = S(k_t), l_t = L(k_t)$*, and* $R_t = S'(k_t)$*.*

(ii) If $r_{t+1} = 0$*, then the net output is below the efficient level,* $y_t \leq S(k_t)$*, and is determined by net aggregate demand,* $y_t = c_t + i_t^k + i_t^h$*. The labor supply is below its efficient level,* $l_t \leq L(k_t)$*, and is determined as the unique solution to,*

$$y_t = F(k_t, l_t) - v(l_t) \text{ over the range } l_t \in [0, L(k_t)]. \tag{11}$$

The rental rate of capital is given by $R_t = R(k_t, y_t) \leq S'(k_t)$*, where the function* $R(k_t, y_t)$ *is strictly decreasing in* k_t *and strictly increasing in* y_t*.*

[12] In our setting, the equilibrium without nominal rigidities would also feature monopoly distortions, which should ideally be corrected by targeted policies such as monopoly subsidies. To simplify the notation, we ignore this distinction and assume the monetary policy attempts to correct for monopoly distortions as well as the distortions due to nominal rigidities.

Part (i) describes the case in which the interest rate is positive and the monetary policy replicates the efficient outcomes. Part (ii) describes the case in which the monetary policy is constrained by the zero lower bound. In this case, the economy experiences a recession with low net output and employment.

Lemma 1 also characterizes the rental rate of capital in each case, which determines the return to investment. To understand these results, consider monopolists' factor demands, captured by the optimality conditions for problem (8),

$$(1-\tau_t)\, F_k(k_t, l_t) = R_t \text{ and } (1-\tau_t)\, F_l(k_t, l_t) = w_t. \tag{12}$$

Here, $\tau_t \geq 0$ denotes the Lagrange multiplier on the demand constraint in (8), which is also *the labor wedge* in this model. If the interest rate is positive, then employment is at its efficient level and the labor wedge is zero, $\tau_t = 0$. In this case, the factors earn their marginal products. If instead the interest rate is zero, then the employment is below its efficient level and the labor wedge is positive, $\tau_t > 0$. In this case, the demand shortage lowers capital's (as well as labor's) rental rate relative to the efficient benchmark. The second part of the lemma shows further that the return to capital in this case can be written as a function of the capital stock and net output. Greater k_t reduces the rental rate due to diminishing returns, whereas greater y_t increases it due to greater demand. These effects will play a central role in our analysis of the investment response in Section 4.

Lemma 1 implies further that the capital stock is bounded from above,

$$k_{t+1} \leq \overline{k} \text{ for each } t\text{, where } S'\left(\overline{k}\right) - \delta^k = 0. \tag{13}$$

Here, the upper bound $\overline{k}$ is the level of capital that delivers a net return of zero absent a demand shortage. Investing beyond this level would never be profitable given the lower bound to the cost of capital implied by (7), as well as the possibility of a demand shortage. Intuitively, there are only so many projects that can be undertaken without violating the lower bound on the interest rate (this would be the case even if the cost of capital included a risk premium—see Remark 3).

Dynamic equilibrium with investment overhang We next characterize the dynamic equilibrium under the assumption that the economy starts with too much housing capital,

$$h_0 = (1 + b_0)\, h^*\text{, where } b_0 > 0. \tag{14}$$

Here, b_0 parameterizes the degree of past overbuilding. Eq. (3) then implies,

$$i_0^h = h^* - \left(1 - \delta^h\right) h_0 = \left(\delta^h - b_0 \left(1 - \delta^h\right)\right) h^*. \tag{15}$$

Note that housing investment at date 0 is below the level required to maintain the target, $i_0^h < \delta^h h^*$. Thus, overbuilding represents a negative shock to housing investment. The equilibrium depends on how the remaining components of aggregate demand, i_0^k and c_0, respond to this shock.

To characterize this response, we solve the equilibrium backwards. Suppose the economy reaches date 1 with $h_1 = h^*$ and some capital level $k_1 \leq \overline{k}$. Since the housing capital has already adjusted, the zero lower bound does not bind in the continuation equilibrium, that is, $r_{t+1} > 0$ for each $t \geq 1$. Consequently, monetary policy replicates the efficient benchmark starting date 1 given $h_1 = \overline{h}$ and $k_1 \leq \overline{k}$. Appendix B.1 shows that the solution converges to a steady-state (k^*, l^*, y^*, c^*), characterized by

$$\begin{aligned} S'(k^*) - \delta^k &= 1/\beta - 1, \text{ and} \\ l^* &= L(k^*), y^* = S(k^*), c^* = S(k^*) - \delta^k k^* - \delta^h h^*. \end{aligned} \tag{16}$$

The initial consumption is given by $c_1 = C(k_1)$, for an increasing function $C(\cdot)$.

Next consider the equilibrium at date 0. The key observation is that aggregate demand is bounded from above due to the constraint on the interest rate. We have already seen in Eq. (13) that capital is bounded from above, which implies that nonhousing investment is bounded,

$$i_1^k \leq \overline{k} - \left(1 - \delta^k\right) k_0.$$

Likewise, consumption is bounded by the Euler equation at the zero interest rate,

$$c_0 \leq \overline{c}_0, \text{ where } u'(\overline{c}_0) = \beta u'\left(C\left(\overline{k}\right)\right). \tag{17}$$

Combining the bounds in (13) and (17) with the demand shock in (15), the aggregate demand (and output) at date 0 is also bounded,

$$y_0 \leq \overline{y}_0 \equiv \overline{k} - \left(1 - \delta^k\right) k_0 + \overline{c}_0 + \left(\delta^h - b_0 \left(1 - \delta^h\right)\right) h^*. \tag{18}$$

The equilibrium depends on the comparison between the maximum demand and the efficient level, i.e., whether $\overline{y}_0 < S(k_0)$. This in turn depends on whether the amount of overbuilding b_0 exceeds a threshold level,

$$\bar{b}_0 \equiv \frac{\bar{k} - \left(1-\delta^k\right) k_0 + \bar{c}_0 + \delta^h h^* - S\left(k_0\right)}{\left(1-\delta^h\right) h^*}. \tag{19}$$

Proposition 1 (Overbuilding and the Demand-driven Recession)**.** *Suppose* $b_0 > 0$.

(i) If $b_0 \leq \bar{b}_0$*, then, the date* 0 *equilibrium features,*

$$r_1 \geq 0, y_0 = S\left(k_0\right) \text{ and } l_0 = L\left(k_0\right).$$

(ii) If $b_0 > \bar{b}_0$*, then, the date* 0 *equilibrium features a demand-driven recession with,*

$$r_1 = 0, k_1 = \bar{k}, y_0 = \bar{y}_0 < S\left(k_0\right) \text{ and } l_0 < L\left(k_0\right).$$

Moreover, the net output y_0 *and the labor supply* l_0 *are decreasing in* b_0.

In either case, the continuation allocations starting at date 1 *feature positive interest rates and solve a neoclassical planning problem. The economy converges to a steady state* (k^*, l^*, y^*, c^*) *given by* (16).

Part (i) describes the equilibrium for the case in which the initial overbuilding is not too large. In this case, the economy does not experience a demand-driven recession. Low investment in housing capital is offset by a reduction in the interest rate and an increase in nonhousing investment and consumption, leaving the output and employment determined by productivity. The left part of the panels in Figure 3 (the range corresponding to $b_0 \leq \bar{b}_0$) illustrate this outcome.

Part (ii), our main result, characterizes the case in which the initial overbuilding is sufficiently large. In this case, the reduction in aggregate demand due to low housing investment is large enough to plunge the economy into a demand-driven recession. The lower bound on the interest rate prevents the nonhousing investment and consumption from expanding sufficiently to pick up the slack, which leads to low output and employment. Figure 3 illustrates this result. Greater overbuilding triggers a deeper recession.

Comparative statics of the recession When is a given amount of overbuilding, b_0, more likely to trigger a demand-driven recession? As illustrated by Eq. (19), factors that reduce aggregate demand at date 0, such as a higher discount factor β (that lowers $\bar{c}_0$), increase the incidence of the demand-driven recession in our setting. More generally, other frictions that reduce aggregate demand during the decumulation phase, such as household deleveraging or the financial crisis, are also complementary to our mechanism. Intuitively, this is because the zero lower bound represents a

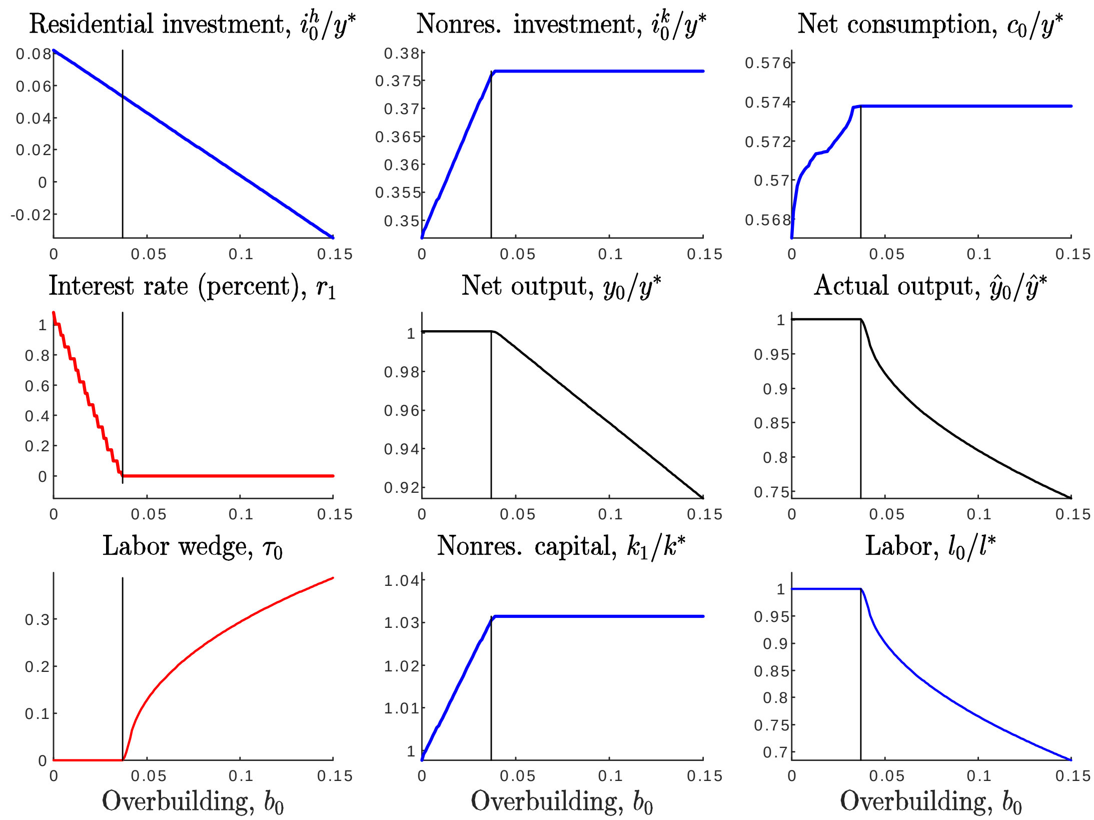

Figure 3: Date 0 equilibrium variables as a function of the initial overbuilding b_0 (measured as a fraction of the steady-state residential capital stock, h^*).

nonlinear constraint on monetary policy. A demand shock that lowers the interest rate leaves a smaller slack for monetary policy, increasing the potency of other demand shocks such as overbuilding.

Eq. (19) illustrates further that overbuilding of the two types of capital is complementary in terms of triggering a demand-driven recession: that is, greater initial level of nonhousing capital stock k_0 increases the incidence of a demand-driven recession. A high level of k_0 affects the equilibrium at date 0 through two channels. First, it reduces nonhousing investment at date 0 and lowers aggregate demand—similar to a high level of h_0. Second, it also increases the efficient output, $S(k_0)$, and makes a demand shortage more likely.

A distinguishing feature of housing capital is its high durability relative to many other types of capital. In the appendix, we consider a slight variant of the model to investigate whether high durability is conducive to triggering a demand-driven recession. The extension features two types of housing capital, one more durable (i.e., depreciates more slowly) than the other. The analysis reveals that, controlling for the total amount of overbuilding in both types of capital, overbuilding durable capital is more likely to trigger a demand-driven recession. Intuitively, depreciation helps to "erase" the overbuilt capital, reducing the impact of past overbuilding on future aggregate demand. Since durable capital depreciates more slowly, once overbuilt, it tends to stay around for longer and reduce aggregate demand by a larger amount.

This observation suggests that overbuilding is a greater concern when it hits durable capital such as housing, structures, or infrastructure (e.g., railroads), as opposed to less durable capital such as equipment or machinery. A previous literature has empirically investigated whether the overbuilding of information technology (IT) equipment during the boom years of late 1990s and 2000 might have contributed to the 2001 recession in the US (see Desai and Goolsbee (2004) and the references therein). Note, however, that the IT equipment such as computers tend to depreciate very quickly. A more fruitful research direction could be to empirically investigate episodes that feature overbuilding of more durable capital.

Other constraints on monetary policy For concreteness, we focus on the zero lower bound constraint on monetary policy. While the ZLB is relevant for the Great Recession, monetary policy can also be constrained for many other reasons. In economies with fixed exchange rates (such as the Euro Zone), the monetary policy is often constrained because it is linked with the policies of other countries. In economies under the gold or silver standard, which has been historically common, the

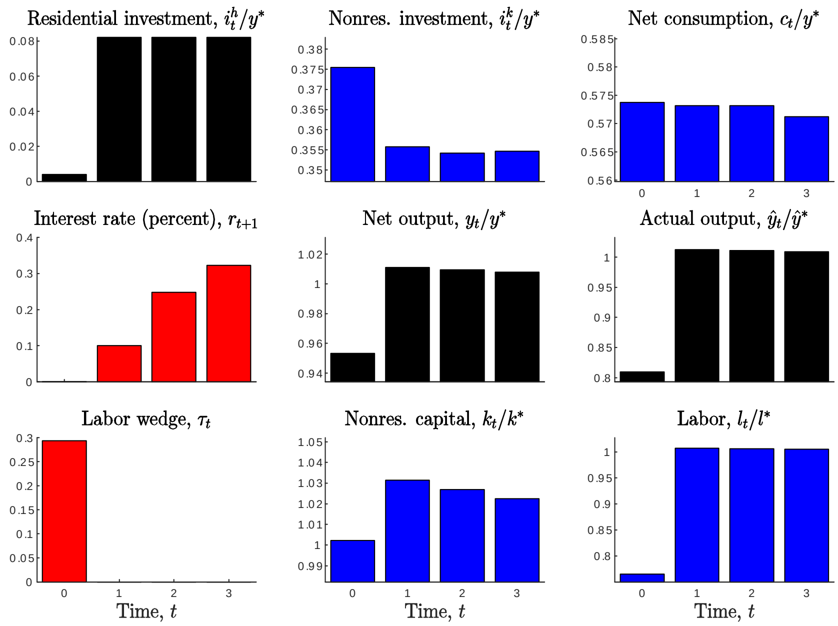

Figure 4: The evolution of equilibrium variables over time, starting with $b_0 > \bar{b}_0$.

monetary policy is constrained since the money supply is linked to the local quantity of precious metals. As it should be clear from our analysis, our main result (as well as our results in subsequent sections) continues to apply when the monetary policy is constrained for other reasons than the zero lower bound.

Appendix A.2 illustrates this point by deriving a version of Proposition 1 in an environment in which the money supply follows an exogenous path. In this setting, the interest rate is determined by the money supply and the household liquidity preferences. Since these forces are largely exogenous, the interest rate does not decline sufficiently to meet the decline in aggregate demand. Consequently, excessive overbuilding triggers a demand-driven recession as in our baseline analysis. In fact, the recession is often more severe because, while the interest rate during the recession declines to zero in the baseline model, it can remain above zero when the money supply is exogenous. Hence, our investment hangover mechanism is widely applicable and it can shed some light on some historical business cycles other than the Great Recession (see the introduction for a discussion of the American business cycle of 1879-1885).

Aftermath of the slump How does the economy behave in the aftermath of a demand-driven slump associated with overbuilding? Figure 4 plots the full dynamic

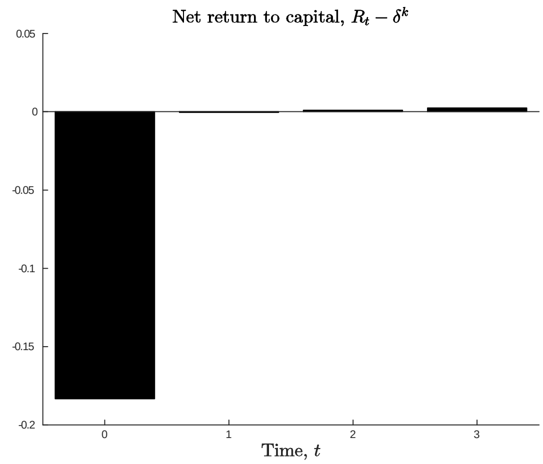

Figure 5: The evolution of net return to capital over time, starting with $b_0 > \bar{b}_0$.

equilibrium that features a demand-driven recession (characterized in Proposition 1). Starting date 1 onwards, the economy has fully recovered from the recession and experiences a neoclassical adjustment to the steady state. The interest rate gradually increases, and it might remain below its steady-state level for several periods. This is because the economy accumulates capital at date 0 thanks to the low cost of capital. The economy decumulates this capital only gradually over time, which leaves the interest rate low even after the recovery. These low rates are reminiscent of the secular stagnation hypothesis, recently revived by Summers (2013). According to this hypothesis, the economy could permanently remain depressed with low interest rates due to a chronic demand shortage (see Eggertsson and Mehrotra (2014) for a formalization). In our model, the economy eventually recovers, but it remains fragile to another demand shock after the recovery in view of the low rates. Intuitively, the economy has used much of its investment capacity to fight the demand shock at date 0, which leaves little capacity to fight another demand shock going forward.

Figure 4 illustrates further that, while there is a recession at date 0, several components of aggregate demand—especially nonhousing investment—actually expand. The recession is confined to the overbuilt sector. This prediction is inconsistent with facts in major recessions, such as the Great Recession, in which most components of aggregate demand decline simultaneously. To address this puzzle, we next analyze the investment response in more detail.

4 Investment response and the accelerator

This section investigates a variant of the model in which the demand-driven slump persists over multiple periods. We show how the decumulation of overbuilt capital can induce an initial bust in investment in other types of capital followed by a boom. We also discuss the relationship of our model to the acceleration principle of investment. We finally briefly discuss how our model can be further extended to generate an initial reduction in consumption.

The analysis is motivated by Figure 5, which illustrates the evolution of the net return to capital $R_t - \delta^k$ corresponding to the equilibrium plotted in Figure 4. The near-zero return during the recovery phase reflects the high level of accumulated capital. The figure illustrates that the net return at date 0, given the predetermined capital stock k_0, is even lower. Intuitively, the recession at date 0 lowers not only the output but also factor returns (see Lemma 1). This suggests that, if nonhousing investment could respond to the shock during period 0, it could also fall.

To investigate this possibility, we modify the model so that the decumulation of housing capital is spread over many periods. One way to ensure this is to assume that there is a lower bound on housing investment at every period.

Assumption 1. The household problem (5) features an additional constraint, $i_t^h \geq i^h$ for each t, for some $i^h < \delta^h h^*$.

For instance, the special case $i^h = 0$ captures the irreversibility of housing investment. More generally, the lower bound provides a tractable model of downward adjustment costs. To simplify the exposition, we also assume that the initial housing capital, $h_0 = h^* (1 + b_0)$, is such that the economy adjusts to the target level in exactly $T \geq 1$ periods.

Assumption 2. $\delta^h h^* = \left(\delta^h h_0\right) \left(1 - \delta^h\right)^T + i^h \left(1 - \left(1 - \delta^h\right)^T\right)$ for an integer $T \geq 1$.

With these assumptions, the optimal housing investment path is given by

$$i_t^h = \begin{cases} i^h < \delta^h h^* & \text{if } t \in \{0, ..., T-1\} \\ \delta^h h^* & \text{if } t \geq T \end{cases}, \tag{20}$$

For future reference, note that the parameter i^h also provides an (inverse) measure of the severity of the housing investment shock.

We characterize the equilibrium backwards. The economy reaches date T with housing capital $h_T = h^*$ and some $k_T \leq \bar{k}$. As before, the continuation equilibrium is neoclassical. In particular, consumption is given by $c_T = C(k_T)$, where recall that

$C(\cdot)$ is an increasing function.

Next consider the equilibrium during the decumulation phase, $t \in \{0,..,T-1\}$. We conjecture that—under appropriate assumptions—there is an equilibrium that features a demand-driven recession at all of these dates, that is, $r_{t+1} = 0$ for each $t \in \{0,..,T-1\}$. In this equilibrium, the economy reaches date T with the maximum level of capital, $k_T = \overline{k}$ (since the interest rate in the last period is zero, $r_T = 0$). Consumption is also equal to its maximum level, that is, $c_t = \overline{c}_t$ for each t, where

$$u'(\overline{c}_t) = \beta u'(\overline{c}_{t+1}) \text{ for each } t \in \{0,1,..,T-1\}.$$

It remains to characterize the path of the capital stock $\{k_t\}_{t=1}^{T-1}$ during the decumulation phase.

To this end, consider the optimal investment decision at some date $t-1$, which determines the capital stock at date t. The net return from this investment is given by $R(k_t, y_t) - \delta^k$ (cf. Lemma 1). The net cost of investment is given by $r_t = 0$. The economy invests up to the point at which the benefits and costs are equated (see (6)), which implies,

$$R(k_t, y_t) - \delta^k = 0 \text{ for each } t \in \{1,..,T-1\}. \tag{21}$$

Recall that the return function $R(\cdot)$ is decreasing in the capital stock k_t and increasing in net output y_t. Hence, Eq. (21) says that, if the (expected) output at date t is large, then the economy invests more at date $t-1$ and obtains a greater capital stock at date t. Intuitively, higher output in a period is associated with higher investment in the previous period.

Note also that the output at date t is determined by aggregate demand,

$$y_t = \overline{c}_t + k_{t+1} - \left(1-\delta^k\right) k_t + i^h \text{ for each } t \in \{0,...,T-1\}. \tag{22}$$

Intuitively, higher investment (as well as consumption) in a period induces higher output in the same period. Eqs. (21) and (22) represent a difference equation that can be solved backwards starting with $k_T = \overline{k}$. The resulting path corresponds to an equilibrium as long as $S(k_0) > y_0$, so that there is a recession in the first period as we have conjectured. The next result establishes that this is the case if the shock is sufficiently severe, as captured by low i^h, and characterizes the behavior of nonhousing capital in equilibrium.

Proposition 2 (Nonresidential Investment Response)**.** *Consider the model with the adjustment length $T \geq 2$. Suppose Assumptions 1-2 and Assumption 3 in Appendix*

B hold.[13]

(i) There exists $i^{h,1}$ such that if $i^h < i^{h,1}$, then there is a unique equilibrium path $\{k_t, y_{t-1}\}_{t=1}^T$, which solves Eqs. (21) − (22) *along with $k_T = \bar{k}$. The equilibrium features a demand-driven recession at each date $t \in \{0, .., T-1\}$ with $r_{t+1} = 0$ and $y_t < S(k_t)$.*

(ii) There exists $i^{h,2} \leq i^{h,1}$ such that, if $i^h < i^{h,2}$, then the nonhousing capital declines at date 1*, and then increases before date T:*

$$k_0 > k_1 \text{ and } k_1 < k_T = \bar{k}.$$

The main result of this section is the second part, which establishes conditions under which the nonhousing capital (and investment) follows a non-monotone path during the recession: falling initially, but eventually increasing.

To understand the drop in investment, note that a negative shock to housing investment reduces aggregate demand and output. This in turn lowers nonhousing investment as captured by the break-even condition (21). When the shock is sufficiently severe, the aggregate demand at date 1 is sufficiently low that capital declines. Intuitively, the economy is optimally responding to the low return to capital depicted in Figure 5.

In later periods, aggregate demand and output gradually increase in anticipation of the eventual recovery. As this happens, the low cost of capital becomes the dominant factor for nonhousing investment. Consequently, the economy starts reaccumulating capital, and in fact—exits the slump with the maximum level of capital $\bar{k}$ as in the earlier model.

Figure 6 illustrates the dynamic evolution of the equilibrium variables for the case $T = 2$. The parameters are chosen so that the figure can be compared to Figure 4 after replacing a single period with two periods. The lower panels on the left illustrate the non-monotonic response of capital and investment identified in Proposition 2. The figure illustrates that the recession can be roughly divided into two phases. In the first phase, captured by date 0, both types of investment fall. This induces a particularly severe recession with low output and employment. In the second phase, captured by date 1 in the figure (and dates $t \in \{1, .., T-1\}$ more generally), housing investment

[13] If the condition $i^h < i_1^h$ is violated, then there is an alternative equilibrium in which there is a partial liquidity trap at dates $t \in \{T_b - 1, .., T-1\}$ for some $T_b \geq 2$. We omit the characterization of these equilibria for brevity.

Assumption 3 is a regularity condition on shocks and parameters that ensures an interior liquidity trap equilibrium at date 0 with positive output. This assumption is satisfied for all of our numerical simulations and is relegated to the appendix.

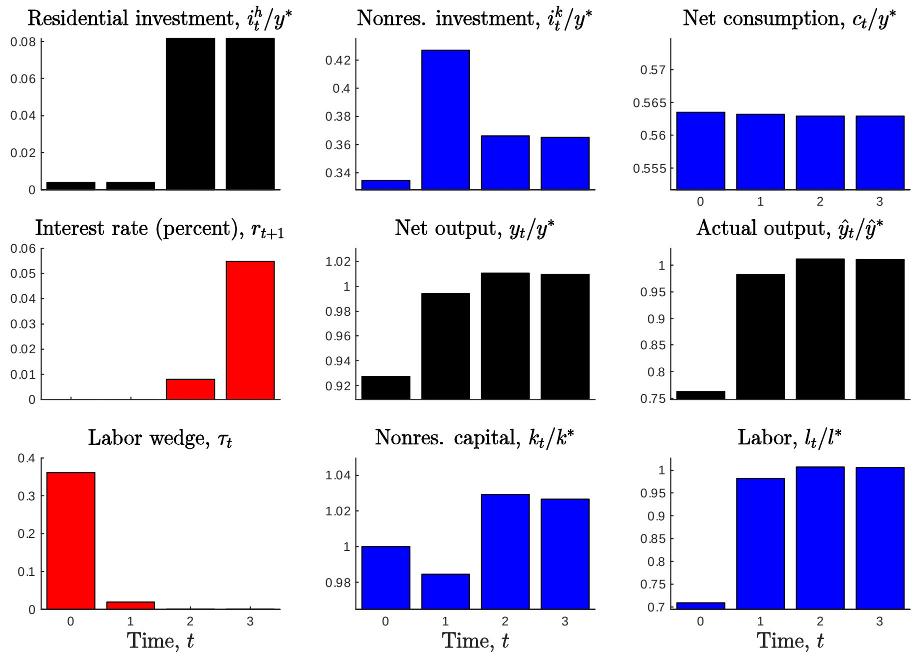

Figure 6: The evolution of equilibrium variables over time, given the length of decumulation $T = 2$.

remains low whereas the nonhousing investment gradually recovers and eventually booms. The investment response also raises aggregate demand. Hence, the second phase of the recession in our model represents a partial and asymmetric recovery in which the housing sector is left behind, similar to the aftermath of the Great Recession (see Figure 1).

4.1 Comparison with the acceleration principle

Our analysis of nonhousing investment bears a certain resemblance with the accelerator theory of investment (see Clark (1917)). To illustrate the similarities, let us linearize Eq. (21) around $(k, y) \simeq \left(\overline{k}, S\left(\overline{k}\right)\right)$, to obtain the approximation

$$k_t \simeq \alpha + \beta E_{t-1}\left[y_t\right] \text{ for each } t \in \{1, .., T-1\},$$

where $\beta = -R_y/R_k > 0, \alpha = \overline{k} - \beta S\left(\overline{k}\right)$, and $E_{t-1}\left[y_t\right] = y_t$. We introduce the (redundant) expectations operator to compare our rational expectations approach with the previous literature. Taking the first differences of this expression, and assuming that the depreciation rate is small, $\delta^k \simeq 0$, we further obtain

$$i_t^k \simeq k_{t+1} - k_t \simeq \beta (y_{t+1} - y_t) \text{ for each } t \in \{1, .., T-2\}. \tag{23}$$

Starting at date 1, our model implies a version of the acceleration principle, which posits that investment is proportional to changes in output (see Eckaus (1953) for a review).

Our model, however, has several important differences. First, the accelerator theory posits a relationship between the investment flow and the changes in output (or consumption) flows, without explicitly keeping track of the capital stock. In contrast, the capital stock plays an important role in our analysis. In fact, our main point is that a high level of the initial capital stock reduces aggregate demand. To see this, consider the analog of Eq. (23) for date 0,

$$i_0^k \simeq k_1 - k_0 \simeq \alpha + \beta y_1 - k_0.$$

Similar to Eq. (23), a reduction in expected output, y_1, or an increase in the initial capital stock, k_0, reduces investment. However, unlike Eq. (23), the initial stock, k_0, is a given of the model and is not necessarily related to the initial output, y_0. In fact, Eq. (18) shows that these two variables are *inversely* related. This is because an increase in the initial stock of capital reduces aggregate demand via the investment hangover mechanism. Our analysis, thus, suggests that the acceleration principle should be qualified for the early stages of demand-driven recessions (or booms) in which the initial capital stock might be inappropriate for the current level of economic activity.

A second difference is that the relationship in (23) is mechanically assumed in the accelerator literature, whereas we obtain Eq. (21) by combining the optimal investment behavior with the constrained monetary policy. In particular, our analysis illustrates that a constrained interest rate is important for obtaining strong accelerator effects. Otherwise, the endogenous interest rate response would typically dampen the accelerator effects (e.g., if the monetary policy focuses on output stabilization).[14]

A third difference is that the agents in our economy hold rational expectations, whereas the macroeconomic applications of the accelerator theory often use versions of Eq. (23) with backward looking expectations (for instance, $E_{t-1}[y_t] = y_{t-2}$). In particular, our model does not feature the periodic oscillations of output emphasized

[14]In his review of the accelerator theory, Caballero (1999) notes: "the absence of prices (the cost of capital, in particular) from the right-hand side of the flexible accelerator equation has earned it disrespect despite its empirical success." The liquidity trap provides a theoretical rationale for excluding the cost of capital from the investment equation.

in Samuelson (1939) or Metzler (1941), which are driven by adaptive expectations.

4.2 Consumption response

While our model can account for the decline in investment in the earlier part of the recession, it cannot generate a similar behavior for consumption. As Figure 6 illustrates, (net) consumption expands during the recession due to the Euler equation.[15] However, the Euler equation—and the permanent income hypothesis that it implies—cannot fully capture the behavior of consumption in response to income changes in the data. After reviewing the vast empirical literature on this topic, Jappelli and Pistaferri (2010) note "there is by now considerable evidence that consumption appears to respond to anticipated income increases, over and above by what is implied by standard models of consumption smoothing."

To make consumption more responsive to income, Appendix A.3 extends the model by introducing additional households that have high marginal propensities to consume (MPC) out of income. The main result shows that, if there are sufficiently many high-MPC households, then aggregate consumption initially declines. Intuitively, the low output earlier in the recession lowers all households' incomes, which in turn reduces aggregate consumption due to the high-MPC households. As output increases later in the recession, so does consumption. Hence, consumption also responds non-monotonically to overbuilding.

Appendix A.3 also shows that the model with high-MPC households features a Keynesian income multiplier with two implications. First, the recession is more severe than in the baseline model, because the decline of consumption exacerbates the reduction in aggregate demand and output. Second, the accelerator effects are more pronounced in the sense that investment decreases more early in the recession, while also increasing more later in the recession. In this sense, the multiplier and the accelerator effects reinforce one another.

5 Policy implications

We next investigate the welfare implications of our analysis. Since our model features a liquidity trap, several policies that have been discussed in the literature are also

[15] Actual consumption, $c_t = \hat{c}_t + v(l_t)$, might fall in view of the reduction in employment, l_t. We do not emphaisze this result since it is mainly driven by the GHH functional form for the preferences, which we adopt for expositional simplicity.

relevant in this context.[16] We skip a detailed analysis of these policies for brevity. Instead, we focus on constrained policy interventions directed towards controlling investment (housing and nonhousing), which plays the central role in our analysis. We first discuss ex-post policies by which the government can improve welfare once the overbuilding is realized. We then discuss ex-ante policies that the government can implement (prior to date 0) as a precaution.

The policy implications are driven by *aggregate demand externalities*, which are best illustrated by Figure 3 in the baseline setting. In the region, $b_0 \geq \bar{b}_0$, increasing the initial stock of housing, b_0, does not change the initial net consumption, c_0, which is a sufficient statistic for welfare (because it also takes into account labor costs). That is, starting the economy with more housing capital (or conversely, destroying some housing capital) neither raises nor lowers welfare. Intuitively, giving one unit of housing capital to a household raises her welfare (see Eq. (24) below), but it also lowers housing investment. This in turn reduces aggregate demand and employment, and reduces other households' welfare. In the baseline setting, these demand externalities are so strong that they completely undo the direct value of housing capital.

The externalities are very powerful in part because of the GHH preferences in (2). To provide a more transparent cost-benefit analysis for policy interventions, in this section we work with a slight modification of the model (all of the results also hold in the baseline setting). Suppose at date 0, and only at this date, households' preferences over consumption and labor are given by the separable form, $u(c_0) - v_0(l_0)$, as opposed to the GHH form, $u(\hat{c}_0 - v(l_0))$. With a slight abuse of terminology, we use c_0 to denote consumption at date 0 as opposed to net consumption, and $y_0 = F(k_0, l_0)$ to denote output at date 0 as opposed to net output. We also abstract away from adjustment costs so that the competitive equilibrium decumulates the excess capital in a single period. Lemma 2 in Appendix A.4 establishes that a sufficiently high level of overbuilding triggers a demand-driven recession also in this setting.

5.1 Ex-post policies: Slowing down disinvestment

A natural question in this environment concerns the optimal government policy regarding housing investment. On the one hand, since overbuilding is associated with housing, it might sound intuitive that the planner should not interfere with the decu-

[16] In particular, welfare can be improved with unconventional monetary policies as in Eggertsson and Woodford (2003), or unconventional tax policies as in Correia et al. (2013). Once we modify the model appropriately to include government spending, welfare can also be improved by increasing government spending during the recession as in Werning (2012) and Christiano et al. (2011).

mulation of this type of capital. On the other hand, policies that support the housing market have been widely used during and after the Great Recession (see Berger et al. (2016) for an evaluation of some of these policies). We next formally analyze the desirability of these types of policies.

We start by revisiting the representative household's equilibrium trade-off for housing investment, which provides a useful benchmark for the planner's trade-off. Imagine a household who already invested up to the target level, $h_1 = h^*$, and who is considering to invest an additional unit. Appendix A.4 defines the value function, $W_0(h_1)$, for this household and shows that,

$$\frac{d_+ W_0(h_1)}{dh_1}|_{h_1=h^*} = u'(\bar{c}_0)\left(\frac{1-\delta^h}{1+r_1} - 1\right) < 0. \tag{24}$$

Here, $\frac{d_+ W_0(h_1)}{dh_1}$ denotes the right derivative, and the inequality follows since $r_1 = 0$. The household assigns a positive value, $1-\delta^h$, to the excess unit of housing capital: Even though she does not receive any flow utility in the short run, she will benefit from the nondepreciated housing in the future. Nonetheless, she chooses $h_1 = h^*$ in equilibrium because the benefit is lower than the private cost of capital.[17]

Next consider a constrained planner who can fully determine housing investment at date 0, but cannot interfere with the remaining market allocations either at date 0 or in the future. Appendix A.4 defines the value function, $W_{0,pl}(h_1)$, for this constrained planner and shows that,

$$\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*} = u'(\bar{c}_0)\left(\left(1-\delta^h\right) - (1-\tau_0) + \frac{dc_0}{dh_1}\tau_0\right). \tag{25}$$

Here, $\tau_0 > 0$ is the labor wedge, which captures the severity of the demand shortage (as in the baseline model). Comparing Eqs. (24) and (25) illustrates that the (direct) social benefit of building is the same as the private benefit, $1-\delta^h > 0$. However, the social cost is lower, $1-\tau_0 < 1$, which leads to the following result.

Proposition 3 (Slowing Down Disinvestment)**.** *Consider the equilibrium characterized in Lemma 2. There exists $\tilde{b}_0$ such that, if $b_0 > \tilde{b}_0$, then the planner chooses a higher level of housing investment than the target level, $h_{1,pl} > h^*$.*

The planner recognizes that housing investment increases aggregate demand and employment. This is socially beneficial, and the benefits are captured by the labor

[17] The right derivative, $\frac{d_+ W_0(h_1)}{dh_1}$, is strictly less than zero at the optimal housing level, $h_1 = h^*$, since the household preferences in (2) feature a kink at this level.

wedge, τ_0, because employment is below its efficient level. Thus, the demand externalities lower the social cost of building. The reduced cost, by itself, does not create sufficient rationale for intervention—the planner also compares the cost with the benefit. Proposition 3 shows that the planner intervenes as long as the initial overbuilding is sufficiently large. Eqs. (24) and (25) suggest further that this is more likely if the overbuilt capital is more durable, so that $1 - \delta^h$ is higher. Intuitively, durable capital—such as housing—has a relatively high value, even if it is overbuilt in the short run, because it helps to economize on future investment.

Eq. (25) illustrates an additional benefit of investing in durable capital, captured by the nonnegative term, $\frac{dc_0}{dh_1}\tau_0$ (see Appendix A.4). Intuitively, bringing the non-depreciated part of the capital to date 1 creates a future wealth effect that raises consumption not only at date 1, but also at date 0, which further increases employment. This channel is reminiscent of the forward guidance policies that create a similar wealth effect by committing to low interest rates in the future.[18]

Our model, thus, provides a rationale for policies that support housing investment during an investment hangover. In practice, the planner can do this by increasing housing demand, e.g., with mortgage subsidies or modifications, or by increasing housing supply, e.g., with construction subsidies. Both types of policies can internalize the inefficiency in our model. Note, however, that the demand side policies tend to increase house prices, whereas the supply side policies tend to decrease them. The demand side interventions might be more appropriate if one considers additional ingredients, such as financial frictions, that are left out of our analysis.

To isolate the trade-offs, we focused on a planner who can only influence housing investment. In practice, the policymakers can use various other tools to fight a demand-driven slump. Eq. (25) would also apply in variants of the model in which the planner optimally utilizes multiple policies. In those variants, the equation would imply that the planner should stimulate housing investment as long as she cannot substantially mitigate the demand shortage (i.e., lower the labor wedge, τ_0) by using only the other feasible policies. This prediction is arguably applicable to various developed economies in recent years, e.g., the US and Europe, that have featured zero nominal interest rates with low employment and output despite utilizing various stimulus policies.

[18] In fact, increasing h_1 also lowers the future interest rate, r_2, in our setting. Note, however, that future output remains efficient in our model, $y_1 = s(k_1)$, whereas it exceeds the efficient level in environments with forward guidance (Werning, 2012).

5.2 Ex-ante policies: Restricting investment

We next analyze whether the planner can improve welfare via ex-ante interventions. To this end, consider the baseline model with an ex-ante period, date -1. Suppose also that the economy can be in one of two states at date 0, denoted by $s \in \{H, L\}$. State L is a low-demand state in which the target level of housing capital is h^* as before (and the planner has no tools for ex-post intervention). State H is a high-demand state in which the utility function in (2) is modified so that the target level of housing capital is $\left(1+\lambda^H\right) h^*$ for some $\lambda^H > 0$. Let $\pi^H \in (0,1)$ denote the ex-ante probability of the high-demand state at date 0. The economy starts with $h_{-1} = \left(1+\lambda^H\right) h^*$ and $k_{-1} = k^*$.

The model captures a situation in which the housing demand has recently increased relative to its historical level, and the economy has already adjusted to this new level. However, there is a possibility that the current state is not sustainable and the housing demand will revert back to its historical average.[19] We also envision that π^H is large, so that the representative household believes the high-demand state is likely to persist, but also that $\pi^H < 1$ so that there is room for precautionary policies.

We first characterize the choice of h_0 and k_0 in the competitive equilibrium, which we then compare with the constrained efficient allocations. The preferences in (2) imply that the opportunity cost of consuming housing services below target is very large. Consequently, households invest in housing capital according to their demand in state H, that is, $h_0 = \left(1+\lambda^H\right) h^*$. Thus, the degree of overbuilding in state L is now endogenized, $b_0 = \lambda^H$.[20] Nonhousing investment, k_0, is in turn determined by a standard optimality condition,

$$u'\left(c_{-1}\right) = \beta\left(\pi^H\left(R_0^H + 1 - \delta^k\right) u'\left(c_0^H\right) + \left(1-\pi^H\right)\left(R_0^L + 1 - \delta^k\right) u'\left(c_0^L\right)\right). \quad (26)$$

Appendix A.4 completes the characterization, and establishes that there is a demand-driven recession in state L of date 0 if λ^H and π^H are sufficiently high.

Next consider a constrained planner that can determine households' date -1 allocations, including h_0, k_0, but cannot interfere with equilibrium allocations starting date 0. Like households, the planner also optimally chooses $h_{0,pl} = \left(1+\lambda^H\right) h^*$. However, the planner's choice of nonhousing capital, $k_{0,pl}$, is potentially different. Appen-

[19] An alternative interpretation is to think of state H as capturing the historical housing demand. In this case, state L represents a "new normal" in which housing demand is permanently lower.

[20] The feature that overbuilding is determined exactly by the demand in state H is extreme. However, a similar outcome would also obtain in less extreme versions as long as π^H is sufficiently large.

dix A.4 describes the constrained planning problem and characterizes the planner's optimality condition as,

$$u'(c_{-1}) = \beta\left(\pi^H\left(R_0^H + 1 - \delta^k\right)u'\left(c_0^H\right) + \left(1-\pi^H\right)\left(R_0^L + (1-\tau_0)\left(1-\delta^k\right)\right)u'\left(c_0^L\right)\right). \tag{27}$$

Conditions (26) and (27) are similar except that the planner penalizes the nondepreciated part of the capital in state L, since $1-\tau_0 < 1$, which leads to the following.

Proposition 4 (Restricting Ex-ante Investment)**.** *Consider the setup with an ex-ante period, described in Lemma 3 in Appendix A.4. The constrained planner chooses a lower level of investment compared to the competitive equilibrium,* $k_{0,pl} < k_0$.

Intuitively, some of the capital invested at date -1 remains nondepreciated at date 0, which in turn lowers aggregate demand and exacerbates the recession in state L. Private agents do not internalize these negative externalities and overinvest in capital from a social point of view. In our stylized model, the inefficiency does not show up in housing capital, because the extreme preferences in (2) imply a corner solution for both the private sector and the planner. In alternative formulations with somewhat elastic housing demand, the planner would optimally restrict ex-ante investment in both types of capital. In fact, Eq. (27) suggests that the externality is particularly strong for more durable types of capital such as housing, because the inefficiency is driven by the nondepreciated part.

Proposition 4 is reminiscent of the results in a recent literature, e.g., Korinek and Simsek (2015) and Farhi and Werning (2013), which investigate the implications of aggregate demand externalities for ex-ante macroprudential policies in financial markets. For instance, Korinek and Simsek (2015) show that, in the run-up to liquidity traps, private agents take on too much debt, because they do not internalize that leverage reduces aggregate demand. We complement this analysis by showing that aggregate demand externalities also create inefficiencies for ex-ante *physical* investment. Our model highlights a distinct mechanism, and generates policy implications that are not the same as the macroprudential policies typically emphasized in this literature. We provide a rationale for restricting ex-ante investment regardless of whether investment is financed by debt or other means.

In practice, the planner could restrict investment by using a variety of direct policies, e.g., taxes, quantity restrictions, or financing restrictions. A natural question is whether the planner should also use the monetary policy. The US Fed has been criticized for keeping the interest rate low in the run-up to the Great Recession. Our next result investigates whether a contractionary policy that raises the interest rate at date -1 above its natural level might be desirable.

Proposition 5 (Jointly Optimal Monetary and Investment Policy)**.** *Consider the setup in Proposition 4. Suppose the planner chooses the interest rate, r_0, at date -1, in addition to controlling the household's ex-ante allocations. It is optimal for this planner to set $r_0 = r_0^*$ and implement $y_{-1} = S(k_{-1})$.*

Put differently, once the investment (restricting) policies are in place, it is optimal for the monetary policy to pursue its myopic output stabilization goal described in (10). The constrained efficient outcome (characterized by condition (27)) is to reduce the investment at date -1 while increasing consumption, so that there is some reallocation but not a recession at date -1. The investment policies implement this outcome by allowing the interest rate to be determined in equilibrium so as to clear the goods market (the zero lower bound constraint does not bind at date -1 under our assumptions). In contrast, setting a high level of the interest rate, $r_0 > r_0^*$, reduces investment while also creating an inefficient recession at date -1.[21]

Let us summarize the insights from our welfare analyses in this section. Ex-post, once the economy is in the demand-driven slump, welfare can be improved by policies that stimulate investment, including investment in the overbuilt capital. Ex-ante, before the economy enters the slump, welfare can be improved by policies that restrict investment. The optimal ex-post and ex-ante policies share the broad principle that they intertemporally substitute investment from periods that feature efficient outcomes to periods (or states) that feature deficient demand. Intertemporal substitution is less costly for more durable types of capital that deliver a utility flow over a long horizon of time. Hence, our analysis also suggests that the policy interventions are particularly desirable for more durable types of capital.

6 Conclusion

We have presented a model of investment hangover motivated by the Great Recession that combines both Austrian and Keynesian features. On the Austrian side, the recession is precipitated by overbuilding of durable capital such as housing, which

[21] That said, if the planner does not have access to the investment policies described above, or faces additional costs in implementing these policies, then she might want to resort to constrained monetary policy as a second-best measure.

necessitates a reallocation of resources to other sectors. On the Keynesian side, a constrained monetary policy prevents the interest rate from declining sufficiently, which slows down reallocation and creates an aggregate demand shortage. The demand shortage can also reduce investment in other types of capital that are not overbuilt, leading to a severe recession. Eventually, other types of investment recovers, but the slump in the overbuilt sector continues for a long time.

The model yields predictions that are consistent with the broad trends of GDP, housing investment, and nonhousing investment in the Great Recession. In particular, the model explains why housing investment collapsed and has not really recovered, and why other types of investment also declined initially, but then recovered much more robustly. We need both Keynesian and Austrian features to obtain these empirically accurate predictions.

The model also features aggregate demand externalities, with several policy implications for investment. Welfare can be improved by ex-post policies that slow down the decumulation of housing capital, as well as ex-ante policies that restrict the accumulation of capital. These policies intertemporally substitute investment towards periods that feature deficient demand.

Although we have focused on the Great Recession, the model is more widely applicable in environments in which the key assumptions hold: that is, if durable capital is overbuilt and the monetary policy is constrained. During the times of Hayek and Keynes, speculative overbuilding was seen as a critical impetus to recessions, but the focus was more on railroads and perhaps industrial plant than on housing. In the recent European context, overbuilding of houses (or structures) might have contributed to the macroeconomic slump in many countries, and the monetary policy was constrained by the currency union as well as the liquidity trap. We leave an elaboration of these applications of our model for future work.

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A Online Appendix: Omitted extensions

This appendix completes the analysis of the extensions of the baseline model discussed in the main text. The online appendix B contains the proofs of omitted results in the main text as well as this appendix.

A.1 Comparative statics with respect to durability

A distinguishing feature of housing capital is its durability relative to other types of capital. A natural question is whether durability is conducive to triggering a demand-driven recession driven by overbuilding. In this section, we address this question in an extension of the baseline model with two types of housing capital, one more durable than the other. We show that overbuilding the more durable capital (relative to the less durable capital) is more likely to trigger a demand-driven recession.

Consider a slight variant of the model in Section 3 in which there are two types of housing capital that depreciate at different rates given by δ^{h^d} and δ^{h^n}, with $\delta^{h^d} < \delta^{h^n}$. Thus, type d (durable) housing capital has a lower depreciation rate than type n (nondurable) housing capital. Suppose the preferences in (2) are modified so that each type has a target level $h^*/2$. Suppose also that $\left(\delta^{h^d} + \delta^{h^n}\right)/2 = \delta^h$ so that the average depreciation rate is the same as before. Let $h_0^d = \left(1 + b_0^d\right)(h^*/2)$ and $h_0^n = (1 + b_0^n)(h^*/2)$, so that b_0^d and b_0^n capture the overbuilding in respectively durable and nondurable capital. The case with symmetric overbuilding, $b_0^d = b_0^n = b_0$, results in the same equilibrium as in Section 3. Our next result investigates the effect of overbuilding one type of capital more than the other.

Proposition 6 (Role of Durability)**.** *Consider the model with two types of housing capital with different depreciation rates. Given the average overbuilding $b_0 = \left(b_0^d + b_0^n\right)/2$, the incidence of a demand-driven recession $1\left[l_0 < L\left(k_0\right)\right]$ is increasing in overbuilding of the more durable housing capital b_0^d.*

To obtain an intuition, consider the maximum aggregate demand at date 0, which can be written as [cf. Eq. (18)],

$$\overline{y}_0 = \overline{k} - \left(1 - \delta^k\right) k_0 + \overline{c}_0 + \delta^h h^* - b_0^d \left(1 - \delta^{h^d}\right) \frac{h^*}{2} - b_0^n \left(1 - \delta^{h^n}\right) \frac{h^*}{2}. \tag{A.1}$$

Note that $1 - \delta^{h^d} > 1 - \delta^{h^n}$, and thus, overbuilding of the durable housing capital (relative to the nondurable capital) induces a greater reduction in aggregate demand at date 0. Intuitively, depreciation helps to "erase" the overbuilt capital naturally, thereby inducing a smaller reduction in investment and aggregate demand.

A.2 Investment hangover with exogenous monetary policy

A key ingredient of our analysis is constrained monetary policy. In the main text, we focus on the zero lower bound (ZLB) as the source of the constraint. In this section, we derive the analogue of our main result in Section 3 in an environment in which the money supply is determined by exogenous forces.

To introduce the money supply, we modify household preferences to introduce the demand for money explicitly. Specifically, the household's optimization problem can now be written as,

$$\max_{\{l_t,\hat{c}_t,a_{t+1},M_t\}_t} \sum_{t=0}^{\infty} \beta^t u\left(\hat{c}_t - v\left(l_t\right) + \eta\left(\frac{M_t}{P_t}\right)\right) + u^h \mathbf{1}\left[h_t \geq h^*\right] \tag{A.2}$$
$$\text{s.t. } P_t\left(\hat{c}_t + a_{t+1} + i_t^h\right) + M_t = P_t\left(w_t l_t + a_t\left(1+r_t\right) + \Pi_t\right) + M_{t-1},$$
$$\text{and } h_{t+1} = h_t\left(1-\delta^h\right) + i_t^h.$$

Here, P_t denotes the aggregate price level. The household money balances are denoted by M_t, and the real money balances are given by M_t/P_t. The function, $\eta(\cdot)$ is strictly increasing, which captures the transaction services provided by additional real money balances. The household problem is the same as in Section 2 except for the presence of money balances in preferences as well as the budget constraint. The optimality condition for money balances, M_t, implies a money demand equation,

$$\eta'\left(\frac{M_t}{P_t}\right) = \frac{r_{t+1}^n}{1+r_{t+1}^n}. \tag{A.3}$$

Here, $1 + r_{t+1}^n = \left(1+r_{t+1}\right)\frac{P_{t+1}}{P_t}$ denotes the nominal interest rate, which captures the opportunity cost of holding money balances (as opposed to interest-bearing assets). The left hand side captures the marginal benefit of holding money balances.[22] The rest of the equilibrium is as described before.

We assume the money supply follows an exogenous path, $\left\{\overline{M}_t\right\}_{t=0}^{\infty}$. For analytical tractability, we focus on the case in which the money supply is fixed, $\overline{M}_t = \overline{M}$

[22] With our specification, the marginal benefit does not depend the household's consumption or aggregate output. This is slightly different than conventional specifications of money demand but it does not play an important role beyond providing analytical tractability.

for each t (the general case is similar). As before, the aggregate price level is also predetermined and constant, $P_t = P$ for each t. Combining these assumptions with Eq. $(A.3)$ implies that the nominal interest rate is also constant. There is one degree of freedom because different choices for the aggregate price level (which is a given of this model) lead to different levels for the interest rate. We assume the aggregate price level is such that the interest rate is equal to its steady-state level, that is,[23]

$$r_{t+1} = r_{t+1}^n = 1/\beta - 1 \text{ for each } t. \tag{A.4}$$

The characterization of the remaining equilibrium allocations then parallels the baseline analysis. We conjecture an equilibrium in which, starting date 1 onwards, the employment and output are at their efficient levels. As before, this implies capital earns its marginal contribution to supply, $R_1 = S'(k_1)$ [cf. (9)]. Combining this with Eq. (6), and using $(A.4)$, we obtain $k_1 = k^*$. That is, the economy reaches the steady-state level of capital in a single period. This determines the investment at date 0 as

$$i_0^k = k_1 - \left(1 - \delta^k\right) k_0.$$

Next consider (net) consumption at date 0. Since the economy reaches the steady-state at date 1, we have $c_1 = c^*$. Combining this with the Euler equation and Eq. $(A.4)$, we also obtain $c_0 = c^*$. It follows that aggregate demand and output at date 0 is given [cf. Eq. (18)]:

$$y_0 = k^* - \left(1 - \delta^k\right) k_0 + c^* + \left(\delta^h - b_0 \left(1 - \delta^h\right)\right) h^*.$$

When $y_0 < S(k_0)$, the economy features a demand-driven recession at date 0. This is the case as long as the amount of overbuilding b_0 exceeds a threshold level [cf. (19)]:

$$\bar{b}_0 \equiv \frac{k^* - \left(1 - \delta^k\right) k_0 + c^* + \delta^h h^* - S(k_0)}{\left(1 - \delta^h\right) h^*}.$$

It can also be checked that, it the initial capital stock is at its steady-state level $k_0 = k^*$, then the threshold is zero, $\bar{b}_0 = 0$: that is, any amount of overbuilding triggers a recession.

[23] This price level can be justified by assuming that the prices were set at a point in the past at which the economy was (and was expected to remain) at a steady state. In view of a New-Keynesian Phillips curve, the firms would not want to change their prices only if they expected the discounted sum of the output gaps to be equal to zero. When the economy is at a steady state, this implies a zero output gap for each period and the interest rate given by $(A.4)$.

Hence, our main result generalizes to a setting with exogenous (and fixed) money supply. Intuitively, the key to the argument is that the monetary policy is constrained and cannot lower the interest rate sufficiently to counter the aggregate demand reduction due to overbuilding. When the monetary policy is exogenous—as in the case of an exogenous money supply, it is naturally constrained and cannot lower the interest rate in response to shocks. In fact, overbuilding in this case leads to a deeper recession because the nominal interest rate remains above zero during the recession, whereas the monetary policy is the main text partially fights the recession by lowering the nominal interest rate to zero.

A.3 Consumption response and the Keynesian multiplier

In the main text, we assume a representative household whose consumption satisfies the Euler equation. However, the Euler equation cannot fully capture the behavior of consumption in response to income changes in the data (see Jappelli and Pistaferri, 2010). We next modify the model by introducing constrained agents that have high MPCs out of income. We show that this version of the model can account for the drop in consumption earlier in the recession. The model also features a Keynesian income multiplier, which exacerbates the recession and reinforces the investment accelerator mechanism.

Suppose, in addition to the representative household analyzed earlier, there is an additional mass l^{tr} of households which we refer to as *income-trackers*. These agents are excluded from financial markets so that they consume all of their income, that is, their MPC is equal to 1 (for simplicity). Each income-tracker inelastically supplies 1 unit of labor in a competitive market for a wage level w_t^{tr}, which provides her only source of income. Consequently, total consumption is now given by $c_t + w_t^{tr} l^{tr}$, where c_t is the consumption of the representative household and $w_t^{tr} l^{tr}$ denotes the consumption of income-trackers.

The aggregate production function can generally be written as $\tilde{F}(k_t, l_t, l^{tr})$, where l_t is the labor supply by the representative household and l^{tr} is the total labor supply by income-trackers. To simplify the analysis, we focus on the special case $\tilde{F}(k_t, l_t, l^{tr}) = F(k_t, l_t) + \eta^{tr} l^{tr}$, where F is a neoclassical production function and $\eta^{tr} > 0$ is a scalar. We continue to use the notation $y_t = F(k_t, l_t) - v(l_t)$ and $\hat{y}_t = F(k_t, l_t)$ to refer to respectively the net and the actual output excluding the supply of income-trackers. Total net and actual output are respectively given by $\hat{y}_t + \eta^{tr} l^{tr}$ and $y_t + \eta^{tr} l^{tr}$. The rest of the model is the same as in Section 4 (that also features the endogenous investment response).

In view of these assumptions, the economy is subject to the resource constraint,

$$c_t + i_t^k + i_t^h + w_t^{tr} l^{tr} = y_t + \eta^{tr} l^{tr} \leq S(k_t) + \eta^{tr} l^{tr}. \tag{A.5}$$

Lemma 4 in Appendix B characterizes the income-trackers' wage level as

$$w_t^{tr} = \psi(k_t, y_t) \eta^{tr}. \tag{A.6}$$

Here, $\psi(k_t, y_t) \in [0, 1]$ is a measure of efficient resource utilization (more specifically, $\psi = 1 - \tau$ where τ is the labor wedge). It is an increasing function of y_t and satisfies $\psi = 1$ when the output is at its efficient level, $y_t = S(k_t)$. Intuitively, the demand shortage lowers factor returns, including the wages of income trackers.

Combining Eqs. $(A.5)$ and $(A.6)$ implies

$$y_t = c_t + k_{t+1} - (1 - \delta) k_t + i_t^h + (\psi(k_t, y_t) - 1) \eta^{tr} l^{tr}, \tag{A.7}$$

for each $t \in \{0, 1, .., T-1\}$. This expression illustrates a *Keynesian cross* as well as a *Keynesian income multiplier* in our setting. The equilibrium obtains when the actual and demanded net outputs are equal, as in a typical the Keynesian cross. Moreover, total demand depends on the output y_t through income-trackers' consumption, illustrating the multiplier. Consider, for instance, a shock to aggregate demand that lowers net output. This lowers income-trackers' income and their consumption, which in turn induces a second round reduction in aggregate demand and output, and so on.

Next consider a housing investment shock that lasts T periods as in the previous section. We conjecture an equilibrium with a demand-driven recession at all dates $t \in \{0, 1, .., T-1\}$. As before, the break-even condition (21) holds. Eqs. $(A.7)$ and (21) can then be solved backwards starting with $k_T = \bar{k}$. The next result establishes the existence of an equilibrium, and characterizes the behavior of consumption in equilibrium.

Proposition 7 (Consumption Response)**.** *Consider the model with mass l^{tr} of income-trackers and the adjustment length $T \geq 1$. Suppose Assumptions 1-2 and Assumption 3^{tr} in Appendix B hold.*

(i) There exists $i^{h,1}$ such that if $i^h < i^{h,1}$, then there is an equilibrium path $\{k_t, y_{t-1}\}_{t=1}^{T}$, which solves Eqs. (21) and $(A.7)$ along with $k_T = \bar{k}$. Any equilibrium features a demand-driven recession at each date $t \in \{0, .., T-1\}$ with $r_{t+1} = 0$ and $y_t < S(k_t)$.

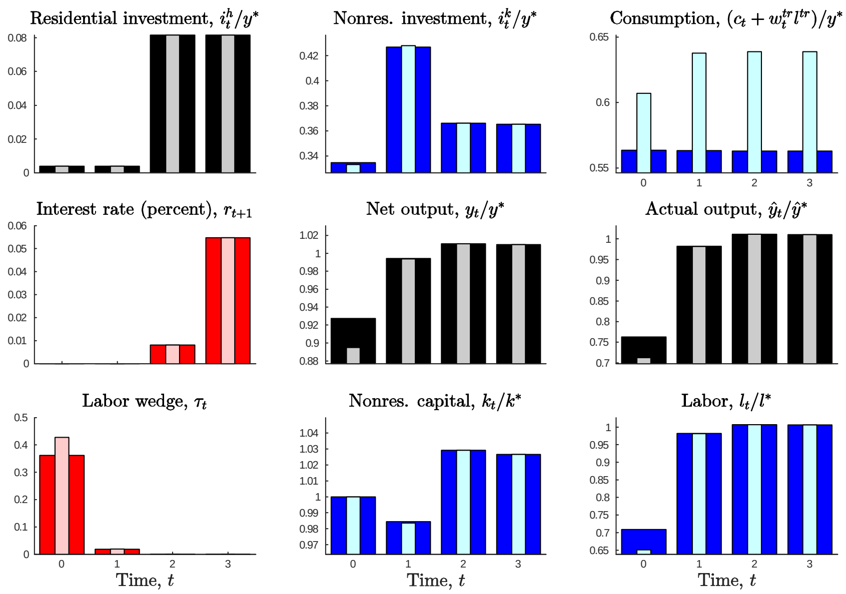

Figure 7: The dynamic equilibrium with income-trackers (light bars) compared to the equilibrium without income-trackers (dark bars). The variables definition are the same as in Figure 3, except that the top right panel in this case illustrates the total net consumption—including the consumption of income-trackers.

(ii) There exists l_1^{tr} such that if $l^{tr} > l_1^{tr}$, then total consumption at date 0 *(in any equilibrium) is below its steady-state level, that is,*

$$c_0 + w_0^{tr} l^{tr} < c^* + \eta^{tr} l^{tr}.$$

The main result of this section is the second part, which establishes conditions under which overbuilding also lowers total consumption at date 0 in any equilibrium.[24] When the economy is in a demand-driven recession, the income-trackers' consumption declines in view of the decline in output. With sufficiently many income-trackers, this also reduces total consumption in contrast to the baseline model. The light bars in Figure 7 illustrate this result by plotting the dynamic equilibrium using the same parameters as before except for the new parameters $\eta^{tr}, l^{tr} > 0$. Consumption declines early in the recession, and it recovers later in the recession due to the recovery in output. Hence, this version of the model can generate a nonmonotonic response in consumption, similar to the nonmonotonic response of investment identified in

[24] The equilibrium is unique in all of our numerical simulations. However, there could in principle be multiple equilibria because Eq. $(A.7)$ represents an intersection of two increasing curves in Y_t.

Proposition 2.

It follows that this version of the model can explain the asymmetric recovery from the Great Recession depicted in Figure 1. In the first phase of the recession, consumption as well as nonhousing investment simultaneously fall, triggering a deep recession. In the second phase, the boom in nonhousing investment increases output, which also increases consumption. Hence, the second phase is a partial and asymmetric recovery in which the housing sector is left behind, as in the aftermath of the Great Recession.

Figure 7 also contrasts this equilibrium with the earlier equilibrium without income-trackers, which is plotted with dark bars. Note that the equilibrium in this section features a greater drop in output and employment, as well as a greater labor wedge. Intuitively, the Keynesian income multiplier aggravates the recession. Perhaps less obviously, the equilibrium also features a more severe drop in investment at date 0, followed by a stronger recovery at date 1. Intuitively, a more severe recession implies a lower return to capital, which in turn lowers investment at date 0. Put differently, the Keynesian income multiplier exacerbates the investment accelerator mechanism. The decline in investment at date 0 further lowers net output and consumption, aggravating the Keynesian income multiplier. In this sense, the multiplier and the accelerator mechanisms reinforce each other.

A.4 Policy analysis with separable preferences

We next complete the analysis of the model with separable preferences described and used in Section 5. We first establish the analog of Proposition 1 for this setting. To this end, let $\overline{c}_0$ and $\overline{k}$ respectively denote the maximum level of consumption and investment characterized in Section 3. The aggregate demand is then bounded from above, $y_0 \leq \overline{y}_0$, where

$$\overline{y}_0 = \overline{y}_0 \equiv \overline{k} - \left(1 - \delta^k\right) k_0 + \overline{c}_0 + \left(\delta^h - b_0 \left(1 - \delta^h\right)\right) h^*. \tag{A.8}$$

as in Eq. (18) in the main text.

Next consider the efficient level of employment at date 0. The efficiency implies the household's intratemporal condition holds, $w_0 u'(c_0) = v'(l_0)$, and the equilibrium wage level is determined by the labor's marginal product, $w_0 = F_l(k_0, l_0)$. Combining these conditions is equivalent to setting the labor wedge to zero, where the labor wedge is now given by,

$$\tau_0 = 1 - \frac{v_0'(l_0)}{u'(c_0) F_l(k_0, l_0)}. \tag{A.9}$$

Let $L_0(k_0)$ denote the efficient level of output at date 0 (when there is a liquidity trap) characterized by setting $\tau_0 = 0$ when $c_0 = \overline{c}_0$. This also implies an efficient level of output denoted by, $S_0(k_0) = F(k_0, L_0(k_0))$.

As in Section 3, the equilibrium depends on a comparison of the maximum level of demand, $\overline{y}_0$, with the efficient supply, $S_0(k_0)$. Let $\overline{b}_0^{sep}$ denote the threshold level of overbuilding that ensures $\overline{y}_0 = S_0(k_0)$, that is,

$$\overline{b}_0^{sep} = \frac{\overline{k} - \left(1-\delta^k\right) k_0 + \overline{c}_0 + \delta^h h^* - S_0(k_0)}{\left(1-\delta^h\right) h^*}. \tag{A.10}$$

We then have the following analogue of Proposition 1.

Lemma 2. *Consider the modified model with separable preferences at date* 0*. The competitive equilibrium decumulates the excess housing capital in a single period,* $h_1 = h^*$*. If the overbuilding is sufficiently large,* $b_0 > \overline{b}_0^{sep}(k_0)$*, then the date* 0 *equilibrium features a demand-driven recession with,*

$$r_1 = 0, \quad \tau_0 > 0, \quad y_0 = \overline{y}_0 < S_0(k_0), \text{ and } l_0 < L_0(k_0).$$

A.4.1 Ex-post welfare analysis

Next suppose the overbuilding is sufficiently large so that the economy is in a recession. We next respectively define the household's and the planner's value functions and derive their optimality conditions. Note that choosing $h_1 < h^*$ is sub-optimal in view of the preferences (2). We thus consider the value functions over the region $h_1 \geq h^*$.

The household's problem can then be written as (cf. problem (5)),

$$\begin{aligned} W_0(h_1) &= \max_{\{c_t, a_{t+1}\}_t} \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \text{s.t. } & \quad c_t + a_{t+1} + h_{t+1} = e_t + a_t(1+r_t) + \Pi_t + \left(1-\delta^h\right) h_t \\ \text{given } h_0 &\geq h^*, h_1 \geq h^* \text{ and } h_t = h^* \text{ for each } t \geq 2. \end{aligned}$$

Using the envelope theorem, we obtain,

$$\frac{dW_0(h_1)}{dh_1}|_{h_1=h^*} = \beta u'(c_1)\left(1-\delta^h\right) - u'(c_0).$$

Combining this with the Euler equation, $u'(c_0) = \beta(1+r_1)u'(c_1)$, establishes Eq. (24).

Next consider a constrained planner who can (only) control housing investment at date 0. When h_1 is in a neighborhood of h^*, the constrained planning problem can be written as,

$$W_{0,pl}(h_1) = \max_{c_0,k_1,y_0,l_0} u(c_0) - v_0(l_0) + \beta V(k_1,h_1), \tag{A.11}$$
$$\text{s.t. } k_1 = \overline{k} \text{ and } u'(c_0) = \beta u'(C(h_1)),$$
$$\text{and } y_0 = F(k_0,l_0) = k_1 - \left(1-\delta^k\right)k_0 + c_0 + h_1 - \left(1-\delta^h\right)(1+b_0)h^* \tag{A.12}$$

Here, $V(k_1,h_1)$ denotes the efficient value function characterized as the solution to problem $(B.1)$, and $C(h_1)$ denotes the efficient level of consumption. The second line captures the zero lower bound constraint, which implies that consumption and non-housing investment are determined by the zero interest rate. The third line captures that output and employment are determined by the aggregate demand at date 0. Importantly, the output is increasing in h_1 because a greater level of housing investment increases aggregate demand.

To derive the optimality condition for problem $(A.11)$, note that the capital stocks k_0 and $k_1 = \overline{k}$ are constant, and that the remaining variables, $c_0(h_1), y_0(h_1), l_0(h_1)$, are determined as implicit functions of h_1. Implicitly differentiating the aggregate demand constraint $(A.12)$ with respect to h_1, we obtain,

$$\frac{dl_0}{dh_1} = \frac{1+\frac{dc_0}{dh_1}}{F_l(k_0,l_0)} = \left(1+\frac{dc_0}{dh_1}\right)\frac{(1-\tau_0)u'(c_0)}{v'(l_0)}.$$

Here, the second equality substitutes the labor wedge from Eq. $(A.9)$. Using problem $(B.1)$ along with the envelope theorem, we also obtain,

$$\frac{dV_1(k_1,h_1)}{dh_1} = \left(1-\delta^h\right)u'(c_1) = \left(1-\delta^h\right)\frac{u'(c_0)}{\beta}.$$

Here, the second equality uses the Euler equation. Differentiating the objective function of problem $(A.11)$ with respect to h_1, and using these expressions, we obtain,

$$\begin{aligned}\frac{dW_{0,pl}(h_1)}{dh_1} &= u'(c_0)\frac{dc_0}{dh_1} - v_0'(l_0)\frac{dl_0}{dh_1} + \beta\frac{dV_1(k_1,h_1)}{dh_1}, \\ &= u'(c_0)\left(\frac{dc_0}{dh_1} - \left(1+\frac{dc_0}{dh_1}\right)(1-\tau_0) + 1 - \delta^h\right).\end{aligned}$$

Rearranging terms establishes Eq. (25). Using this expression, Appendix B proves

Proposition 3 and completes the welfare analysis in Section 5.1.

A.4.2 Ex-ante welfare analysis

Next consider the ex-ante welfare analysis in Section 5.2. Recall that the representative household optimally chooses $h_0 = h^* \left(1 + \lambda^H\right)$, along with k_0 characterized as the solution to (26). The representative household recognizes that the rental rate of capital in state L, R_0^L, is below its efficient level (due to the demand shortage). This might induce her to choose a lower level of k_0 as a precaution. A sufficiently low level of k_0 can, in turn, raise the aggregate demand and prevent the demand-driven recession [cf. Eq. $(A.10)$]. Nonetheless, the following result establishes that the economy experiences a recession in state L, as long as the probability of the state is sufficiently low, and the demand for housing in the counterfactual state H is sufficiently high.

Lemma 3. *Consider the modified model with the ex-ante date* -1*, with the initial conditions,* $h_{-1} = h^* \left(1 + \lambda^H\right)$ *and* $k_{-1} = k^*$*. Suppose* $\lambda^H > \overline{b}_0^{sep}(k^*)$*, where* $\overline{b}_0^{sep}(k^*)$ *denotes the overbuilding threshold in* $(A.10)$ *given* $k_0 = k^*$*. There exists* $\overline{\pi} < 1$ *such that, if* $\pi^H \in (\overline{\pi}, 1)$*, then the equilibrium features a demand-driven recession in state* L *of date* 0 *(but not in any other dates or states).*

The equilibrium path starting the high-demand state H of date 0 is straightforward. It solves the neoclassical planning problem $(B.1)$ with a steady level of housing investment given by, $i_t^h = \delta \left(1 + \lambda^H\right) h^*$ for each $t \geq 0$. The zero lower bound does not bind and the rental rate of capital is given by $R_0^H = S'(k_0)$. The equilibrium path starting the low-demand state L of date 0 is characterized as in Lemma 2 given the (endogenous) level of overbuilding, $b_0 = \lambda^H$.

Next consider a constrained planner who can (only) control households' date -1 allocations. As described in the main text, the planner optimally chooses $h_{0,pl} = h_0 = \left(1 + \lambda^H\right) h^*$. However, the planner's choice of nonhousing capital, $k_{0,pl}$, is potentially different. To characterize this choice, let $V_0^H(k_0, h_0)$ and $V_0^L(k_0, h_0)$ denote the welfare of the representative household in respectively states H and L of date 0. The ex-ante constrained planning problem can then be written as,

$$\max_{c_{-1}, k_0} u(c_{-1}) + \beta \left(\pi^H V_0^H(k_0, h_0) + \left(1 - \pi^H\right) V_0^L(k_0, h_0)\right), \qquad (A.13)$$

$$\text{s.t.} \qquad c_{-1} + k_0 + h_{0,pl} = S(k_{-1}) + \left(1 - \delta^k\right) k_{-1} + \left(1 - \delta^h\right) h_{-1}.$$

In particular, the planner optimally trades off the ex-ante consumption, c_{-1}, with investment, k_0, evaluating the benefits of the latter in the competitive equilibrium that will obtain in each state. The optimality condition for the problem is then given by

$$u'(c_{-1}) = \beta \left(\pi^H \frac{dV_0^H(k_0, h_0)}{dk_0} + \left(1 - \pi^H\right) \frac{dV_0^L(k_0, h_0)}{dk_0} \right). \tag{A.14}$$

We next derive $\frac{dV_0^H(k_0,h_0)}{dk_0}$ and $\frac{dV_0^L(k_0,h_0)}{dk_0}$, and establish Eq. (27). If state H is realized, then the equilibrium solves the analog of problem $(B.1)$ (with appropriate modifications to capture the higher target level, $\left(1+\lambda^H\right)h^*$). Then, the envelope theorem implies,

$$\frac{dV_0^H(k_0, h_0)}{dk_0} = \left(S'(k_0) + 1 - \delta^k\right) u'\left(c_0^H\right).$$

Suppose instead state L is realized. We conjecture (and verify in Proposition 3) that the planner's allocation also features a demand-driven recession in this state. The continuation allocation is characterized by Lemma 2, and it solves problem $(A.11)$ with $h_1 = h^*$ (since we rule out ex-post policies). This problem implies that the following variables are constant, $k_1 = \overline{k}, c_0 = \overline{c}_0, h_1 = h^*$ (and thus, the continuation value V_1 is also constant). In contrast, output and employment, $y_0(k_0), l_0(k_0)$, are determined as implicit functions of k_0. Implicitly differentiating the aggregate demand constraint $(A.12)$ with respect to k_0, we obtain,

$$\frac{dl_0}{dk_0} = -\frac{F_k(k_0, l_0) + \left(1 - \delta^k\right)}{F_l(k_0, l_0)} = -\left(F_k(k_0, l_0) + \left(1 - \delta^k\right)\right) \frac{(1-\tau_0)\, u'(\overline{c}_0)}{v'(l_0)}.$$

Here, the second equality substitutes the labor wedge from Eq. $(A.9)$. Differentiating the objective function with respect to k_0, and using this expression, we further obtain,

$$\frac{dV_0^L(k_0, h_0)}{dk_0} = -v_0'(l_0) \frac{dl_0}{dh_1} = (1-\tau_0)\left(F_k(k_0, l_0) + \left(1 - \delta^k\right)\right) u'(\overline{c}_0).$$

Plugging in $R_0^L = (1-\tau_0) F_k(k_0, l_0)$ from Lemma 2 implies,

$$\frac{dV_0^L(k_0, h_0)}{dk_0} = \left(R_0^k + (1-\tau_0)\left(1 - \delta^k\right)\right) u'(\overline{c}_0).$$

Plugging the expressions for $\frac{dV_0^H(k_0,h_0)}{dk_0}$ and $\frac{dV_0^L(k_0,h_0)}{dk_0}$ into $(A.14)$ implies the planner's optimality condition (27). Appendix B proves Propositions 4 and 5, and completes the welfare analysis in Section 5.2.

B Online Appendix: Omitted proofs

B.1 Proofs for the baseline model

This section presents the proofs of the results for the baseline model and its variants analyzed in Sections 2, 3 and 4.

Efficient Benchmark. Consider a planner that maximizes households' welfare starting date t onwards, given the initial state h_t, k_t, and the feasibility constraints of the economy. The planner's problem can then be written as,

$$\begin{aligned}
&\max_{\left\{\hat{c}_{\tilde{t}}, l_{\tilde{t}}, k_{\tilde{t}+1}, \tilde{h}_{t+1}, [l_{\tilde{t}}(\nu), k_{\tilde{t}}(\nu)]_\nu\right\}_{\tilde{t}=t}^{\infty}} \sum_{\tilde{t}=t}^{\infty} \beta^{\tilde{t}} \left(u\left(\hat{c}_{\tilde{t}} - \nu\left(l_{\tilde{t}}\right)\right) + u^h \mathbf{1}\left[h_t \geq h^*\right]\right), \\
&\text{s.t. } \hat{c}_{\tilde{t}} + k_{\tilde{t}+1} + h_{\tilde{t}+1} \leq \hat{y}_{\tilde{t}} + \left(1-\delta^k\right) k_{\tilde{t}} + \left(1-\delta^h\right) h_{\tilde{t}}, \text{ where} \\
\hat{y}_{\tilde{t}} \;=\; &\left(\int_0^1 \left(F\left(k_{\tilde{t}}\left(\nu\right), l_{\tilde{t}}\left(\nu\right)\right)\right)^{\frac{\varepsilon-1}{\varepsilon}} d\nu\right)^{\varepsilon/(\varepsilon-1)}, k_{\tilde{t}} = \int k_{\tilde{t}}\left(\nu\right) d\nu, \text{ and } l_{\tilde{t}} = \int l_{\tilde{t}}\left(\nu\right) d\nu.
\end{aligned}$$

By concavity, the planner chooses $k_{\tilde{t}}(\nu) = k_{\tilde{t}}$ and $l_{\tilde{t}}(\nu) = l_{\tilde{t}}$ for each $\tilde{t}$. The optimality condition for labor then implies Eq. (9). Combining these observations, the planner's problem reduces to the neoclassical planning problem,

$$\begin{aligned}
V\left(k_t, h_t\right) \quad &= \quad \max_{\left\{c_{\tilde{t}}, k_{\tilde{t}+1}, h_{t+1}\right\}_{\tilde{t}=t}^{\infty}} \sum_{\tilde{t}=t}^{\infty} \beta^{\tilde{t}} \left(u\left(c_{\tilde{t}}\right) + u^h \mathbf{1}\left[h_t \geq h^*\right]\right), \qquad \text{(B.1)} \\
&\text{s.t.} \quad c_{\tilde{t}} + k_{\tilde{t}+1} - \left(1-\delta^k\right) k_{\tilde{t}} + h_{\tilde{t}+1} - \left(1-\delta^h\right) h_{\tilde{t}} = S\left(k_{\tilde{t}}\right).
\end{aligned}$$

Here, the function $S(\cdot)$ describes the supply-determined net output defined in (9).

Equilibrium in the Aftermath of Overbuilding. Suppose the economy reaches date 1 with $h_1 = h^*$ and $k_1 \leq \overline{k}$. We claim that the continuation equilibrium is the same the efficient benchmark. To this end, consider the solution to the planner's problem $(B.1)$ starting with $h_1 = h^*$ and $k_1 \leq \bar{k}$. We conjecture a solution in which $h_{t+1} = h^*$ for each $t \geq 1$, as in (3), and the remaining allocations are characterized as the solution to the neoclassical system,

$$\begin{aligned}
S\left(k_{\tilde{t}}\right) \quad &= \quad c_{\tilde{t}} + k_{\tilde{t}+1} - \left(1-\delta^k\right) k_{\tilde{t}} + \delta^h h^*, \qquad \text{(B.2)} \\
u'\left(c_{\tilde{t}}\right) \quad &= \quad \beta\left(1 + S'\left(k_{\tilde{t}}\right) - \delta^k\right) u'\left(c_{\tilde{t}+1}\right),
\end{aligned}$$

together with a standard transversality condition. The steady-state to this system is characterized by,

$$\beta\left(1-\delta^k+S'\left(k^*\right)\right)=1 \text{ and } S\left(k^*\right)=c^*+\delta^k k^*+\delta^h h^*.$$

We assume the parameters satisfy, $\min\left(S\left(k_0\right), S\left(k^*\right)\right)>\delta^k k^*+\delta h^*$, which ensures that the economy can afford the required investment at all periods. Then, using standard arguments, there is a unique interior path that solves the system in $(B.2)$ and converges to the steady state. Moreover, since capital converges monotonically to its steady-state level, and since we have $k_1 \leq \overline{k}$ and $k^* < \overline{k}$, we also have $k_{t+1} \leq \overline{k}$ for each $t \geq 1$. This in turn implies the interest rate satisfies, $r_{t+1} = S'\left(k_{t+1}\right) - 1 \geq S'\left(\overline{k}\right) - 1 = 0$ for each $t \geq 1$.

In particular, the implied real interest rate is nonnegative along the socially optimal path, which has two implications. First, the planner finds it optimal to choose $h_{t+1} = h^*$ as we have conjectured (since the gross return on investment, $1 + r_{t+1}$, exceeds the return on empty houses, $1 - \delta^h$). Second, and more importantly, the lower bound constraint (7) does not bind along the socially optimal path. This implies that the monetary policy rule in (10) replicates the dynamically efficient outcomes. That is, the competitive equilibrium from date 1 onwards (starting $h_1 = h^*$ and $k_1 \leq \overline{k}$) coincides with the efficient benchmark. This completes the characterization of the equilibrium in the aftermath of overbuilding. □

Proof of Lemma 1. First consider the case $r_{t+1} > 0$. In this case, the monetary policy implements the efficient allocation with $l_t = L\left(k_t\right)$ and $y_t = S\left(k_t\right)$**.** In addition, the first order conditions for problems (9) and (4) further imply, $F_l\left(k_t, L\left(k_t\right)\right) = v'\left(L\left(k_t\right)\right) = w_t$. Combining this with Eq. (12) implies that the labor wedge is zero, $\tau_t = 0$. Combining Eqs. (12) and (9) then imply the rental rate of capital is given by $F_k\left(k_t, L\left(k_t\right)\right) = S'\left(k_t\right)$, completing the proof for the first part.

Next consider the case $r_{t+1} = 0$. In this case, Eq. (12) implies $F_l\left(k_t, l_t\right) \geq v'\left(l_t\right)$. This in turn implies that $l_t \in \left[0, L\left(k_t\right)\right]$. By feasibility, net output satisfies

$$y_t = c_t + i_t^h + i_t^h = F\left(k_t, l_t\right) - v\left(l_t\right).$$

This right hand side is strictly increasing in l_t over the range $\left[0, L\left(k_t\right)\right]$. The minimum and the maximum are respectively given by 0 and $S\left(k_t\right)$, which implies $y_t \in \left[0, S\left(k_t\right)\right]$. Moreover, given y_t that satisfies these resource constraints, there is a unique solution to (11), which we denote by $L^d\left(k_t, y_t\right)$. Combining this with Eq. (12), we further obtain the labor wedge as,

$$1-\tau_t = \frac{v'(l_t)}{F_l(k_t, l_t)} = \frac{v'\left(L^d(k_t, y_t)\right)}{F_l\left(k_t, L^d(k_t, y_t)\right)}.$$

Plugging this into Eq. (12) for capital, we obtain the rental rate of capital as,

$$R_t = \frac{v'\left(L^d(k_t, y_t)\right)}{F_l\left(k_t, L^d(k_t, y_t)\right)} F_k\left(k_t, L^d(k_t, y_t)\right) \equiv R(k_t, y_t),$$

where the last equality defines the function $R(\cdot)$. Note that $R(k_t, y_t) \leq S'(k_t)$ since the labor wedge is nonnegative. It can also be checked that $R_k < 0$ and $R_y > 0$, completing the proof. □

Proof of Proposition 1. As we have shown above, the equilibrium at date 1 starting with $h_1 = h^*$ and $k_1 \leq \bar{k}$ coincides with the efficient benchmark. Note also that, by standard arguments, the neoclassical system in $(B.2)$ can be described by an increasing consumption function, $c_1 = C(k_1)$.

To characterize the equilibrium at date 0, we define $K_1(r_0)$ for each $r_0 \geq 0$ as the solution to

$$S'(K_1(r_0)) - \delta^k = r_0.$$

Note that $K_1(r_0)$ is decreasing in the interest rate, with $K_1(0) = \bar{k}$ and $\lim_{r_0 \to \infty} K_1(r_0) = 0$. Similarly, define the function $C_0(r_0)$ as the solution to the Euler equation

$$u'(C_0(r_0)) = \beta(1+r_0) u'(C(K_1(r_0))).$$

Note that $C_0(r_0)$ is decreasing in the interest rate, with $C_0(0) = \bar{c}_0$ and $\lim_{r_0 \to \infty} C_0(r_0) = 0$. Finally, define the aggregate demand function

$$Y_0(r_0) = C_0(r_0) + K_1(r_0) - \left(1-\delta^k\right) k_0 + i_0^h.$$

Note that $Y_0(r_0)$ is also decreasing in the interest rate, with

$$Y_0(0) = \bar{y}_0 \text{ and } \lim_{r_0 \to \infty} Y_0(r_0) = i_0^h - \left(1-\delta^k\right) k_0.$$

Next consider the date 0 equilibrium for the case $b_0 \leq \bar{b}_0$. Note that this implies $S(k_0) \leq \bar{y}_0 = Y_0(0)$, and that we also have $\lim_{r_0 \to \infty} Y_0(r_0) < S(k_0)$ (since we assume housing investment is feasible). By the intermediate value theorem, there is a unique equilibrium interest rate $r_0 \in [0, \infty)$ such that $Y_0(r_0) = S(k_0)$. The equilibrium features $c_0 = C_0(r_0)$ and $K_1(r_0) = k_1$, along with $y_0 = S(k_0)$ and $l_0 = L(k_0)$.

Next consider the date 0 equilibrium for the case $b_0 > \bar{b}_0$. In this case, $Y_0(0) < S(k_0)$. Thus, the unique equilibrium features $r_0 = 0$ and $y_0 = \overline{y}_0 < S(k_0)$. Consumption and investment are given by $c_0 = \overline{c}_0$ and $k_1 = \overline{k}_1$. Labor supply l_0 is determined as the unique solution to (11) over the range $l_0 \in (0, L(k_0))$. Finally, Eq. $(A.8)$ implies the equilibrium output, $y_0 = \overline{y}_0$, is declining in the initial overbuilding b_0.

In either case, it can also be checked that the economy reaches date 1 with $h_1 = h^*$ and $k_1 \geq \min(k_0, k^*)$. Thus, the continuation equilibrium is characterized as described above, completing the proof. □

Proof of Proposition 6. Note that the recession is triggered if $\overline{y}_0 < S(k_0)$, where $\overline{y}_0$ is given by Eq. $(A.1)$. Since $1 - \delta^{h^d} > 1 - \delta^{h^n}$, increasing b_0^d (while keeping $b_0 = (b_0^d + b_0^n)/2$ constant) reduces $\overline{y}_0$, proving the result. □

Proposition 2 also requires the following assumption.

Assumption 3. (i) $i^h \in [-\overline{c}_T, S(\overline{k}) - \delta\overline{k} - \overline{c}_0)$ and (ii) $R(k_0, \tilde{y}_0) < \delta^k$, where $\tilde{y}_0 = \overline{c}_0 - \overline{c}_T + \overline{k} - (1 - \delta^k) k_0$

Part (i) ensures that i^h is not too low to induce zero aggregate demand, but also not too high so that a demand shortage is possible. Part (ii) ensures that the worst possible shock $i^h = -\overline{c}_T$ is sufficient to induce a demand shortage at date 0.

Proof of Proposition 2. We first claim that the solution to Eq. (21) can be written as $k_t = K(y_t)$, where $K(\cdot)$ is an increasing function over $(0, S(\overline{k}))$. To this end, consider some $y \in (0, S(\overline{k}))$. Let $\tilde{k} < \overline{k}$ denote the unique capital level such that $y = S(\tilde{k})$. Note that,

$$R(\tilde{k}, y) = S'(\tilde{k}) > \delta^k \text{ and } R(\overline{k}, y) < S'(\overline{k}) = \delta^k.$$

Here, the former inequality follows since $\tilde{k} < \overline{k}$, and the latter inequality follows from Lemma 1 since $y < S(\overline{k})$. Since $R_k < 0$, there exists a unique $K(y) \in (\tilde{k}, \overline{k})$ such that $R(K(Y), Y) - \delta^k = 0$. Thus, the function $K(\cdot)$ is well defined. Note also that $K(\cdot)$ is continuous and strictly increasing. Note also that $\lim_{y \to 0} K(y) = 0$ and $K(S(\overline{k})) = \overline{k}$.

Given the function $K(\cdot)$, the path of capital can be written as the solution to the system,

$$
\begin{aligned}
k_t &= K(y_t) \qquad \text{(B.3)}\\
\text{and } y_t &= Y_t(k_t) \equiv \overline{c}_t + k_{t+1} + i^h - \left(1-\delta^k\right) k_t.
\end{aligned}
$$

Here, the second equation defines the function $Y_t(k_t)$, which is strictly decreasing in k_t. Hence, the current level of output and capital are satisfied as the intersection of a strictly increasing and a strictly decreasing relation. We next claim that, given $\overline{c}_t$ and $k_{t+1} \in (0, \overline{k}]$, there is a unique solution to the system in $(B.3)$. To see this, first note that the boundary conditions at $y_t = 0$ and $k_t = 0$ respectively satisfy,

$$\lim_{y \to 0} K(y) = 0, \ Y_t(0) \geq 0,$$

where the inequality follows since Assumption 3(i) implies $\overline{c}_t + i^h \geq 0$ for each t. Next note the following boundary conditions at $y_t = S\left(\overline{k}\right)$ and $k_t = \overline{k}$,

$$K\left(S\left(\overline{k}\right)\right) = \overline{k} \text{ and } Y_t\left(\overline{k}\right) \leq \overline{c}_t + \delta\overline{k} + i^h < S\left(\overline{k}\right),$$

where the strict inequality follows by using Assumption 3(i) together with $\overline{c}_t < \overline{c}_0$. It follows that there is a unique solution to the system $(B.3)$ which also satisfies $k_t \in \left(0, \overline{k}\right)$ and $y_t \in \left(0, S\left(\overline{k}\right)\right)$. It can also be seen that k_t and y_t are both strictly increasing in i^h.

Since the solution satisfies $k_t < \overline{k}$, we can reiterate the same analysis to solve for $k_{t-1} \in \left(0, \overline{k}\right)$ and $y_{t-1} \in \left(0, S\left(\overline{k}\right)\right)$. By induction, we obtain a unique equilibrium path $\{k_1, y_1\}_{t=0}^{T-1}$. Note also that k_0 is given, and output at date 0 is determined by

$$y_0 = Y_0(k_0) = \overline{c}_0 + k_1 + i^h - \left(1-\delta^k\right) k_0.$$

Note that the path $\{k_1, y_1\}_{t=0}^{T-1}$ as well as the initial output y_0 are strictly increasing in i^h.

We next claim there is a demand shortage at date 0, $y_0 < S(k_0)$, as long as the housing investment is below a threshold. Note that $y_0 < S(k_0)$ if and only if $R(k_0, y_0) < \delta^k$. First consider the claim for the worst possible shock, $i^h = -\overline{c}_T$. In this case, the output at date 0 satisfies,

$$y_0 \leq \overline{c}_0 - \overline{c}_T + \overline{k} - \left(1-\delta^k\right) k_0.$$

Combining this with Assumption 3(ii), we obtain $R(k_0, y_0) < \delta^k$, proving the claim. Since y_0 is strictly increasing in i^h, there exists $i_1^h > -\bar{c}_T$ such that $y_0 = S(k_0)$. It follows that $y_0 < S(k_0)$ whenever $i^h < i^{h,1}$, proving the first part of the proposition.

Similarly, we claim that the worst allowed shock $i^h = -\bar{c}_T$ induces $k_1 < k_0$. To see this, consider the output at date 1 given by,

$$y_1 = \bar{c}_1 - \bar{c}_T + k_2 - \left(1 - \delta^k\right) k_1 \leq \bar{c}_0 - \bar{c}_T + \bar{k} - \left(1 - \delta^k\right) k_1.$$

Combining this with Assumption 3(ii), we obtain $R(k_0, y_1) < \delta^k$. This in turn implies $k_1 = K(y_1) < k_0$, proving the claim. Since k_1 is strictly increasing in i^h, there exists $i^{h,2} \leq i^{h,1}$ and $i^{h,2} > -\bar{c}_T$ such that $k_1 = k_0$. It follows that $k_1 < k_0$ whenever $i^h < i^{h,2}$, completing the proof. □

B.2 Proofs for the extension with income-trackers

Lemma 4. *The income-trackers' wage level is given by Eq.* (*A.6*) *for some function* $\psi(k_t, y_t)$, *which has the following properties:*

(i) $\psi(k_t, y_t) = 1 - \tau_t = \frac{v'(l_t)}{F_l(k_t, l_t)}$,

(ii) $\psi(k_t, y_t) = 1$ *if* $r_{t+1} > 0$, *and* $\psi(k_t, y_t) \in [0, 1]$ *if* $r_{t+1} = 0$,

(iii) $\psi(k_t, y_t)$ *is strictly decreasing in* k_t, *and strictly increasing in* y_t.

Proof. As in the proof of Lemma 1, let $L^d(k, y)$ denote the labor supply corresponding to capital level $k \in (0, \bar{k}]$ and net demand $y \in (0, S(k)]$. Next consider the analogue of Problem (8) that also includes firms' demand for hand-to-mouth labor. The firm's optimization in this case implies

$$w^{tr}(k_t, y_t) = (1 - \tau_t) \eta^{tr},$$

where $\tau_t \geq 0$ is the Lagrange multiplier on the demand constraint. As before, the same problem also implies that τ_t is equal to the labor wedge, that is:

$$1 - \tau_t = \frac{v'\left(L^d(k_t, y_t)\right)}{F_l\left(k_t, L^d(k_t, y_t)\right)} \equiv \psi(k_t, y_t).$$

Here, the last line defines the function $\psi(k_t, y_t)$. Combining these expressions proves the first part. Recall that the labor wedge satisfies $\tau_t = 0$ if $r_{t+1} = 0$, and $\tau_t \in [0, 1]$ if $r_{t+1} > 0$, proving the second part. It can also be checked that $\psi_k < 0$ and $\psi_y > 0$, completing the proof. □

Proposition **7** requires a strengthening of Assumption 3(i). Assumption 3(ii) remains unchanged.
Assumption 3$^{\text{tr}}$.(i) $i^h \in \left[-\left(\bar{c}_T - \eta^{tr} l^{tr}\right), S\left(\bar{k}\right) - \delta \bar{k} - \bar{c}_0\right)$.

Proof of Proposition 7. Let $K(y)$ denote the function defined in the proof of Proposition 2 that describes the break-even capital level $k_t = K(y_t)$ given output y_t. Eqs. (21) and $(A.7)$can then be written as, $y_t = f(y_t)$ for each $t \geq 1$, where

$$f(y_t) \equiv \bar{c}_t + k_{t+1} - \left(1 - \delta^k\right) K(y_t) + i^h + \left(\psi\left(K(y_t), y_t\right) - 1\right) \eta^{tr} l^{tr}. \tag{B.4}$$

The output at date 0 is separately characterized as the solution to Eq. $(A.7)$ with the initial k_0 (as opposed to $K(y_0)$).

We next claim that, given $k_{t+1} \in (0, \bar{k}]$, there exists a solution to $(B.4)$ over the range $y_t \in \left(0, S\left(\bar{k}\right)\right)$. To see this, note that

$$\lim_{y_t \to 0} f(y_t) > \bar{c}_T + i^h - \eta^{tr} l^{tr} \geq 0,$$

where the first inequality uses $\bar{c}_t \geq \bar{c}_T, k_{t+1} > 0$ and $\psi \geq 0$, and the second inequality uses Assumption 3$^{\text{tr}}$(i). Next note that

$$f\left(S\left(\bar{k}\right)\right) \leq \bar{c}_0 + \bar{k} - \left(1 - \delta^k\right) k_0 + i^h < S\left(\bar{k}\right),$$

where the first inequality uses $\bar{c}_t \leq \bar{c}_0, k_{t+1} \leq \bar{k}$ and $\psi \leq 1$, and the second inequality uses Assumption 3$^{\text{tr}}$(ii). Combining the last two inequalities implies the existence of a solution $y_t \in \left(0, S\left(\bar{k}\right)\right)$ along with $k_t = K(y_t) \in \left(0, \bar{k}\right)$. Applying the same argument recursively, we obtain the path $\{k_t, y_t\}_{t=1}^{T-1}$. By the same argument, there exists $y_0 \in \left(0, S\left(\bar{k}\right)\right)$ that solves Eq. $(A.7)$ with the initial k_0.

Note that there could be multiple solutions to Eq. $(B.4)$ [and Eq. $(A.7)$ for date 0], which could generate multiple equilibria. We establish the desired results for the "best" equilibrium that has the highest capital and net output, which also implies the results for any other equilibrium. To this end, let y_t^b denote the supremum over all y_t's that solve Eq. $(B.4)$ [and Eq. $(A.7)$ for date 0] given k_{t+1}^b. Then let $k_t^b = K\left(y_t^b\right)$. By induction, we obtain a particular solution to Eq. $(B.4)$ [and Eq. $(A.7)$ for date 0]. It is easy to show that this is the "best" solution in the sense that any other solution satisfies $k_t \leq k_t^b$ and $y_t \leq y_t^b$ for each t.

We next claim that there is a demand shortage at date 0, $y_0^b < S(k_0)$, or equivalently $R\left(k_0, y_0^b\right) < \delta^k$, as long as i^h is below a threshold. First consider the claim for the worst possible shock $i^h = -\left(\bar{c}_T - \eta^{tr} l^{tr}\right)$. In this case, output at time 0 satisfies,

$$
\begin{aligned}
y_0^b &= \bar{c}_0 - \bar{c}_T + k_1^b - \left(1-\delta^k\right) k_0 + (\psi - 1)\,\eta^{tr} l^{tr} \\
&\leq \bar{c}_0 - \bar{c}_T + \bar{k} - \left(1-\delta^k\right) k_0.
\end{aligned}
$$

Combining this with Assumption 3(ii), we obtain $R\left(k_0, y_0^b\right) < \delta^k$, proving the claim for the worst possible shock. As in the proof of Proposition 2, this further implies that there exists $i^{h,1} > -\left(\bar{c}_T - \eta^{tr} l^{tr}\right)$ such that there is a demand shortage at date 0 if and only if $i^h < i^{h,1}$. This completes the proof of the first part of the proposition.

To prove the second part, first note that $y_t^b < S\left(k_t^b\right)$ also implies $\psi_t\left(k_t^b, y_t^b\right) < 1$ for each $t \in \{0, .., T-1\}$. Eqs. $(B.4)$ and $(A.7)$ then imply that y_t^b is strictly decreasing in l^{tr} for each $t \in \{0, .., T-1\}$. Next note that the required inequality can be rewritten as,

$$
\bar{c}_0 - c^* < \left(1 - \psi\left(k_0, y_0^b\right)\right) \eta^{tr} l^{tr}. \tag{B.5}
$$

Since y_0^b is strictly decreasing in l^{tr}, so is the expression $\psi\left(k_0^b, y_0^b\right)$. Thus, there exists l_1^{tr} such that $(B.5)$ holds for the best equilibrium path, $\left\{k_t^b, y_{t-1}^b\right\}_{t=0}^{T-1}$ if and only if $l^{tr} > l_1^{tr}$. Note also that any other equilibrium features $y_0 \leq y_0^b$, and thus $\psi\left(k_0, y_0\right) \leq \psi\left(k_0, y_0^b\right)$. It follows that, as long as $l^{tr} > l_1^{tr}$, the inequality in $(B.5)$ holds for any equilibrium, completing the proof. □

B.3 Proofs for the policy analysis in Section 5 and Appendix A.4

Proof of Lemma 2. Most of the proof is described in Appendix A.4. If $b_0 < \bar{b}_0^{sep}$, then the maximum aggregate demand is above the efficient level, $\bar{y}_0 > S_0\left(k_0\right)$. In this case, the zero lower bound constraint does not bind and outcomes are efficient. If instead $b_0 > \bar{b}_0^{sep}$, then output is below the efficient level and it is determined by aggregate demand, $y_0 = \bar{y}_0 < S_0\left(k_0\right)$. The employment is also below the efficient level, $l_0 < L_0\left(k_0\right)$, and it is characterized by solving, $y_0 = \bar{y}_0 = F\left(k_0, l_0\right)$. The labor wedge is characterized by solving, $1 - \tau_0 = \frac{v_0'(l_0)}{F_l(k_0, l_0) u'(\bar{c}_0)}$, and it satisfies $\tau_0 > 0$.

For future reference, note also that the employment level can also be written as a function, $L_0^d\left(k_0, y_0\right)$, as in the proof of Lemma 1. This also implies the labor wedge can be written as a similar function,

$$\tau(k_0, y_0) = 1 - \frac{v_0'\left(L_0^d(k_0, y_0)\right)}{F_l\left(k_0, L_0^d(k_0, y_0)\right) u'(\overline{c}_0)}.$$

It can be checked that function, $\tau(\cdot)$, is increasing in k_0 and decreasing in y_0. □

Proof of Proposition 3. We first show that the planner's marginal utility, $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*}$, is increasing in the labor wedge, τ_0. Note that the Euler equation in problem $(A.11)$ implies,

$$\frac{dc_0}{dh_1}|_{h_1=h^*} = \frac{\beta u''(C(h^*))}{u''(\overline{c}_0)} C'(h^*) > 0.$$

Here, the inequality follows because the solution to the neoclassical problem $(B.1)$ implies $C'(h^*) > 0$. Note also that the derivative $\frac{dc_0}{dh_1}|_{h_1=h^*}$ is independent of b_0 or τ_0. Combining this with Eq. (25) proves that $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*}$ is increasing τ_0.

Next note from the proof of **2** that the labor wedge, τ_0, is strictly decreasing in aggregate demand, $y_0 = \overline{y}_0$. Since the maximum demand, $\overline{y}_0$, in Eq. $(A.8)$ is strictly decreasing in overbuilding, b_0, this implies that the labor wedge is strictly increasing in overbuilding, b_0. This in turn implies that the planner's marginal utility, $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*}$, is strictly increasing in b_0. It can also be checked that $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*} > 0$ for sufficiently high levels of b_0. Let $\tilde{b}_0 > \overline{b}_0^{sep}$ denote the level of overbuilding such that $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*} = 0$. It follows that, $\frac{d_+ W_{0,pl}(h_1)}{dh_1}|_{h_1=h^*} > 0$ if and only if $b_0 > \tilde{b}_0$. This also implies $h_{1,pl} > h^*$ if and only if $b_0 > \tilde{b}_0$. □

Proof of Lemma 3. First consider the limiting case with $\pi^H = 1$. In this case, given the initial conditions, the economy is at an efficient steady-state with,

$$h_t = h^*\left(1 + \lambda^H\right), k_t = k^* \text{ and } c^* = S(k^*) - \delta^h\left(1 + \lambda^H\right) h^* - \delta^k k^*.$$

In particular, the competitive equilibrium features $k_0 = k^*$. In this equilibrium, the economy does not feature a demand shortage at date 0 or state H of date 1. In fact, we have $r_1 = r_2^H = 1/\beta > 0$. However, since $\lambda^H > \overline{b}_0^{sep}(k^*)$, the economy features a demand shortage in the (zero probability) state L.

Next note that the capital choice in competitive equilibrium is a continuous function of the probability of the high state, $k_0\left(\pi^H\right)$. By Eq. $(A.10)$, $\overline{b}_0^{sep}(k_0)$ is also a continuous function of k_0. It follows that there exists $\overline{\pi}^1$ (which could also be $\overline{\pi}^1 = 0$) such that $\lambda^H > \overline{b}_0^{sep}(k^*)$ if and only if $\pi^H > \overline{\pi}^1$. Similarly, note that the interest rates r_1 and r_2^H are also continuous functions of π^H. Using continuity once again, there exists $\overline{\pi}^2 < 1$ (which could also be $\overline{\pi}^2 = 0$) such that the economy does not feature a demand shortage at date 0 or at state H if and only if $\pi^H > \overline{\pi}^2$. Taking $\overline{\pi} = \max\left(\overline{\pi}^1, \overline{\pi}^2\right)$ proves the statement. □

Proof of Proposition 4. The planner's optimality condition (27) implies $k_{0,pl} < k_0$ since $\tau_0 > 0, \pi^H > 0$, and $1 - \delta^k > 0$. □

Proof of Proposition 5. In this case, the difference is that the planner can also control the ex-ante employment and net output, l_{-1}, y_{-1}, by deviating from the monetary policy in (10). Thus, the analogue of the planner's problem in $(A.13)$ is given by,

$$\max_{l_{-1},y_{-1},c_{-1},k_0} u\left(c_{-1}\right) + \beta\left(\pi^H V_0^H\left(k_0, h_0\right) + \left(1 - \pi^H\right) V_0^L\left(k_0, h_0\right)\right),$$

$$\text{s.t.} \quad c_{-1} + k_0 + h_{0,pl} = S\left(k_{-1}\right) + \left(1 - \delta^k\right) k_{-1} + \left(1 - \delta^h\right) h_{-1},$$

$$\text{and } y_{-1} = F\left(k_{-1}, l_{-1}\right) - v\left(l_{-1}\right) \leq S\left(k_{-1}\right).$$

It is easy to check that the first order conditions maximize the net output, $y_{-1} = S\left(k_{-1}\right)$ and $l_{-1} = L\left(k_{-1}\right)$. This in turn leads to the same problem $(A.13)$ as before, as well as the same first order conditions (27). In particular, the planner sets the interest rate, $r_0 = r_0^*$, which (by definition) replicates the statically efficient allocations at date -1, completing the proof. □